Computer Algebra Independent Integration Tests

Summer 2023 edition

5-Inverse-trig-functions/5.6-Inverse-cosecant/158-5.6.1-u-a+b-arccsc-c-x-^n

Nasser M. Abbasi

September 5, 2023 Compiled on September 5, 2023 at 11:55am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [178]. This is test number [158].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (178)	0.00 (0)
Mathematica	100.00 (178)	0.00 (0)
Maple	82.58 (147)	17.42 (31)
Fricas	69.10 (123)	30.90 (55)
Giac	52.81 (94)	47.19 (84)
Sympy	37.64 (67)	62.36 (111)
Maxima	35.39 (63)	64.61 (115)
Mupad	32.02 (57)	67.98 (121)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

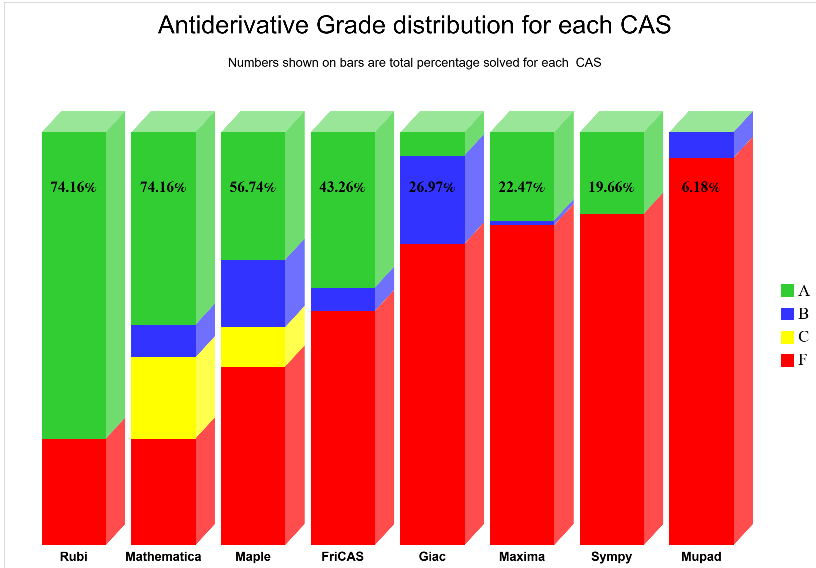
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

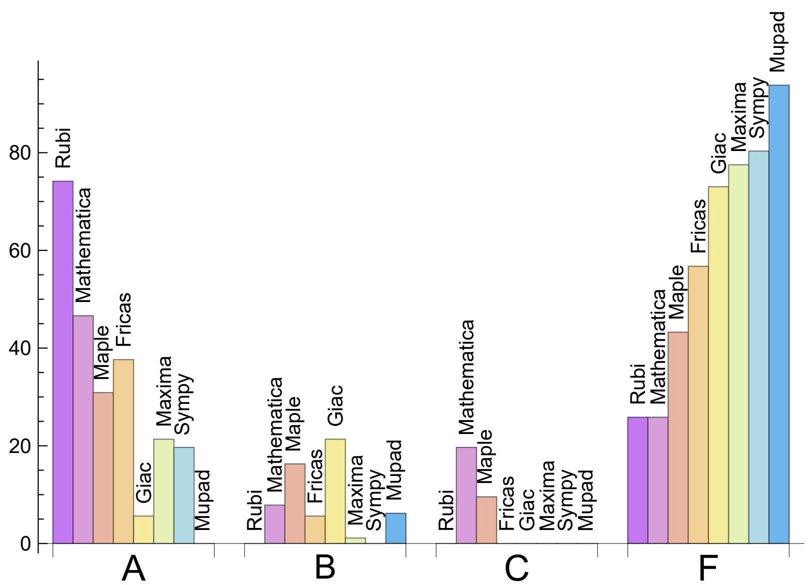
System	% A grade	% B grade	% C grade	% F grade
Rubi	74.157	0.000	0.000	25.843
Mathematica	46.629	7.865	19.663	25.843
Fricas	37.640	5.618	0.000	56.742
Maple	30.899	16.292	9.551	43.258
Maxima	21.348	1.124	0.000	77.528
Sympy	19.663	0.000	0.000	80.337
Giac	5.618	21.348	0.000	73.034
Mupad	0.000	6.180	0.000	93.820

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	31	100.00	0.00	0.00
Fricas	55	92.73	7.27	0.00
Giac	84	65.48	4.76	29.76
Maxima	115	40.87	0.00	59.13
Sympy	111	69.37	30.63	0.00
Mupad	121	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.34
Rubi	0.36
Maxima	0.69
Mupad	1.12
Giac	2.14
Maple	5.54
Mathematica	5.64
Sympy	22.69

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	35.07	1.21	27.00	1.17
Sympy	129.01	1.36	36.00	1.16
Maxima	156.00	4.29	98.00	1.41
Rubi	212.46	1.00	134.50	1.00
Fricas	249.81	1.87	85.00	1.24
Mathematica	306.05	1.22	128.50	1.09
Giac	321.71	2.82	48.00	1.25
Maple	342.34	1.54	170.00	1.19

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

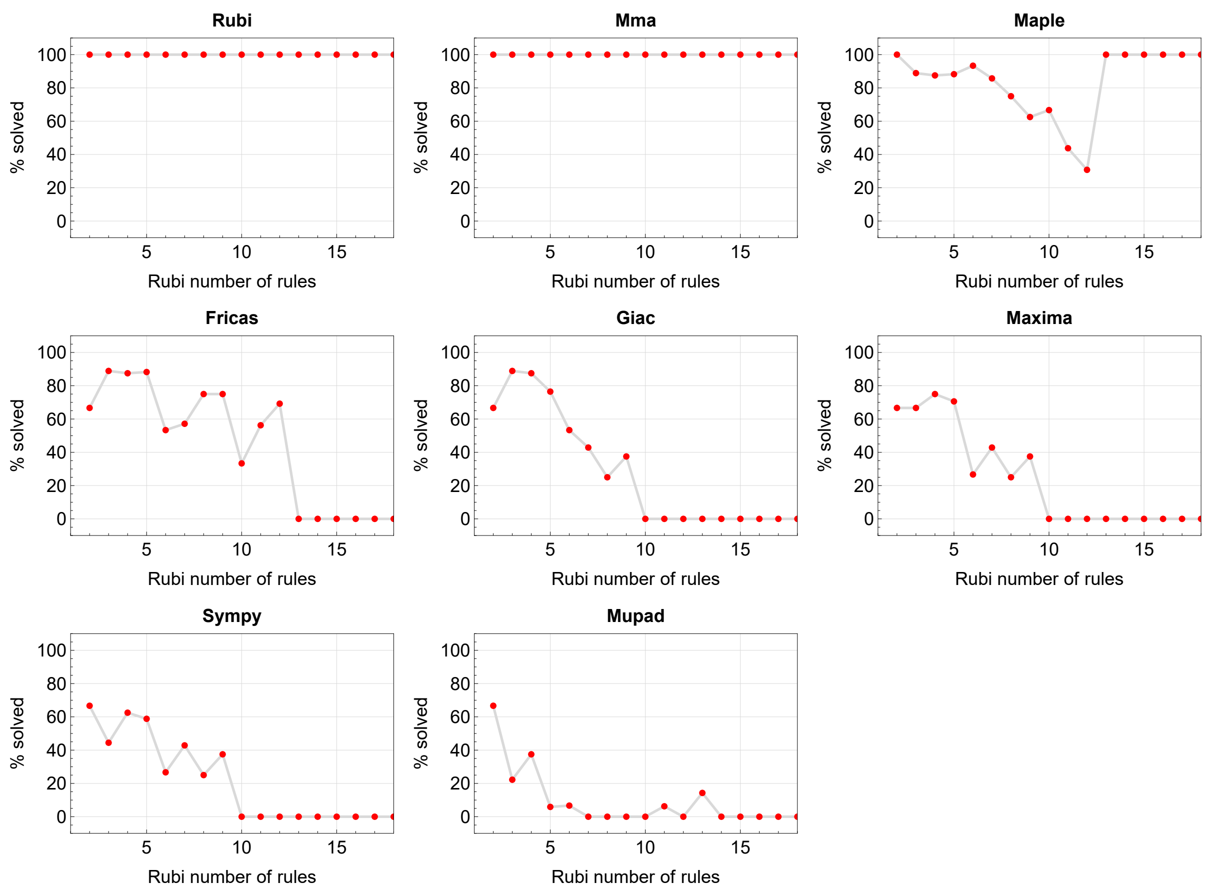


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

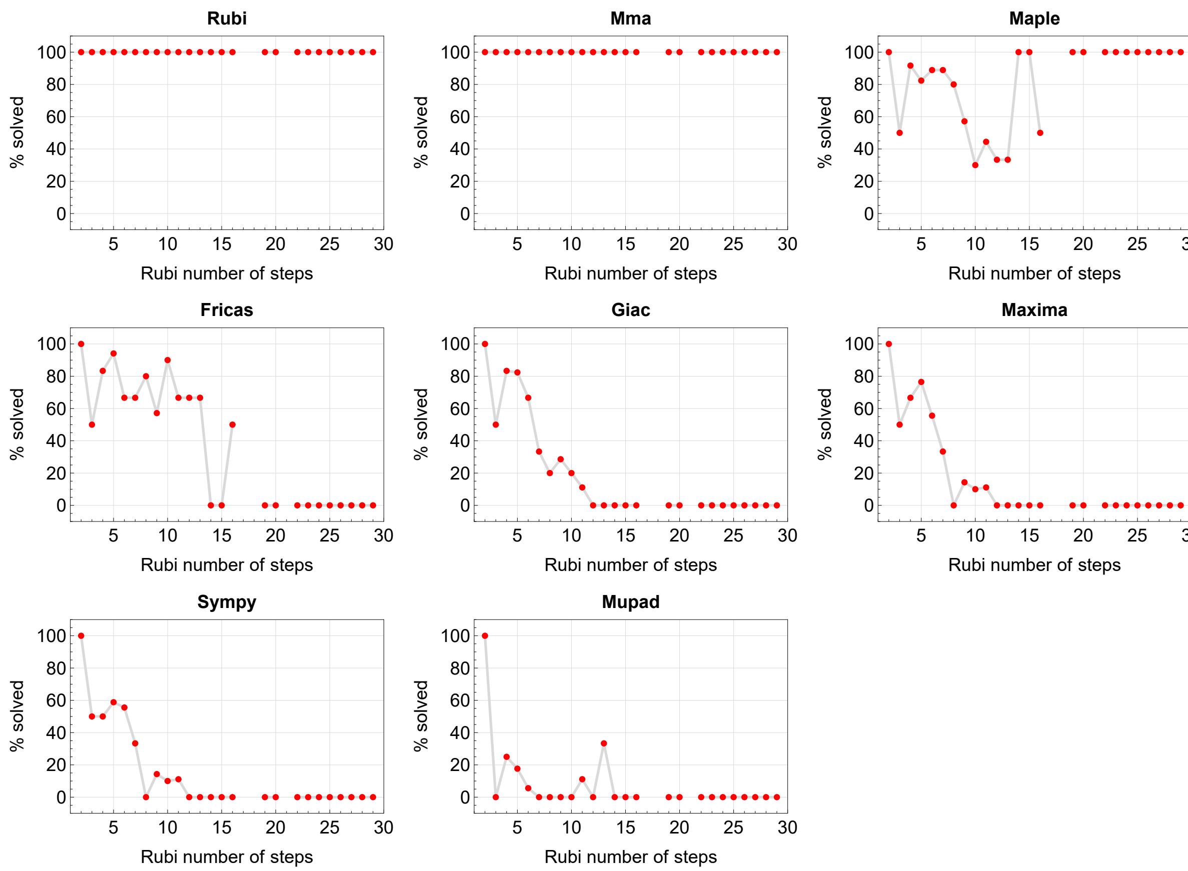


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

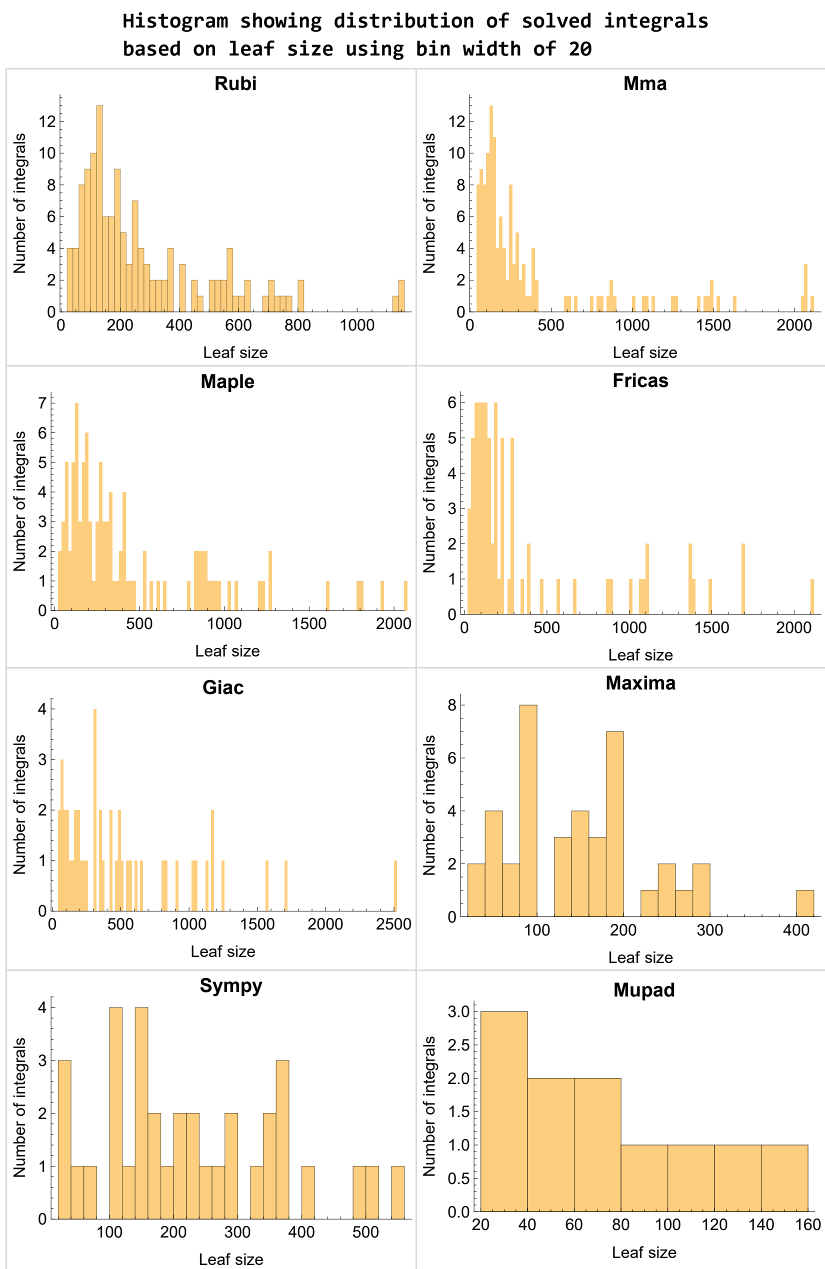


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

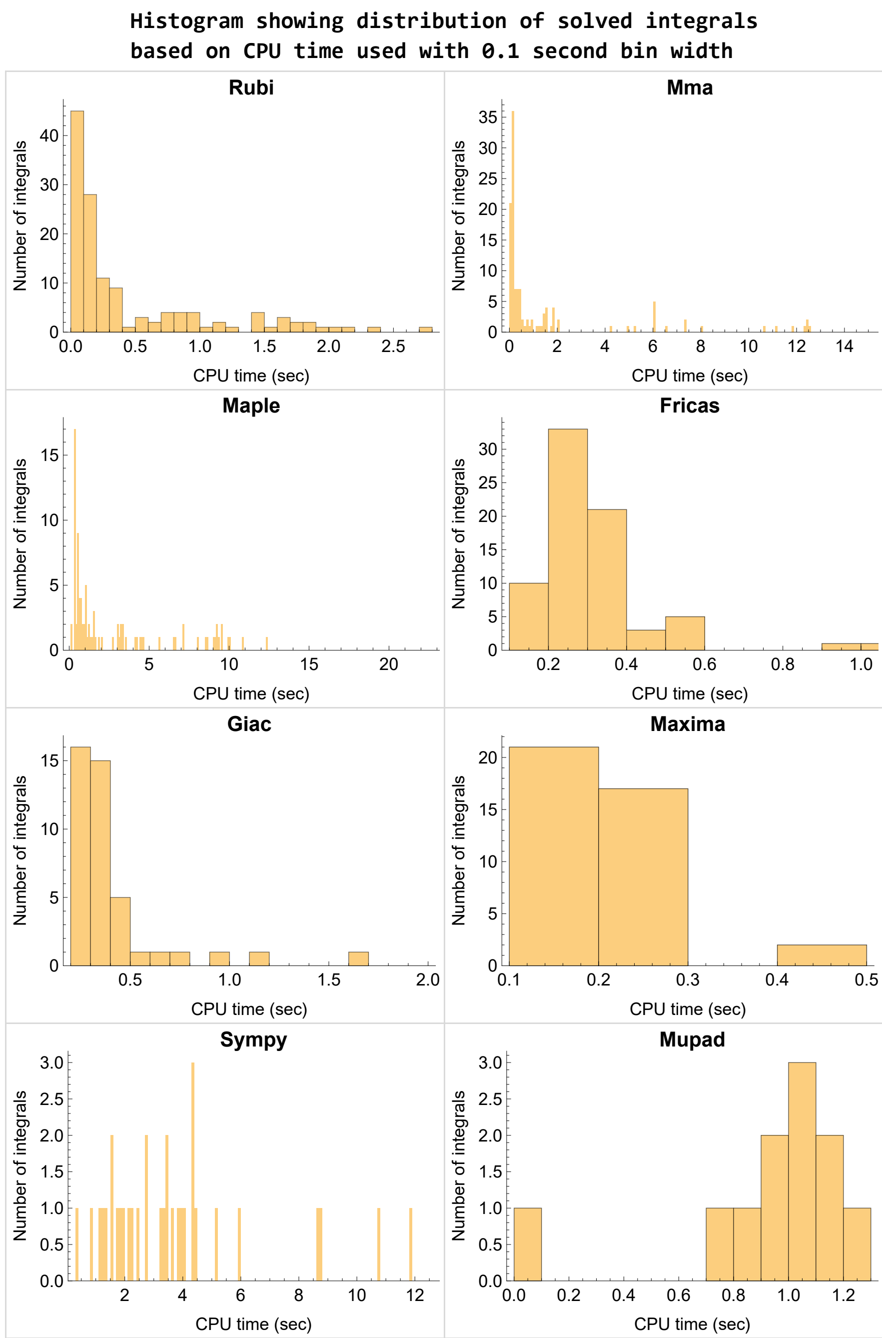


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

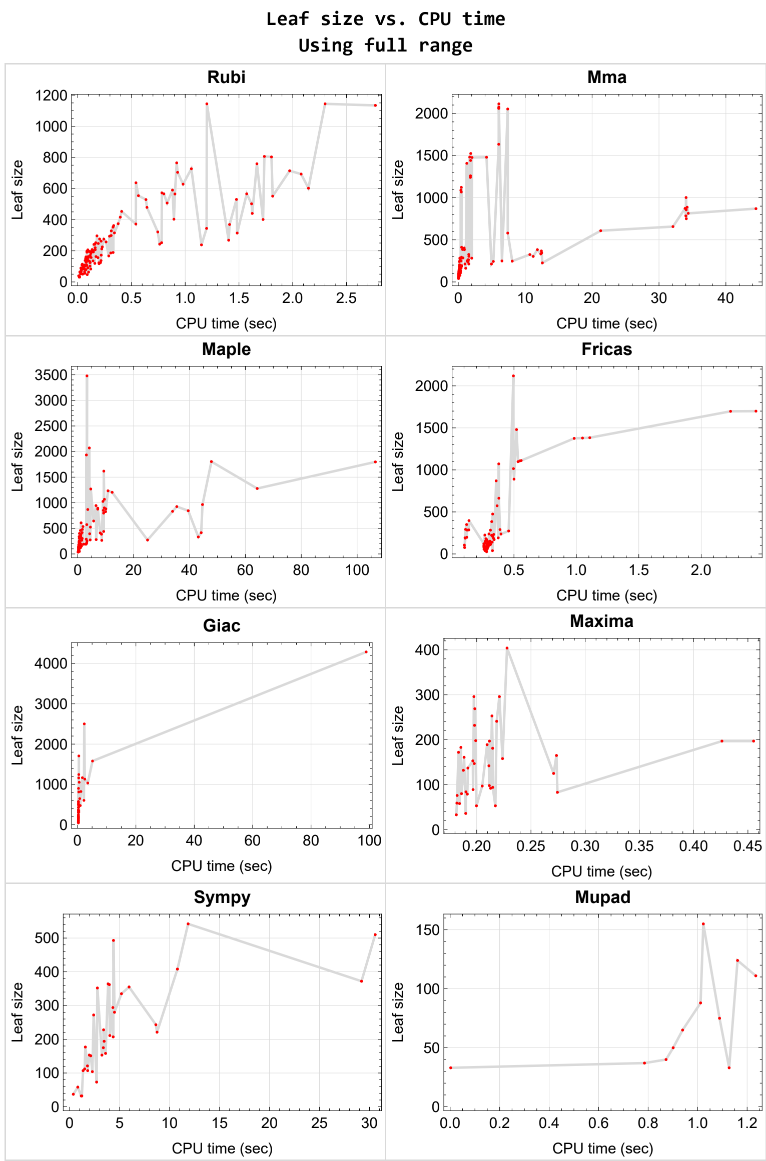


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {25, 51, 53, 57, 60, 72, 75, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117}

Maple {98, 99, 101, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `sleliever` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
x, aa = expr.operator(), expr.operands()
if x is None:
    return 1
else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

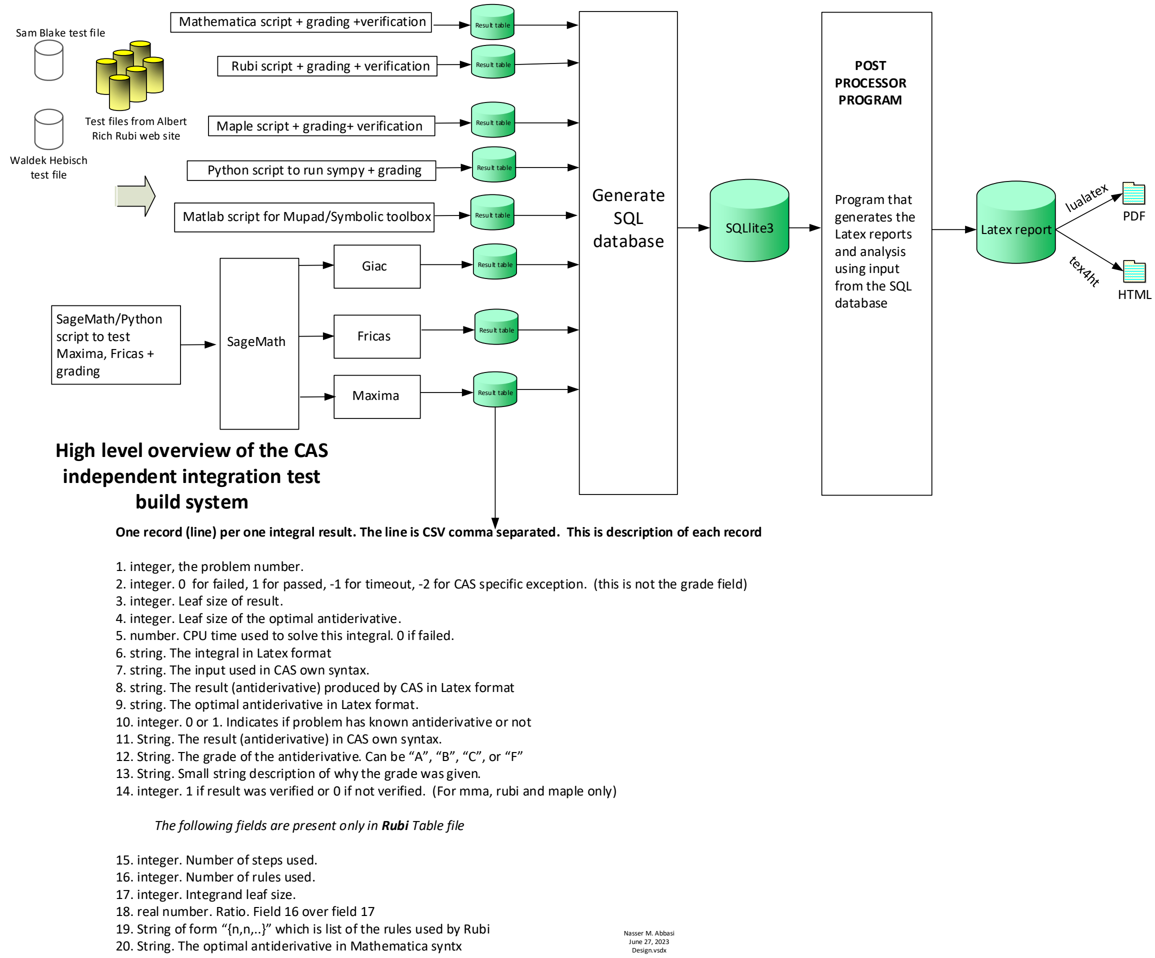
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 59, 60, 66, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 108, 109, 110, 115, 116, 117, 145, 154, 163, 165, 166, 167, 174, 175, 176 }

B grade { 25, 53, 72, 98, 99, 100, 101, 102, 103, 104, 106, 107, 111, 114 }

C grade { 51, 52, 56, 57, 58, 63, 64, 65, 69, 70, 71, 105, 112, 113, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 164 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 18, 22, 24, 25, 27, 36, 37, 38, 46, 47, 49, 53, 57, 58, 59, 60, 63, 64, 65, 66, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 96, 97 }

B grade { 10, 12, 17, 19, 20, 21, 23, 26, 28, 29, 30, 31, 32, 44, 45, 48, 50, 51, 52, 56, 70, 71, 72, 75, 88, 95, 105, 112, 113 }

C grade { 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117 }

F normal fail { 41, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 29, 30, 31, 32, 44, 45, 46, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 105, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 163, 164, 174, 175, 176 }

B grade { 7, 17, 47, 49, 50, 112, 113, 156, 157, 158 }

C grade { }

F normal fail { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 41, 48, 51, 52, 53, 57, 59, 60, 63, 65, 66, 69, 70, 71, 72, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 165, 166, 167 }

F(-1) timedout fail { 56, 58, 64, 75 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 17, 20, 29, 44, 45, 46, 47, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95 }

B grade { 15, 22 }

C grade { }

F normal fail { 8, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 41, 48, 49, 50, 86, 87, 96, 97, 99, 101, 103, 104, 105, 106, 111, 112, 113, 114, 120, 129, 140, 149, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-1) timedout fail { }

F(-2) exception fail { 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 98, 100, 102, 107, 108, 109, 110, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162 }

Giac

A grade { 9, 10, 11, 12, 36, 37, 38, 80, 81, 92 }

B grade { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 17, 20, 21, 22, 23, 29, 30, 31, 32, 44, 45, 46, 47, 76, 77, 78, 79, 82, 83, 84, 85, 88, 89, 90, 91, 93, 94, 95 }

C grade { }

F normal fail { 16, 18, 24, 25, 26, 27, 28, 41, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 87, 97, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 176 }

F(-1) timedout fail { 103, 104, 107, 111 }

F(-2) exception fail { 8, 19, 48, 49, 50, 86, 96, 98, 99, 100, 101, 102, 105, 106, 108, 109, 110, 112, 113, 114, 115, 116, 117, 174, 175 }

Mupad

A grade { }

B grade { 6, 7, 8, 9, 10, 20, 29, 47, 79, 86, 87 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 44, 45, 46, 47, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95 }

B grade { }

C grade { }

F normal fail { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 71, 72, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 105, 107, 108, 109, 110, 118, 119, 120, 126, 127, 129, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 166, 167, 176 }

F(-1) timedout fail { 54, 61, 67, 69, 70, 73, 74, 75, 104, 106, 111, 112, 113, 114, 115, 116, 117, 128, 132, 133, 136, 137, 151, 159, 160, 161, 162, 163, 164, 165, 169, 170, 174, 175 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	107	177	161	115	221	646	0
N.S.	1	1.00	0.94	1.55	1.41	1.01	1.94	5.67	0.00
time (sec)	N/A	0.043	0.102	0.368	0.188	0.313	8.747	0.747	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	72	79	80	62	153	518	0
N.S.	1	1.00	0.81	0.89	0.90	0.70	1.72	5.82	0.00
time (sec)	N/A	0.029	0.092	0.362	0.186	0.287	1.981	0.315	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	97	141	132	106	175	480	0
N.S.	1	1.00	1.09	1.58	1.48	1.19	1.97	5.39	0.00
time (sec)	N/A	0.035	0.056	0.342	0.188	0.288	3.376	0.626	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	70	59	52	107	352	0
N.S.	1	1.00	0.97	1.09	0.92	0.81	1.67	5.50	0.00
time (sec)	N/A	0.019	0.078	0.345	0.182	0.273	1.352	0.304	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	85	94	97	94	107	310	0
N.S.	1	1.00	1.33	1.47	1.52	1.47	1.67	4.84	0.00
time (sec)	N/A	0.027	0.045	0.329	0.205	0.291	1.809	0.452	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	50	61	36	39	58	182	40
N.S.	1	1.00	1.28	1.56	0.92	1.00	1.49	4.67	1.03
time (sec)	N/A	0.009	0.028	0.341	0.190	0.328	0.820	0.288	0.872

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	37	53	64	32	62	33
N.S.	1	1.00	1.87	1.19	1.71	2.06	1.03	2.00	1.06
time (sec)	N/A	0.015	0.038	0.131	0.200	0.278	1.224	0.270	1.127

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	53	136	0	0	0	0	65
N.S.	1	1.00	0.83	2.12	0.00	0.00	0.00	0.00	1.02
time (sec)	N/A	0.064	0.043	0.890	0.000	0.000	0.000	0.000	0.939

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	41	59	33	26	37	42	37
N.S.	1	1.00	1.28	1.84	1.03	0.81	1.16	1.31	1.16
time (sec)	N/A	0.018	0.029	0.352	0.181	0.281	0.375	0.297	0.785

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	66	96	83	40	121	66	50
N.S.	1	1.00	1.29	1.88	1.63	0.78	2.37	1.29	0.98
time (sec)	N/A	0.026	0.035	0.372	0.274	0.278	1.787	0.286	0.901

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	59	71	58	39	112	87	0
N.S.	1	1.00	0.98	1.18	0.97	0.65	1.87	1.45	0.00
time (sec)	N/A	0.028	0.056	0.352	0.184	0.268	1.510	0.276	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	78	138	125	53	194	117	0
N.S.	1	1.00	1.03	1.82	1.64	0.70	2.55	1.54	0.00
time (sec)	N/A	0.034	0.050	0.359	0.271	0.270	3.458	0.277	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	79	76	50	158	149	0
N.S.	1	1.00	0.84	0.96	0.93	0.61	1.93	1.82	0.00
time (sec)	N/A	0.044	0.066	0.329	0.182	0.279	3.601	0.303	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	88	174	165	63	243	174	0
N.S.	1	1.00	0.87	1.72	1.63	0.62	2.41	1.72	0.00
time (sec)	N/A	0.050	0.079	0.361	0.274	0.284	8.632	0.283	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	124	170	197	146	0	811	0
N.S.	1	1.00	1.16	1.59	1.84	1.36	0.00	7.58	0.00
time (sec)	N/A	0.084	0.137	1.028	0.426	0.296	0.000	0.429	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	213	268	0	0	0	0	0
N.S.	1	1.00	1.53	1.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.997	1.439	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	89	122	84	111	0	427	0
N.S.	1	1.00	1.62	2.22	1.53	2.02	0.00	7.76	0.00
time (sec)	N/A	0.065	0.166	1.007	0.190	0.287	0.000	0.350	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	147	176	0	0	0	0	0
N.S.	1	1.00	1.75	2.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	0.197	0.752	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	137	338	0	0	0	0	0
N.S.	1	1.00	1.51	3.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.107	1.091	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	71	115	79	57	0	104	88
N.S.	1	1.00	1.42	2.30	1.58	1.14	0.00	2.08	1.76
time (sec)	N/A	0.046	0.102	0.761	0.191	0.260	0.000	0.296	1.012

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	102	184	0	82	0	163	0
N.S.	1	1.00	1.16	2.09	0.00	0.93	0.00	1.85	0.00
time (sec)	N/A	0.058	0.078	0.589	0.000	0.259	0.000	0.301	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	108	153	197	93	0	224	0
N.S.	1	1.00	1.06	1.50	1.93	0.91	0.00	2.20	0.00
time (sec)	N/A	0.071	0.156	1.345	0.455	0.265	0.000	0.287	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	148	265	0	120	0	304	0
N.S.	1	1.00	1.10	1.98	0.00	0.90	0.00	2.27	0.00
time (sec)	N/A	0.082	0.119	1.270	0.000	0.259	0.000	0.301	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	285	417	0	0	0	0	0
N.S.	1	1.00	1.38	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.163	0.704	1.561	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	580	535	0	0	0	0	0
N.S.	1	1.00	2.64	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	7.394	1.897	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	182	303	0	0	0	0	0
N.S.	1	1.00	1.44	2.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.469	1.589	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	265	378	0	0	0	0	0
N.S.	1	1.00	1.84	2.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.281	1.155	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	242	608	0	0	0	0	0
N.S.	1	1.00	1.95	4.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.115	0.154	1.204	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	135	197	147	98	0	195	155
N.S.	1	1.00	1.69	2.46	1.84	1.22	0.00	2.44	1.94
time (sec)	N/A	0.068	0.135	0.890	0.198	0.259	0.000	0.318	1.023

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	186	318	0	150	0	302	0
N.S.	1	1.00	1.49	2.54	0.00	1.20	0.00	2.42	0.00
time (sec)	N/A	0.080	0.156	1.072	0.000	0.270	0.000	0.316	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	204	299	0	173	0	428	0
N.S.	1	1.00	1.20	1.76	0.00	1.02	0.00	2.52	0.00
time (sec)	N/A	0.116	0.196	1.513	0.000	0.280	0.000	0.314	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	283	479	0	225	0	576	0
N.S.	1	1.00	1.36	2.30	0.00	1.08	0.00	2.77	0.00
time (sec)	N/A	0.134	0.229	1.655	0.000	0.267	0.000	0.315	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	18
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.50
time (sec)	N/A	0.012	2.624	0.842	0.268	0.255	0.358	33.774	0.787

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	16
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.60
time (sec)	N/A	0.005	2.557	0.590	0.294	0.245	0.400	11.266	0.790

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	12	16	20
N.S.	1	1.00	1.14	1.00	1.14	1.07	0.86	1.14	1.43
time (sec)	N/A	0.018	0.236	0.505	0.290	0.275	0.873	1.776	0.841

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	48	0	0	0	54	0
N.S.	1	1.00	0.91	1.02	0.00	0.00	0.00	1.15	0.00
time (sec)	N/A	0.088	0.061	0.593	0.000	0.000	0.000	0.282	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	58	0	0	0	95	0
N.S.	1	1.00	0.89	0.92	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.101	0.059	0.503	0.000	0.000	0.000	0.289	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	91	102	0	0	0	200	0
N.S.	1	1.00	0.78	0.87	0.00	0.00	0.00	1.71	0.00
time (sec)	N/A	0.199	0.139	0.508	0.000	0.000	0.000	0.287	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	1279	44	15	18	22
N.S.	1	1.00	1.12	1.00	79.94	2.75	0.94	1.12	1.38
time (sec)	N/A	0.017	4.271	0.862	15.475	0.286	21.443	0.883	1.086

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	551	30	15	18	22
N.S.	1	1.00	1.12	1.00	34.44	1.88	0.94	1.12	1.38
time (sec)	N/A	0.019	2.799	0.828	6.868	0.273	9.777	0.614	1.119

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	83	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.032	0.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.38
time (sec)	N/A	0.019	0.662	3.506	0.313	0.261	1.137	0.963	0.788

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	684	32	15	18	22
N.S.	1	1.00	1.12	1.00	42.75	2.00	0.94	1.12	1.38
time (sec)	N/A	0.020	1.392	1.998	1.740	0.273	5.312	1.732	0.819

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	165	401	269	290	362	1130	0
N.S.	1	1.00	0.99	2.40	1.61	1.74	2.17	6.77	0.00
time (sec)	N/A	0.289	0.170	0.630	0.198	0.388	3.977	2.480	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	122	304	198	209	228	602	0
N.S.	1	1.00	0.99	2.47	1.61	1.70	1.85	4.89	0.00
time (sec)	N/A	0.197	0.110	0.570	0.199	0.329	3.402	2.191	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	113	110	92	129	104	341	0
N.S.	1	1.00	1.36	1.33	1.11	1.55	1.25	4.11	0.00
time (sec)	N/A	0.121	0.138	0.483	0.213	0.289	2.280	0.444	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	37	53	64	32	62	33
N.S.	1	1.00	1.87	1.19	1.71	2.06	1.03	2.00	1.06
time (sec)	N/A	0.016	0.030	0.163	0.217	0.276	1.168	0.283	0.002

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	411	868	0	0	0	0	0
N.S.	1	1.00	1.60	3.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.494	3.599	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	141	192	0	475	0	0	0
N.S.	1	1.00	1.38	1.88	0.00	4.66	0.00	0.00	0.00
time (sec)	N/A	0.114	0.150	3.020	0.000	0.330	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	250	573	0	1111	0	0	0
N.S.	1	1.00	1.45	3.33	0.00	6.46	0.00	0.00	0.00
time (sec)	N/A	0.219	0.312	3.244	0.000	0.558	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	496	693	870	1204	0	0	0	0	0
N.S.	1	1.40	1.75	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.078	44.493	12.327	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	502	368	826	0	0	0	0	0
N.S.	1	1.24	0.91	2.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.617	12.419	9.987	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	657	386	0	0	0	0	0
N.S.	1	1.00	2.09	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	32.090	8.517	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	73	21	0	21	25
N.S.	1	1.00	1.10	0.90	3.48	1.00	0.00	1.00	1.19
time (sec)	N/A	0.051	38.504	0.716	0.689	0.255	0.000	1.133	0.909

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	88	21	20	21	25
N.S.	1	1.00	1.10	0.90	4.19	1.00	0.95	1.00	1.19
time (sec)	N/A	0.057	5.295	0.582	0.676	0.271	22.263	1.148	0.871

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	333	798	0	0	0	0	0
N.S.	1	1.00	0.90	2.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.541	12.335	9.266	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	714	714	873	1233	0	0	0	0	0
N.S.	1	1.00	1.22	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.971	33.902	10.845	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	530	530	784	850	0	0	0	0	0
N.S.	1	1.00	1.48	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.477	34.012	9.159	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	383	410	0	0	0	0	0
N.S.	1	1.00	1.11	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.199	0.911	8.052	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	243	252	0	0	0	0	0
N.S.	1	1.00	1.15	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	5.235	3.325	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	67	29	0	21	25
N.S.	1	1.00	1.10	0.90	3.19	1.38	0.00	1.00	1.19
time (sec)	N/A	0.056	3.686	0.681	0.904	0.265	0.000	0.679	0.865

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	89	31	20	21	25
N.S.	1	1.00	1.10	0.90	4.24	1.48	0.95	1.00	1.19
time (sec)	N/A	0.061	6.466	0.630	0.638	0.269	19.500	0.701	0.865

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	814	880	0	0	0	0	0
N.S.	1	1.00	1.48	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.813	34.378	10.080	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	750	439	0	0	0	0	0
N.S.	1	1.00	2.03	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.413	34.083	9.296	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	226	280	0	0	0	0	0
N.S.	1	1.00	0.95	1.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.151	12.570	6.606	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	124	215	0	0	0	0	0
N.S.	1	1.00	1.04	1.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	0.198	3.235	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	97	40	0	21	25
N.S.	1	1.00	1.10	0.90	4.62	1.90	0.00	1.00	1.19
time (sec)	N/A	0.066	11.987	0.783	0.697	0.290	0.000	0.684	0.890

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	145	42	20	21	25
N.S.	1	1.00	1.10	0.90	6.90	2.00	0.95	1.00	1.19
time (sec)	N/A	0.068	15.768	0.834	0.703	0.274	84.989	0.696	0.913

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	602	602	887	1067	0	0	0	0	0
N.S.	1	1.00	1.47	1.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.146	34.213	9.564	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	440	856	1026	0	0	0	0	0
N.S.	1	1.00	1.95	2.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.624	34.074	9.030	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	345	900	0	0	0	0	0
N.S.	1	1.00	1.10	2.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.483	12.464	9.512	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	608	875	0	0	0	0	0
N.S.	1	1.00	2.04	2.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	21.287	7.162	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	177	51	0	21	25
N.S.	1	1.00	1.10	0.90	8.43	2.43	0.00	1.00	1.19
time (sec)	N/A	0.069	30.013	0.907	0.780	0.265	0.000	0.695	0.900

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	179	53	0	21	25
N.S.	1	1.00	1.10	0.90	8.52	2.52	0.00	1.00	1.19
time (sec)	N/A	0.085	27.563	0.869	0.760	0.260	0.000	0.696	0.912

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	540	637	1002	1618	0	0	0	0	0
N.S.	1	1.18	1.86	3.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.542	34.051	9.336	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	140	328	296	191	408	1166	0
N.S.	1	1.00	0.68	1.59	1.44	0.93	1.98	5.66	0.00
time (sec)	N/A	0.094	0.164	0.629	0.197	0.374	10.788	1.665	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	121	254	232	170	294	822	0
N.S.	1	1.00	0.75	1.58	1.44	1.06	1.83	5.11	0.00
time (sec)	N/A	0.077	0.130	0.630	0.198	0.341	4.328	1.162	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	149	132	153	141	153	473	0
N.S.	1	1.00	1.37	1.21	1.40	1.29	1.40	4.34	0.00
time (sec)	N/A	0.040	0.169	0.314	0.196	0.325	3.234	0.943	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	104	116	89	125	73	1052	75
N.S.	1	1.00	1.20	1.33	1.02	1.44	0.84	12.09	0.86
time (sec)	N/A	0.050	0.096	0.329	0.197	0.297	2.705	0.565	1.089

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	69	108	94	66	151	136	0
N.S.	1	1.00	0.66	1.03	0.90	0.63	1.44	1.30	0.00
time (sec)	N/A	0.057	0.088	0.322	0.215	0.283	2.118	0.303	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	94	127	137	88	280	245	0
N.S.	1	1.00	0.62	0.84	0.90	0.58	1.84	1.61	0.00
time (sec)	N/A	0.074	0.138	0.327	0.192	0.280	4.492	0.292	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	110	145	172	109	372	367	0
N.S.	1	1.00	0.56	0.74	0.87	0.55	1.89	1.86	0.00
time (sec)	N/A	0.085	0.133	0.352	0.183	0.285	29.200	0.292	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	115	139	183	127	364	1244	0
N.S.	1	1.00	0.59	0.71	0.93	0.65	1.86	6.35	0.00
time (sec)	N/A	0.114	0.173	0.637	0.185	0.309	3.857	0.426	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	97	121	142	106	272	900	0
N.S.	1	1.00	0.63	0.79	0.93	0.69	1.78	5.88	0.00
time (sec)	N/A	0.090	0.161	0.722	0.211	0.305	2.408	0.339	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	78	217	98	85	177	556	0
N.S.	1	1.00	0.57	1.57	0.71	0.62	1.28	4.03	0.00
time (sec)	N/A	0.069	0.101	0.738	0.212	0.294	1.579	0.328	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	108	190	0	0	0	0	111
N.S.	1	1.00	0.87	1.53	0.00	0.00	0.00	0.00	0.90
time (sec)	N/A	0.212	0.082	2.043	0.000	0.000	0.000	0.000	1.235

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	125	191	0	0	0	0	124
N.S.	1	1.00	0.91	1.39	0.00	0.00	0.00	0.00	0.91
time (sec)	N/A	0.217	0.088	2.759	0.000	0.000	0.000	0.000	1.161

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	184	459	404	273	542	1579	0
N.S.	1	1.00	0.73	1.82	1.60	1.08	2.15	6.27	0.00
time (sec)	N/A	0.167	0.231	0.974	0.228	0.458	11.857	5.134	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	151	339	296	237	355	1033	0
N.S.	1	1.00	0.79	1.77	1.55	1.24	1.86	5.41	0.00
time (sec)	N/A	0.086	0.144	0.511	0.221	0.396	5.950	3.505	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	134	250	197	232	207	2502	0
N.S.	1	1.00	0.82	1.53	1.21	1.42	1.27	15.35	0.00
time (sec)	N/A	0.098	0.124	0.521	0.212	0.338	4.362	2.318	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	125	230	158	222	211	4288	0
N.S.	1	1.00	0.80	1.46	1.01	1.41	1.34	27.31	0.00
time (sec)	N/A	0.099	0.175	0.469	0.224	0.321	4.024	98.895	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	127	175	181	126	335	314	0
N.S.	1	1.00	0.69	0.96	0.99	0.69	1.83	1.72	0.00
time (sec)	N/A	0.123	0.176	0.502	0.215	0.276	5.190	0.297	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	153	207	241	158	510	491	0
N.S.	1	1.00	0.63	0.86	1.00	0.66	2.12	2.04	0.00
time (sec)	N/A	0.151	0.165	0.534	0.218	0.283	30.569	0.303	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	159	198	253	186	493	1706	0
N.S.	1	1.00	0.66	0.82	1.05	0.77	2.04	7.05	0.00
time (sec)	N/A	0.160	0.202	1.046	0.214	0.336	4.391	0.450	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	124	352	189	152	352	1160	0
N.S.	1	1.00	0.64	1.81	0.97	0.78	1.81	5.95	0.00
time (sec)	N/A	0.106	0.172	0.972	0.210	0.308	2.782	0.392	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	157	288	0	0	0	0	0
N.S.	1	1.00	0.84	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	0.283	3.061	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	194	274	0	0	0	0	0
N.S.	1	1.00	1.03	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.504	4.461	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	565	565	1260	415	0	0	0	0	0
N.S.	1	1.00	2.23	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.904	1.823	44.193	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	507	507	1123	394	0	0	0	0	0
N.S.	1	1.00	2.21	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.832	0.451	4.277	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	529	1068	272	0	0	0	0	0
N.S.	1	1.00	2.02	0.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.635	0.450	24.994	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	479	479	1089	1934	0	0	0	0	0
N.S.	1	1.00	2.27	4.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.644	0.384	3.107	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	572	572	1241	332	0	0	0	0	0
N.S.	1	1.00	2.17	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.783	1.806	43.161	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	628	628	1480	643	0	0	0	0	0
N.S.	1	1.00	2.36	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.980	4.220	5.692	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	593	590	1442	524	0	0	0	0	0
N.S.	1	0.99	2.43	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.882	1.835	4.549	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	131	286	265	0	385	0	0	0
N.S.	1	0.98	2.13	1.98	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.078	0.553	8.608	0.000	0.322	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	566	566	1408	2071	0	0	0	0	0
N.S.	1	1.00	2.49	3.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.801	1.275	4.158	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	803	803	1634	965	0	0	0	0	0
N.S.	1	1.00	2.03	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.802	6.045	44.659	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	765	765	1482	844	0	0	0	0	0
N.S.	1	1.00	1.94	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.921	1.705	39.557	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	762	759	1477	832	0	0	0	0	0
N.S.	1	1.00	1.94	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.667	2.059	33.948	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	806	806	1525	925	0	0	0	0	0
N.S.	1	1.00	1.89	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.737	1.863	35.461	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	727	727	2053	1269	0	0	0	0	0
N.S.	1	1.00	2.82	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.057	7.390	4.667	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	390	945	0	1015	0	0	0
N.S.	1	1.00	2.48	6.02	0.00	6.46	0.00	0.00	0.00
time (sec)	N/A	0.125	0.761	6.541	0.000	0.496	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	385	894	0	890	0	0	0
N.S.	1	1.00	1.99	4.63	0.00	4.61	0.00	0.00	0.00
time (sec)	N/A	0.146	0.643	7.185	0.000	0.501	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	704	704	2114	3479	0	0	0	0	0
N.S.	1	1.00	3.00	4.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.929	6.064	3.307	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1144	1144	2067	1804	0	0	0	0	0
N.S.	1	1.00	1.81	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.201	6.070	47.847	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1144	1144	2075	1278	0	0	0	0	0
N.S.	1	1.00	1.81	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.301	6.064	64.233	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1134	1134	2060	1798	0	0	0	0	0
N.S.	1	1.00	1.82	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.769	6.048	106.523	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	403	403	326	0	0	1699	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	4.22	0.00	0.00	0.00
time (sec)	N/A	0.895	1.564	0.000	0.000	2.439	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	263	0	0	1379	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	4.69	0.00	0.00	0.00
time (sec)	N/A	0.296	1.442	0.000	0.000	1.049	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	213	0	0	1098	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	5.63	0.00	0.00	0.00
time (sec)	N/A	0.134	1.573	0.000	0.000	0.533	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.87	1.00	1.17
time (sec)	N/A	0.068	6.453	0.493	0.000	0.258	10.391	0.349	1.275

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.074	10.945	0.764	0.000	0.260	14.183	0.369	1.407

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.069	10.764	0.526	0.000	0.259	118.153	0.343	1.394

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.025	17.908	0.439	0.000	0.266	47.880	0.357	1.266

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.065	1.862	0.316	0.000	0.264	7.153	0.356	1.526

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	328	247	0	0	192	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.313	8.064	0.000	0.000	0.108	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	453	453	325	0	0	292	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.409	10.695	0.000	0.000	0.113	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	374	374	304	0	0	1697	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	4.54	0.00	0.00	0.00
time (sec)	N/A	0.376	1.586	0.000	0.000	2.235	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	262	248	0	0	1375	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	5.25	0.00	0.00	0.00
time (sec)	N/A	0.218	1.322	0.000	0.000	0.982	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.87	1.00	1.17
time (sec)	N/A	0.083	7.682	2.917	0.000	0.259	72.838	0.474	1.243

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.090	11.890	2.283	0.000	0.274	63.269	0.363	1.493

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	43	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.87	0.00	1.00	1.17
time (sec)	N/A	0.083	11.079	1.111	0.000	0.261	0.000	0.342	1.514

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	37	0	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.85	0.00	1.00	1.20
time (sec)	N/A	0.029	18.491	1.258	0.000	0.286	0.000	0.387	1.293

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.071	36.147	2.253	0.000	0.267	98.964	0.419	1.686

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.069	5.649	0.400	0.000	0.255	65.772	0.370	1.461

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	416	303	0	0	286	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.396	11.197	0.000	0.000	0.138	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	554	554	383	0	0	397	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.565	11.822	0.000	0.000	0.140	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	321	321	282	0	0	1383	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	4.31	0.00	0.00	0.00
time (sec)	N/A	0.745	2.010	0.000	0.000	1.107	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	225	225	239	0	0	1107	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	4.92	0.00	0.00	0.00
time (sec)	N/A	0.229	1.421	0.000	0.000	0.546	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	107	0	0	870	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	6.59	0.00	0.00	0.00
time (sec)	N/A	0.110	0.326	0.000	0.000	0.358	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	31	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.35	0.87	1.00	1.17
time (sec)	N/A	0.068	1.222	0.518	0.000	0.259	7.655	0.354	1.318

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	33	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.43	0.96	1.00	1.17
time (sec)	N/A	0.083	5.254	0.839	0.000	0.268	27.192	0.355	1.392

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.063	36.978	0.555	0.000	0.269	53.630	0.366	1.422

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.023	0.724	0.545	0.000	0.257	15.699	0.351	1.170

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	247	140	0	0	105	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.191	0.183	0.000	0.000	0.103	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	362	362	249	0	0	198	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.334	6.527	0.000	0.000	0.123	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	252	263	0	0	1480	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	5.87	0.00	0.00	0.00
time (sec)	N/A	0.780	1.528	0.000	0.000	0.521	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	156	162	0	0	1072	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	6.87	0.00	0.00	0.00
time (sec)	N/A	0.188	1.110	0.000	0.000	0.378	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	78	0	0	283	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	3.58	0.00	0.00	0.00
time (sec)	N/A	0.077	0.249	0.000	0.000	0.312	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	42	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.83	0.87	1.00	1.17
time (sec)	N/A	0.088	8.843	2.148	0.000	0.272	75.350	0.359	1.316

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	44	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.91	0.00	1.00	1.17
time (sec)	N/A	0.097	11.368	5.027	0.000	0.273	0.000	0.365	1.486

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.086	14.727	1.467	0.000	0.259	122.610	0.381	1.380

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.069	5.864	1.264	0.000	0.266	32.452	0.422	1.149

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	112	0	0	76	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.066	0.198	0.000	0.000	0.105	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	275	213	0	0	188	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.243	4.957	0.000	0.000	0.110	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	240	0	0	2119	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	8.72	0.00	0.00	0.00
time (sec)	N/A	0.762	1.466	0.000	0.000	0.496	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	162	0	0	663	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	0.170	0.412	0.000	0.000	0.379	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	130	0	0	573	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	4.15	0.00	0.00	0.00
time (sec)	N/A	0.097	0.350	0.000	0.000	0.365	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	53	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.30	0.00	1.00	1.17
time (sec)	N/A	0.098	16.092	3.111	0.000	0.273	0.000	0.366	1.267

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	55	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.39	0.00	1.00	1.17
time (sec)	N/A	0.102	18.717	3.412	0.000	0.277	0.000	0.372	1.460

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.082	15.999	1.654	0.000	0.272	0.000	0.401	1.427

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.076	15.004	1.543	0.000	0.279	0.000	0.446	1.303

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	276	276	185	0	0	284	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.203	0.314	0.000	0.000	0.124	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	296	249	0	0	350	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.179	0.421	0.000	0.000	0.121	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	585	566	402	0	0	0	0	0	0
N.S.	1	0.97	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.572	0.884	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	352	293	0	0	0	0	0	0
N.S.	1	0.95	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	0.480	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	215	202	171	0	0	0	0	0	0
N.S.	1	0.94	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.143	0.341	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.26
time (sec)	N/A	0.053	2.203	4.670	0.572	0.268	35.125	0.319	0.819

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	1.09	1.26
time (sec)	N/A	0.054	4.734	3.287	0.573	0.266	0.000	0.327	0.803

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	42	0	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.68	0.00	1.00	1.16
time (sec)	N/A	0.076	1.063	2.732	0.551	0.286	0.000	0.372	0.897

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.076	0.145	1.367	0.395	0.282	51.607	0.344	0.810

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.069	1.088	1.158	0.408	0.272	21.901	0.357	0.897

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	45	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.80	0.96	1.00	1.16
time (sec)	N/A	0.074	1.222	2.210	0.424	0.279	134.541	0.368	0.982

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	401	401	194	0	0	238	0	0	0
N.S.	1	1.00	0.48	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	1.724	0.271	0.000	0.000	0.300	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	159	0	0	181	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	1.404	0.339	0.000	0.000	0.276	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	135	138	0	0	124	0	0	0
N.S.	1	1.07	1.10	0.00	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.143	0.192	0.000	0.000	0.264	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	88	37	34	26	30
N.S.	1	1.00	1.08	0.92	3.38	1.42	1.31	1.00	1.15
time (sec)	N/A	0.071	0.441	0.372	0.518	0.254	11.160	0.359	1.817

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	112	39	36	26	30
N.S.	1	1.00	1.08	0.92	4.31	1.50	1.38	1.00	1.15
time (sec)	N/A	0.068	8.123	3.034	0.496	0.257	83.530	0.362	1.547

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [69] had the largest ratio of [.8570999999999999973]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	12	0.417
2	A	4	3	1.00	12	0.250
3	A	6	5	1.00	12	0.417
4	A	3	3	1.00	12	0.250
5	A	5	5	1.00	12	0.417
6	A	2	2	1.00	10	0.200
7	A	5	4	1.00	8	0.500
8	A	6	6	1.00	12	0.500
9	A	2	2	1.00	12	0.167
10	A	4	4	1.00	12	0.333
11	A	4	3	1.00	12	0.250
12	A	5	4	1.00	12	0.333
13	A	4	3	1.00	12	0.250
14	A	6	4	1.00	12	0.333
15	A	5	5	1.00	14	0.357
16	A	8	6	1.00	14	0.429
17	A	4	4	1.00	12	0.333
18	A	7	5	1.00	10	0.500
19	A	6	6	1.00	14	0.429
20	A	4	3	1.00	14	0.214
21	A	4	3	1.00	14	0.214
22	A	5	5	1.00	14	0.357
23	A	5	3	1.00	14	0.214
24	A	10	10	1.00	14	0.714

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
25	A	11	8	1.00	14	0.571
26	A	7	7	1.00	12	0.583
27	A	9	6	1.00	10	0.600
28	A	7	7	1.00	14	0.500
29	A	5	3	1.00	14	0.214
30	A	6	6	1.00	14	0.429
31	A	8	6	1.00	14	0.429
32	A	10	6	1.00	14	0.429
33	N/A	0	0	1.00	12	0.000
34	N/A	0	0	1.00	10	0.000
35	N/A	0	0	1.00	14	0.000
36	A	4	4	1.00	14	0.286
37	A	6	6	1.00	14	0.429
38	A	9	5	1.00	14	0.357
39	N/A	0	0	1.00	16	0.000
40	N/A	0	0	1.00	16	0.000
41	A	3	3	1.00	14	0.214
42	N/A	0	0	1.00	16	0.000
43	N/A	0	0	1.00	16	0.000
44	A	11	9	1.00	16	0.562
45	A	10	9	1.00	16	0.562
46	A	9	9	1.00	14	0.643
47	A	5	4	1.00	8	0.500
48	A	4	2	1.00	16	0.125
49	A	7	7	1.00	16	0.438
50	A	8	8	1.00	16	0.500
51	A	31	16	1.40	21	0.762
52	A	24	15	1.24	19	0.790
53	A	15	11	1.00	18	0.611
54	N/A	0	0	1.00	21	0.000
55	N/A	0	0	1.00	21	0.000
56	A	22	13	1.00	18	0.722
57	A	27	17	1.00	21	0.810
58	A	20	15	1.00	21	0.714

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	A	14	13	1.00	19	0.684
60	A	9	9	1.00	18	0.500
61	N/A	0	0	1.00	21	0.000
62	N/A	0	0	1.00	21	0.000
63	A	23	15	1.00	21	0.714
64	A	16	13	1.00	21	0.619
65	A	11	11	1.00	19	0.579
66	A	6	6	1.00	18	0.333
67	N/A	0	0	1.00	21	0.000
68	N/A	0	0	1.00	21	0.000
69	A	31	18	1.00	21	0.857
70	A	25	17	1.00	21	0.810
71	A	19	14	1.00	19	0.737
72	A	12	11	1.00	18	0.611
73	N/A	0	0	1.00	21	0.000
74	N/A	0	0	1.00	21	0.000
75	A	19	14	1.18	18	0.778
76	A	7	7	1.00	19	0.368
77	A	6	7	1.00	19	0.368
78	A	5	5	1.00	16	0.312
79	A	4	5	1.00	19	0.263
80	A	4	5	1.00	19	0.263
81	A	5	6	1.00	19	0.316
82	A	6	6	1.00	19	0.316
83	A	5	5	1.00	19	0.263
84	A	5	5	1.00	19	0.263
85	A	6	5	1.00	17	0.294
86	A	11	11	1.00	19	0.579
87	A	13	13	1.00	19	0.684
88	A	7	8	1.00	21	0.381
89	A	6	7	1.00	18	0.389
90	A	6	7	1.00	21	0.333
91	A	6	7	1.00	21	0.333
92	A	5	6	1.00	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	6	7	1.00	21	0.333
94	A	5	6	1.00	21	0.286
95	A	6	5	1.00	19	0.263
96	A	12	13	1.00	21	0.619
97	A	14	15	1.00	21	0.714
98	A	25	12	1.00	21	0.571
99	A	26	9	1.00	19	0.474
100	A	19	7	1.00	18	0.389
101	A	19	7	1.00	21	0.333
102	A	24	10	1.00	21	0.476
103	A	31	14	1.00	21	0.667
104	A	29	12	0.99	21	0.571
105	A	7	5	0.98	19	0.263
106	A	24	10	1.00	21	0.476
107	A	51	15	1.00	21	0.714
108	A	27	10	1.00	21	0.476
109	A	47	11	1.00	18	0.611
110	A	50	13	1.00	21	0.619
111	A	33	13	1.00	21	0.619
112	A	6	7	1.00	21	0.333
113	A	8	6	1.00	19	0.316
114	A	28	11	1.00	21	0.524
115	A	35	11	1.00	21	0.524
116	A	63	12	1.00	21	0.571
117	A	81	12	1.00	18	0.667
118	A	12	12	1.00	23	0.522
119	A	11	12	1.00	23	0.522
120	A	9	9	1.00	21	0.429
121	N/A	0	0	1.00	23	0.000
122	N/A	0	0	1.00	23	0.000
123	N/A	0	0	1.00	23	0.000
124	N/A	0	0	1.00	20	0.000
125	N/A	0	0	1.00	23	0.000
126	A	11	11	1.00	23	0.478

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
127	A	12	12	1.00	23	0.522
128	A	12	12	1.00	23	0.522
129	A	10	10	1.00	21	0.476
130	N/A	0	0	1.00	23	0.000
131	N/A	0	0	1.00	23	0.000
132	N/A	0	0	1.00	23	0.000
133	N/A	0	0	1.00	20	0.000
134	N/A	0	0	1.00	23	0.000
135	N/A	0	0	1.00	23	0.000
136	A	12	12	1.00	23	0.522
137	A	13	12	1.00	23	0.522
138	A	11	12	1.00	23	0.522
139	A	10	12	1.00	23	0.522
140	A	9	9	1.00	21	0.429
141	N/A	0	0	1.00	23	0.000
142	N/A	0	0	1.00	23	0.000
143	N/A	0	0	1.00	23	0.000
144	N/A	0	0	1.00	20	0.000
145	A	11	11	1.00	23	0.478
146	A	11	12	1.00	23	0.522
147	A	10	11	1.00	23	0.478
148	A	9	11	1.00	23	0.478
149	A	4	4	1.00	21	0.190
150	N/A	0	0	1.00	23	0.000
151	N/A	0	0	1.00	23	0.000
152	N/A	0	0	1.00	23	0.000
153	N/A	0	0	1.00	23	0.000
154	A	5	5	1.00	20	0.250
155	A	10	11	1.00	23	0.478
156	A	10	11	1.00	23	0.478
157	A	7	8	1.00	23	0.348
158	A	5	5	1.00	21	0.238
159	N/A	0	0	1.00	23	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	N/A	0	0	1.00	23	0.000
161	N/A	0	0	1.00	23	0.000
162	N/A	0	0	1.00	23	0.000
163	A	10	10	1.00	23	0.435
164	A	10	11	1.00	20	0.550
165	A	6	7	0.97	23	0.304
166	A	6	7	0.95	23	0.304
167	A	5	6	0.94	21	0.286
168	N/A	0	0	1.00	23	0.000
169	N/A	0	0	1.00	23	0.000
170	N/A	0	0	1.00	25	0.000
171	N/A	0	0	1.00	25	0.000
172	N/A	0	0	1.00	25	0.000
173	N/A	0	0	1.00	25	0.000
174	A	16	11	1.00	26	0.423
175	A	13	11	1.00	26	0.423
176	A	8	9	1.07	26	0.346
177	N/A	0	0	1.00	26	0.000
178	N/A	0	0	1.00	26	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^6(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 114

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^2}{112c^5}+\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)+\frac{5b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}$$

```
[Out] 1/7*x^7*(a+b*arccsc(c*x))+5/112*b*arctanh((1-1/c^2/x^2)^(1/2))/c^7+5/112*b*
x^2*(1-1/c^2/x^2)^(1/2)/c^5+5/168*b*x^4*(1-1/c^2/x^2)^(1/2)/c^3+1/42*b*x^6*
(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5329, 272, 44, 65, 214}

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)+\frac{5b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}+\frac{bx^6\sqrt{1-\frac{1}{c^2x^2}}}{42c}+\frac{5bx^2\sqrt{1-\frac{1}{c^2x^2}}}{112c^5}+\frac{5bx^4\sqrt{1-\frac{1}{c^2x^2}}}{168c^3}$$

```
[In] Int[x^6*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (5*b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(112*c^5) + (5*b*Sqrt[1 - 1/(c^2*x^2)]*x^4)
/(168*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^6)/(42*c) + (x^7*(a + b*ArcCsc[c*x]
))/7 + (5*b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(112*c^7)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)+\frac{b\int \frac{x^5}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{7c}$$

$$=\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)-\frac{b\text{Subst}\left(\int \frac{1}{x^4\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{14c}$$

$$=\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)-\frac{(5b)\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{84c^3}$$

$$= \frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7(a+b\csc^{-1}(cx)) - \frac{(5b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{112c^5}$$

$$= \frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^2}{112c^5} + \frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7(a+b\csc^{-1}(cx)) - \frac{(5b)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{224c^7}$$

$$= \frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^2}{112c^5} + \frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7(a+b\csc^{-1}(cx)) + \frac{(5b)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^5}$$

$$= \frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^2}{112c^5} + \frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c} + \frac{1}{7}x^7(a+b\csc^{-1}(cx)) + \frac{5b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int x^6(a+b\csc^{-1}(cx))\,dx = \frac{ax^7}{7} + b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{5x^2}{112c^5} + \frac{5x^4}{168c^3} + \frac{x^6}{42c}\right) + \frac{1}{7}bx^7\csc^{-1}(cx) + \frac{5b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{112c^7}$$

```
[In] Integrate[x^6*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (a*x^7)/7 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((5*x^2)/(112*c^5) + (5*x^4)/(
168*c^3) + x^6/(42*c)) + (b*x^7*ArcCsc[c*x])/7 + (5*b*Log[x*(1 + Sqrt[(-1 +
 c^2*x^2)/(c^2*x^2)])])/(112*c^7)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.55

method	result
parts	$\frac{a\,x^{7}}{7}+\frac{b\,x^{7}\,\mathrm{arccsc}(cx)}{7}+\frac{b(c^{2}x^{2}-1)x^{4}}{42c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b(c^{2}x^{2}-1)x^{2}}{168c^{5}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b(c^{2}x^{2}-1)}{112c^{7}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b\sqrt{c^{2}x^{2}-1}\,\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{112c^{8}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,x}$
derivativedivides	$\frac{\frac{a\,c^{7}x^{7}}{7}+\frac{b\,c^{7}x^{7}\,\mathrm{arccsc}(cx)}{7}+\frac{b\left(c^{2}x^{2}-1\right)c^{4}x^{4}}{42\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b\left(c^{2}x^{2}-1\right)c^{2}x^{2}}{168\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b\left(c^{2}x^{2}-1\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b\sqrt{c^{2}x^{2}-1}\,\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}}{c^{7}}$
default	$\frac{\frac{a\,c^{7}x^{7}}{7}+\frac{b\,c^{7}x^{7}\,\mathrm{arccsc}(cx)}{7}+\frac{b\left(c^{2}x^{2}-1\right)c^{4}x^{4}}{42\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b\left(c^{2}x^{2}-1\right)c^{2}x^{2}}{168\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b\left(c^{2}x^{2}-1\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b\sqrt{c^{2}x^{2}-1}\,\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{112\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}}{c^{7}}$

```
[In] int(x^6*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a*x^7+1/7*b*x^7*arccsc(c*x)+1/42*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2
)^(1/2)*x^4+5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2+5/112*b
/c^7*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)+5/112*b/c^8*(c^2*x^2-1)^(1/2)/
((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int x^{6}\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{48\,ac^{7}x^{7}-96\,bc^{7}\arctan\left(-cx+\sqrt{c^{2}x^{2}-1}\right)+48\left(bc^{7}x^{7}-bc^{7}\right)\mathrm{arccsc}\left(cx\right)-15\,b\log\left(-cx+\sqrt{c^{2}x^{2}-1}\right)+}{336\,c^{7}}$$

```
[In] integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/336*(48*a*c^7*x^7 - 96*b*c^7*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 48*(b*c^7
*x^7 - b*c^7)*arccsc(c*x) - 15*b*log(-c*x + sqrt(c^2*x^2 - 1)) + (8*b*c^5*x
^5 + 10*b*c^3*x^3 + 15*b*c*x)*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.94

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^7}{7}+\frac{bx^7\operatorname{acsc}(cx)}{7}+\frac{b\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

```
[In] integrate(x**6*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**7/7 + b*x**7*acsc(c*x)/7 + b*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1))
 + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) -
 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2)
 > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 +
 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*
x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.41

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{7}\,ax^7+\frac{1}{672}\left(96\,x^7\operatorname{arccsc}(cx)+\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)b$$

```
[In] integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7 + 1/672*(96*x^7*arccsc(c*x) + (2*(15*(-1/(c^2*x^2) + 1)^(5/2) - 4
0*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) -
 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*log
(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^6)/
c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(96) = 192$.

Time = 0.75 (sec) , antiderivative size = 646, normalized size of antiderivative = 5.67

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{2688}\left(\frac{3\,bx^7\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^7\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{3\,ax^7\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^7}{c}+\frac{bx^6\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^6}{c^2}+\frac{21\,b\right.$$

```
[In] integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/2688*(3*b*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))/c + 3*a*x^7*
(sqrt(-1/(c^2*x^2) + 1) + 1)^7/c + b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c^2
 + 21*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c^3 + 21*a*x^5*(
sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^3 + 9*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/
c^4 + 63*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^5 + 63*a*x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^5 + 45*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1
)^2/c^6 + 105*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^7 + 105*a*
x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^7 + 120*b*log(sqrt(-1/(c^2*x^2) + 1) + 1)/
c^8 - 120*b*log(1/(abs(c)*abs(x)))/c^8 + 105*b*arcsin(1/(c*x))/(c^9*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)) + 105*a/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 45*
b/(c^10*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 63*b*arcsin(1/(c*x))/(c^11*x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 63*a/(c^11*x^3*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^3) - 9*b/(c^12*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 21*b*arcsin(1/(c*
x))/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 21*a/(c^13*x^5*(sqrt(-1/(c^
2*x^2) + 1) + 1)^5) - b/(c^14*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 3*b*arc
sin(1/(c*x))/(c^15*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7) + 3*a/(c^15*x^7*(sqr
t(-1/(c^2*x^2) + 1) + 1)^7))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^6\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^6*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^6*(a + b*asin(1/(c*x))), x)
```

3.2 $\int x^5(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{4b\sqrt{1-\frac{1}{c^2x^2}}\,x}{45c^5}+\frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{45c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)$$

```
[Out] 1/6*x^6*(a+b*arccsc(c*x))+4/45*b*x*(1-1/c^2/x^2)^(1/2)/c^5+2/45*b*x^3*(1-1/
c^2/x^2)^(1/2)/c^3+1/30*b*x^5*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5329, 277, 197}

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{bx^5\sqrt{1-\frac{1}{c^2x^2}}}{30c}+\frac{4bx\sqrt{1-\frac{1}{c^2x^2}}}{45c^5}+\frac{2bx^3\sqrt{1-\frac{1}{c^2x^2}}}{45c^3}$$

```
[In] Int[x^5*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (4*b*Sqrt[1 - 1/(c^2*x^2)]*x)/(45*c^5) + (2*b*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(4
5*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^5)/(30*c) + (x^6*(a + b*ArcCsc[c*x]))/6
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{6c}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{(2b)\int\frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{15c^3}\\
&= \frac{2b\sqrt{1-\frac{1}{c^2x^2}}x^3}{45c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{(4b)\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{45c^5}\\
&= \frac{4b\sqrt{1-\frac{1}{c^2x^2}}x}{45c^5}+\frac{2b\sqrt{1-\frac{1}{c^2x^2}}x^3}{45c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^6}{6}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{4x}{45c^5}+\frac{2x^3}{45c^3}+\frac{x^5}{30c}\right)+\frac{1}{6}bx^6\csc^{-1}(cx)$$

```
[In] Integrate[x^5*(a + b*ArcCsc[c*x]),x]
[Out] (a*x^6)/6 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((4*x)/(45*c^5) + (2*x^3)/(45*
c^3) + x^5/(30*c)) + (b*x^6*ArcCsc[c*x])/6
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

method	result	size
parts	$\frac{x^{6} a}{6}+\frac{b \left(\frac{c^{6} x^{6} \operatorname{arccsc}\left(c x \right)}{6}+\frac{\left(c^{2} x^{2}-1\right) \left(3 c^{4} x^{4}+4 c^{2} x^{2}+8\right)}{90 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}\right)}{c^{6}}$	79
derivativedivides	$\frac{\frac{c^{6} x^{6} a}{6}+b \left(\frac{c^{6} x^{6} \operatorname{arccsc}\left(c x \right)}{6}+\frac{\left(c^{2} x^{2}-1\right) \left(3 c^{4} x^{4}+4 c^{2} x^{2}+8\right)}{90 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}\right)}{c^{6}}$	83
default	$\frac{\frac{c^{6} x^{6} a}{6}+b \left(\frac{c^{6} x^{6} \operatorname{arccsc}\left(c x \right)}{6}+\frac{\left(c^{2} x^{2}-1\right) \left(3 c^{4} x^{4}+4 c^{2} x^{2}+8\right)}{90 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}\right)}{c^{6}}$	83

```
[In] int(x^5*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*a+b/c^6*(1/6*c^6*x^6*arccsc(c*x)+1/90*(c^2*x^2-1)*(3*c^4*x^4+4*c^2*
x^2+8)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int x^5 \left(a+b \csc^{-1}(cx)\right) \, dx = \frac{15 \, b c^{6} x^{6} \operatorname{arccsc}\left(c x\right) + 15 \, a c^{6} x^{6} + {\left(3 \, b c^{4} x^{4} + 4 \, b c^{2} x^{2} + 8 \, b\right)} \sqrt{c^{2} x^{2} - 1}}{90 \, c^{6}}$$

```
[In] integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*b*c^6*x^6*arccsc(c*x) + 15*a*c^6*x^6 + (3*b*c^4*x^4 + 4*b*c^2*x^2
+ 8*b)*sqrt(c^2*x^2 - 1))/c^6
```

Sympy [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.72

$$\int x^5 \left(a+b \csc^{-1}(cx)\right) \, dx = \frac{a x^{6}}{6} + \frac{b x^{6} \operatorname{acsc}{\left(c x \right)}}{6} + \frac{b \left(\begin{cases} \frac{x^{4} \sqrt{c^{2} x^{2} - 1}}{5 c} + \frac{4 x^{2} \sqrt{c^{2} x^{2} - 1}}{15 c^{3}} + \frac{8 \sqrt{c^{2} x^{2} - 1}}{15 c^{5}} & \text{for}\: \left|{c^{2} x^{2}}\right| > 1 \\\frac{i x^{4} \sqrt{- c^{2} x^{2} + 1}}{5 c} + \frac{4 i x^{2} \sqrt{- c^{2} x^{2} + 1}}{15 c^{3}} + \frac{8 i \sqrt{- c^{2} x^{2} + 1}}{15 c^{5}} & \text{otherwise} \end{cases}\right)}{6 c}$$

```
[In] integrate(x**5*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**6/6 + b*x**6*acsc(c*x)/6 + b*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c)
 + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5),
Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c*
*2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{6}\,ax^6+\frac{1}{90}\left(15\,x^6\,\mathrm{arccsc}\left(cx\right)+\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)b$$

```
[In] integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2)
+ 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(75) = 150.

Time = 0.32 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.82

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{5760}\left(\frac{15\,bx^6\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^6\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{15\,ax^6\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^6}{c}+\frac{6\,bx^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5}{c^2}+\right.$$

```
[In] integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/5760*(15*b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c + 15*a*x^
6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 6*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5
/c^2 + 90*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 90*a*x
^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^3 + 50*b*x^3*(sqrt(-1/(c^2*x^2) + 1) +
1)^3/c^4 + 225*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^5 + 2
25*a*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 300*b*x*(sqrt(-1/(c^2*x^2) +
1) + 1)/c^6 + 300*b*arcsin(1/(c*x))/c^7 + 300*a/c^7 - 300*b/(c^8*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)) + 225*b*arcsin(1/(c*x))/(c^9*x^2*(sqrt(-1/(c^2*x^2) +
```

```
 1) + 1)^2) + 225*a/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 50*b/(c^10*x
^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 90*b*arcsin(1/(c*x))/(c^11*x^4*(sqrt(-
1/(c^2*x^2) + 1) + 1)^4) + 90*a/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) -
 6*b/(c^12*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 15*b*arcsin(1/(c*x))/(c^13
*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 15*a/(c^13*x^6*(sqrt(-1/(c^2*x^2) +
1) + 1)^6))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b \csc^{-1}(cx)\right) dx = \int x^5 \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right) dx$$

```
[In] int(x^5*(a + b*asin(1/(c*x))),x)
[Out] int(x^5*(a + b*asin(1/(c*x))), x)
```

3.3 $\int x^4(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^4(a+b\csc^{-1}(cx))\,dx=\frac{3b\sqrt{1-\frac{1}{c^2x^2}}x^2}{40c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c}+\frac{1}{5}x^5(a+b\csc^{-1}(cx))+\frac{3b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}$$

```
[Out] 1/5*x^5*(a+b*arccsc(c*x))+3/40*b*arctanh((1-1/c^2/x^2)^(1/2))/c^5+3/40*b*x^
2*(1-1/c^2/x^2)^(1/2)/c^3+1/20*b*x^4*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5329, 272, 44, 65, 214}

$$\int x^4(a+b\csc^{-1}(cx))\,dx=\frac{1}{5}x^5(a+b\csc^{-1}(cx))+\frac{3b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}+\frac{bx^4\sqrt{1-\frac{1}{c^2x^2}}}{20c}+\frac{3bx^2\sqrt{1-\frac{1}{c^2x^2}}}{40c^3}$$

```
[In] Int[x^4*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (3*b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(40*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^4)/(2
0*c) + (x^5*(a + b*ArcCsc[c*x]))/5 + (3*b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(
40*c^5)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x^3}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{5c}\\
&= \frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)-\frac{b\text{Subst}\left(\int\frac{1}{x^3\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{10c}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c}+\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)-\frac{(3b)\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{40c^3}
\end{aligned}
$$

$$= \frac{3b\sqrt{1-\frac{1}{c^2x^2}}x^2}{40c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c} + \frac{1}{5}x^5(a+b\csc^{-1}(cx)) - \frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{80c^5}$$

$$= \frac{3b\sqrt{1-\frac{1}{c^2x^2}}x^2}{40c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c} + \frac{1}{5}x^5(a+b\csc^{-1}(cx)) + \frac{(3b)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^3}$$

$$= \frac{3b\sqrt{1-\frac{1}{c^2x^2}}x^2}{40c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c} + \frac{1}{5}x^5(a+b\csc^{-1}(cx)) + \frac{3b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int x^4(a+b\csc^{-1}(cx))\,dx = \frac{ax^5}{5} + b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{3x^2}{40c^3}+\frac{x^4}{20c}\right) + \frac{1}{5}bx^5\csc^{-1}(cx) + \frac{3b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{40c^5}$$

```
[In] Integrate[x^4*(a + b*ArcCsc[c*x]),x]
[Out] (a*x^5)/5 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((3*x^2)/(40*c^3) + x^4/(20*c)
) + (b*x^5*ArcCsc[c*x])/5 + (3*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])
])/(40*c^5)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.58

method	result	size
parts	$\frac{a\,x^5}{5}+\frac{x^5 b\,\operatorname{arccsc}(cx)}{5}+\frac{b(c^2x^2-1)x^2}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b(c^2x^2-1)}{40c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40c^6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$	141
derivativedivides	$\frac{\frac{a\,c^5x^5}{5}+\frac{\operatorname{arccsc}(cx)b\,c^5x^5}{5}+\frac{b\left(c^2x^2-1\right)c^2x^2}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\left(c^2x^2-1\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^5}$	148
default	$\frac{\frac{a\,c^5x^5}{5}+\frac{\operatorname{arccsc}(cx)b\,c^5x^5}{5}+\frac{b\left(c^2x^2-1\right)c^2x^2}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\left(c^2x^2-1\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^5}$	148

```
[In] int(x^4*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+1/5*x^5*b*arccsc(c*x)+1/20*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2
)^(1/2)*x^2+3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)+3/40*b/c^6*(
c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19

$$\int x^4\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{8\,ac^5x^5 - 16\,bc^5\arctan\left(-cx+\sqrt{c^2x^2-1}\right) + 8\left(bc^5x^5 - bc^5\right)\operatorname{arccsc}\left(cx\right) - 3\,b\log\left(-cx+\sqrt{c^2x^2-1}\right) + (2\dots}{40\,c^5}$$

```
[In] integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*c^5*x^5 - 16*b*c^5*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 8*(b*c^5*x^
5 - b*c^5)*arccsc(c*x) - 3*b*log(-c*x + sqrt(c^2*x^2 - 1)) + (2*b*c^3*x^3 +
 3*b*c*x)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.97

$$\int x^4\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{ax^5}{5} + \frac{bx^5\operatorname{acsc}\left(cx\right)}{5} + \frac{b\left(\begin{cases} \frac{cx^5}{4\sqrt{c^2x^2-1}} + \frac{x^3}{8c\sqrt{c^2x^2-1}} - \frac{3x}{8c^3\sqrt{c^2x^2-1}} + \frac{3\operatorname{acosh}\left(cx\right)}{8c^4} & \text{for}\ \left|c^2x^2\right| > 1 \\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}} - \frac{ix^3}{8c\sqrt{-c^2x^2+1}} + \frac{3ix}{8c^3\sqrt{-c^2x^2+1}} - \frac{3i\operatorname{asin}\left(cx\right)}{8c^4} & \text{otherwise} \end{cases}\right)}{5c}$$

```
[In] integrate(x**4*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**5/5 + b*x**5*acsc(c*x)/5 + b*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1))
 + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*ac
osh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1))
 - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1))
- 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.48

$$\int x^4\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{5}\,ax^5+\frac{1}{80}\left(16\,x^5\,\mathrm{arccsc}\,(cx)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)b$$

```
[In] integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/80*(16*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*s
qrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) +
 c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1
) - 1)/c^4)/c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(75) = 150$.

Time = 0.63 (sec) , antiderivative size = 480, normalized size of antiderivative = 5.39

$$\int x^4\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{320}\left(\frac{2\,bx^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{2\,ax^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5}{c}+\frac{bx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c^2}+\frac{10\,bx}{}\right.$$

```
[In] integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/320*(2*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c + 2*a*x^5*(
sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^2
+ 10*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^3 + 10*a*x^3*(s
qrt(-1/(c^2*x^2) + 1) + 1)^3/c^3 + 8*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c
^4 + 20*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 20*a*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)/c^5 + 24*b*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 24
*b*log(1/(abs(c)*abs(x)))/c^6 + 20*b*arcsin(1/(c*x))/(c^7*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)) + 20*a/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 8*b/(c^8*x^2*(
sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 10*b*arcsin(1/(c*x))/(c^9*x^3*(sqrt(-1/(c^
2*x^2) + 1) + 1)^3) + 10*a/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - b/(c^
```

```
10*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 2*b*arcsin(1/(c*x))/(c^11*x^5*(sqr
t(-1/(c^2*x^2) + 1) + 1)^5) + 2*a/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)
)*c
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^4\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^4*(a + b*asin(1/(c*x))),x)
[Out] int(x^4*(a + b*asin(1/(c*x))), x)
```

3.4 $\int x^3(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int x^3(a+b\csc^{-1}(cx))\,dx=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{4}x^4(a+b\csc^{-1}(cx))$$

```
[Out] 1/4*x^4*(a+b*arccsc(c*x))+1/6*b*x*(1-1/c^2/x^2)^(1/2)/c^3+1/12*b*x^3*(1-1/c
^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5329, 277, 197}

$$\int x^3(a+b\csc^{-1}(cx))\,dx=\frac{1}{4}x^4(a+b\csc^{-1}(cx))+\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}}{12c}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{6c^3}$$

```
[In] Int[x^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x)/(6*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(12*c)
+ (x^4*(a + b*ArcCsc[c*x]))/4
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{4c}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{6c^3}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int x^3\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^4}{4}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{x}{6c^3}+\frac{x^3}{12c}\right)+\frac{1}{4}bx^4\csc^{-1}(cx)$$

```
[In] Integrate[x^3*(a + b*ArcCsc[c*x]),x]
[Out] (a*x^4)/4 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*(x/(6*c^3) + x^3/(12*c)) + (b*
x^4*ArcCsc[c*x])/4
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

method	result	size
parts	$\frac{x^4 a}{4}+\frac{b\left(\frac{c^4 x^4 \operatorname{arccsc}(cx)}{4}+\frac{(c^2x^2-1)(c^2x^2+2)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	70
derivativedivides	$\frac{\frac{c^4x^4a}{4}+b\left(\frac{c^4 x^4 \operatorname{arccsc}(cx)}{4}+\frac{(c^2x^2-1)(c^2x^2+2)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	74
default	$\frac{\frac{c^4x^4a}{4}+b\left(\frac{c^4 x^4 \operatorname{arccsc}(cx)}{4}+\frac{(c^2x^2-1)(c^2x^2+2)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	74

```
[In] int(x^3*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*a+b/c^4*(1/4*c^4*x^4*arccsc(c*x)+1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^2
*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int x^3\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{3\,bc^4x^4\operatorname{arccsc}\left(cx\right)+3\,ac^4x^4+(bc^2x^2+2\,b)\sqrt{c^2x^2-1}}{12\,c^4}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] 1/12*(3*b*c^4*x^4*arccsc(c*x) + 3*a*c^4*x^4 + (b*c^2*x^2 + 2*b)*sqrt(c^2*x^
2 - 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67

$$\int x^3\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^4}{4}+\frac{bx^4\operatorname{acsc}\left(cx\right)}{4}+\frac{b\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

```
[In] integrate(x**3*(a+b*acsc(c*x)),x)
[Out] a*x**4/4 + b*x**4*acsc(c*x)/4 + b*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c)
 + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*
x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int x^3(a+b\csc^{-1}(cx))\,dx = \frac{1}{4}ax^4 + \frac{1}{12}\left(3\,x^4\operatorname{arccsc}(cx) + \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)b$$

```
[In] integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3
*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(54) = 108.

Time = 0.30 (sec) , antiderivative size = 352, normalized size of antiderivative = 5.50

$$\int x^3(a+b\csc^{-1}(cx))\,dx = \frac{1}{192}\left(\frac{3\,bx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{3\,ax^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c} + \frac{2\,bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c^2} + \frac{12}{}\right.$$

```
[In] integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a*x^4*(
sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 2*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^
2 + 12*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 12*a*x^2*
(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 18*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^
4 + 18*b*arcsin(1/(c*x))/c^5 + 18*a/c^5 - 18*b/(c^6*x*(sqrt(-1/(c^2*x^2) +
1) + 1)) + 12*b*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) +
12*a/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 2*b/(c^8*x^3*(sqrt(-1/(c^2*
x^2) + 1) + 1)^3) + 3*b*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) +
1)^4) + 3*a/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a + b \csc^{-1}(cx)) \, dx = \int x^3 \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int(x^3*(a + b*asin(1/(c*x))),x)
[Out] int(x^3*(a + b*asin(1/(c*x))), x)
```

3.5 $\int x^2(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

```
[Out] 1/3*x^3*(a+b*arccsc(c*x))+1/6*b*arctanh((1-1/c^2/x^2)^(1/2))/c^3+1/6*b*x^2*
(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5329, 272, 44, 65, 214}

$$\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}+\frac{bx^2\sqrt{1-\frac{1}{c^2x^2}}}{6c}$$

```
[In] Int[x^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(6*c) + (x^3*(a + b*ArcCsc[c*x]))/3 + (b*ArcT
anh[Sqrt[1 - 1/(c^2*x^2)]])/(6*c^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{3c}\\
&= \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)-\frac{b\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{6c}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)-\frac{b\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{12c^3}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\text{Subst}\left(\int\frac{1}{c^2-c^2x^2}\,dx,x,\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

$$\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^3}{3}+\frac{bx^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c}+\frac{1}{3}bx^3\csc^{-1}(cx)+\frac{b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

```
[In] Integrate[x^2*(a + b*ArcCsc[c*x]),x]
[Out] (a*x^3)/3 + (b*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + (b*x^3*ArcCsc[c*
x])/3 + (b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

method	result	size
parts	$\frac{x^3a}{3}+\frac{b\left(\frac{c^3x^3\,\mathrm{arccsc}(cx)}{3}+\frac{\sqrt{c^2x^2-1}\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	94
derivativedivides	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\mathrm{arccsc}(cx)}{3}+\frac{\sqrt{c^2x^2-1}\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	98
default	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\,\mathrm{arccsc}(cx)}{3}+\frac{\sqrt{c^2x^2-1}\left(cx\sqrt{c^2x^2-1}+\ln\left(cx+\sqrt{c^2x^2-1}\right)\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^3}$	98

```
[In] int(x^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*a+b/c^3*(1/3*c^3*x^3*arccsc(c*x)+1/6*(c^2*x^2-1)^(1/2)*(c*x*(c^2*x^
2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

$$\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{2\,ac^3x^3-4\,bc^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}bcx+2\left(bc^3x^3-bc^3\right)\operatorname{arccsc}\left(cx\right)-b\log\left(-cx+\sqrt{c}\right.}{6\,c^3}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3 - 4*b*c^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^2
- 1)*b*c*x + 2*(b*c^3*x^3 - b*c^3)*arccsc(c*x) - b*log(-c*x + sqrt(c^2*x^2
- 1)))/c^3
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67

$$\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^3}{3}+\frac{bx^3\operatorname{acsc}\left(cx\right)}{3}+\frac{b\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}\left(cx\right)}{2c^2} & \text{for}\ \left|c^2x^2\right|>1\\-\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}\left(cx\right)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate(x**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**3/3 + b*x**3*acsc(c*x)/3 + b*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) +
acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1
)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.52

$$\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{3}\,ax^3+\frac{1}{12}\left(4\,x^3\operatorname{arccsc}\left(cx\right)+\frac{\frac{2\,\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)b$$

```
[In] integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^
2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2
*x^2) + 1) - 1)/c^2)/c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(54) = 108$.

Time = 0.45 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.84

$$\int x^2(a + b\csc^{-1}(cx))\ dx$$
$$= \frac{1}{24}\left(\frac{bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{ax^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c} + \frac{bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^2} + \frac{3\,bx\left(\sqrt{\right.}}{}\right.$$

```
[In] integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/24*(b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + a*x^3*(sqrt(
-1/(c^2*x^2) + 1) + 1)^3/c + b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 3*b
*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 3*a*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^3 + 4*b*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 4*b*log(1/(ab
s(c)*abs(x)))/c^4 + 3*b*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)
) + 3*a/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - b/(c^6*x^2*(sqrt(-1/(c^2*x^2
) + 1) + 1)^2) + b*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 + a/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a + b\csc^{-1}(cx))\ dx = \int x^2\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\ dx$$

```
[In] int(x^2*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^2*(a + b*asin(1/(c*x))), x)
```

3.6 $\int x(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)$$

```
[Out] 1/2*x^2*(a+b*arccsc(c*x))+1/2*b*x*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5329, 197}

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2c}$$

```
[In] Int[x*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c) + (x^2*(a + b*ArcCsc[c*x]))/2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
```

```
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{2c}$$
$$=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^2}{2}+\frac{bx\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+\frac{1}{2}bx^2\csc^{-1}(cx)$$

```
[In] Integrate[x*(a + b*ArcCsc[c*x]),x]
[Out] (a*x^2)/2 + (b*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*ArcCsc[c*x]
)/2
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.56

method	result	size
parts	$\frac{a\,x^2}{2}+\frac{b\left(\frac{c^2x^2\,\operatorname{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	61
derivativedivides	$\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\,\operatorname{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	65
default	$\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\,\operatorname{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	65

```
[In] int(x*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arccsc(c*x)+1/2/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/
x*(c^2*x^2-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{bc^2x^2\operatorname{arccsc}(cx)+ac^2x^2+\sqrt{c^2x^2-1}\,b}{2\,c^2}$$

```
[In] integrate(x*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] 1/2*(b*c^2*x^2*arccsc(c*x) + a*c^2*x^2 + sqrt(c^2*x^2 - 1)*b)/c^2
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^2}{2}+\frac{bx^2\operatorname{acsc}(cx)}{2}+\frac{b\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

```
[In] integrate(x*(a+b*acsc(c*x)),x)
[Out] a*x**2/2 + b*x**2*acsc(c*x)/2 + b*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**
2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\operatorname{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)b$$

```
[In] integrate(x*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(33) = 66$.

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.67

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{8}\left(\frac{bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{ax^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c}+\frac{2\,bx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{2\,b\arcsin}{c^3}\right.$$

```
[In] integrate(x*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + a*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2/c + 2*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 2*b*ar
csin(1/(c*x))/c^3 + 2*a/c^3 - 2*b/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + b*
arcsin(1/(c*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a/(c^5*x^2*(sqrt
(-1/(c^2*x^2) + 1) + 1)^2))*c
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{2}+\frac{b\,x\,\sqrt{1-\frac{1}{c^2\,x^2}}}{2\,c}$$

```
[In] int(x*(a + b*asin(1/(c*x))),x)
```

```
[Out] (a*x^2)/2 + (b*x^2*asin(1/(c*x)))/2 + (b*x*(1 - 1/(c^2*x^2))^(1/2))/(2*c)
```

3.7 $\int (a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int (a+b\csc^{-1}(cx))\,dx = ax+bx\csc^{-1}(cx)+\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] a*x+b*x*arccsc(c*x)+b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5323, 272, 65, 214}

$$\int (a+b\csc^{-1}(cx))\,dx = ax+\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+bx\csc^{-1}(cx)$$

```
[In] Int[a + b*ArcCsc[c*x],x]
```

```
[Out] a*x + b*x*ArcCsc[c*x] + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5323

```
Int[ArcCsc[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsc[c*x], x] + Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \csc^{-1}(cx)\,dx \\
&= ax + bx\csc^{-1}(cx) + \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\,dx}{c} \\
&= ax + bx\csc^{-1}(cx) - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2c} \\
&= ax + bx\csc^{-1}(cx) + (bc)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right) \\
&= ax + bx\csc^{-1}(cx) + \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \left(a + b\csc^{-1}(cx)\right)\,dx = ax + bx\csc^{-1}(cx) + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

```
[In] Integrate[a + b*ArcCsc[c*x],x]
[Out] a*x + b*x*ArcCsc[c*x] + (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

method	result	size
default	$ax+bx\ \mathrm{arccsc}\left(cx\right)+\frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)}{c}$	37
parts	$ax+bx\ \mathrm{arccsc}\left(cx\right)+\frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)}{c}$	37
derivativedivides	$\frac{acx+b\left(\mathrm{arccsc}(cx)cx+\ln\left(cx+cx\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)\right)}{c}$	40

```
[In] int(a+b*arccsc(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b*x*arccsc(c*x)+b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(29) = 58.

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{acx-2\,bc\arctan\left(-cx+\sqrt{c^{2}x^{2}-1}\right)+(bcx-bc)\,\mathrm{arccsc}\left(cx\right)-b\log\left(-cx+\sqrt{c^{2}x^{2}-1}\right)}{c}$$

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="fricas")
[Out] (a*c*x - 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arccsc(c*x)
 - b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx = ax+b\left(x\,\mathrm{acsc}\left(cx\right)+\frac{\begin{cases}\mathrm{acosh}\left(cx\right) & \text{for}\ \left|c^{2}x^{2}\right|>1\\ -i\,\mathrm{asin}\left(cx\right) & \text{otherwise}\end{cases}}{c}\right)$$

```
[In] integrate(a+b*acsc(c*x),x)
[Out] a*x + b*(x*acsc(c*x) + Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \left(a + b\csc^{-1}(cx)\right)\,dx = ax + \frac{\left(2\,cx\,\mathrm{arccsc}\,(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)\right)b}{2\,c}$$

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="maxima")
[Out] a*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(
-1/(c^2*x^2) + 1) + 1))*b/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.00

$$\int \left(a + b\csc^{-1}(cx)\right)\,dx = \frac{1}{2}\,bc\left(\frac{2\,x\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2} + 1} + 1\right)}{c^2}\right) + ax$$

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="giac")
[Out] 1/2*b*c*(2*x*arcsin(1/(c*x))/c + (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sq
rt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \left(a + b\csc^{-1}(cx)\right)\,dx = a\,x + b\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right) + \frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

```
[In] int(a + b*asin(1/(c*x)),x)
[Out] a*x + b*x*asin(1/(c*x)) + (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

3.8 $\int \frac{a+b\csc^{-1}(cx)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx=\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b}-\left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

```
[Out] 1/2*I*(a+b*arccsc(c*x))^2/b-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/
2))^2)+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5327, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx=\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b}-\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

```
[In] Int[(a + b*ArcCsc[c*x])/x,x]
```

```
[Out] ((I/2)*(a + b*ArcCsc[c*x])^2)/b - (a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcC
sc[c*x])] + (I/2)*b*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5327

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcSin[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b} + 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b} - \left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\
&\quad + b\text{Subst}\left(\int \log\left(1-e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)
\end{aligned}
$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2b} - (a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right) - \frac{1}{2}(ib)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2b} - (a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right) + \frac{1}{2}ib\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int\frac{a+b\csc^{-1}(cx)}{x}\,dx = -b\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + a\log(x) + \frac{1}{2}ib\left(\csc^{-1}(cx)^2 + \text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\right)$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/x,x]
[Out] -(b*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])]) + a*Log[x] + (I/2)*b*(ArcCs
c[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.12

method	result
parts	$a\ln(x) + b\Big(\frac{i\,\text{arccsc}(cx)^2}{2} - \text{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right) + i\,\text{polylog}\left(2, \frac{i}{cx}+\sqrt{1-\right.$
derivativedivides	$a\ln(cx) + b\Big(\frac{i\,\text{arccsc}(cx)^2}{2} - \text{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right) + i\,\text{polylog}\left(2, \frac{i}{cx}+\sqrt{1\right.$
default	$a\ln(cx) + b\Big(\frac{i\,\text{arccsc}(cx)^2}{2} - \text{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right) + i\,\text{polylog}\left(2, \frac{i}{cx}+\sqrt{1\right.$

```
[In] int((a+b*arccsc(c*x))/x,x,method=_RETURNVERBOSE)
[Out] a*ln(x)+b*(1/2*I*arccsc(c*x)^2-arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+
I*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)
^(1/2))+I*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{a + b \csc^{-1}(cx)}{x} \, dx = \int \frac{b \operatorname{arccsc}(cx) + a}{x} \, dx$$

```
[In] integrate((a+b*arccsc(c*x))/x,x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{a + b \csc^{-1}(cx)}{x} \, dx = \int \frac{a + b \operatorname{acsc}(cx)}{x} \, dx$$

```
[In] integrate((a+b*acsc(c*x))/x,x)
[Out] Integral((a + b*acsc(c*x))/x, x)
```

Maxima [F]

$$\int \frac{a + b \csc^{-1}(cx)}{x} \, dx = \int \frac{b \operatorname{arccsc}(cx) + a}{x} \, dx$$

```
[In] integrate((a+b*arccsc(c*x))/x,x, algorithm="maxima")
[Out] (c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) + a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b + a*log(x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \csc^{-1}(cx)}{x} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/x,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expansi
on Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx = \frac{b\,\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,1\mathrm{i}}{2}+\frac{b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)^2\,1\mathrm{i}}{2} + a\,\ln\left(x\right)-b\,\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right)$$

```
[In] int((a + b*asin(1/(c*x)))/x,x)
```

```
[Out] (b*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 + (b*asin(1/(c*x))^2*1i)/2 + a*l
og(x) - b*log(1 - exp(asin(1/(c*x))*2i))*asin(1/(c*x))
```

3.9 $\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx=-bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{x}$$

```
[Out] (-a-b*arccsc(c*x))/x-b*c*(1-1/c^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5329, 267}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx=-\frac{a+b\csc^{-1}(cx)}{x}-bc\sqrt{1-\frac{1}{c^2x^2}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/x^2,x]
```

```
[Out] -(b*c*Sqrt[1 - 1/(c^2*x^2)]) - (a + b*ArcCsc[c*x])/x
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
```

```
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\csc^{-1}(cx)}{x} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^3}\,dx}{c}$$
$$= -bc\sqrt{1-\frac{1}{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{x}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx = -\frac{a}{x} - bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{x}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^2,x]
[Out] -(a/x) - b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/x
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

method	result	size
parts	$-\frac{a}{x}+bc\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)$	59
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$	63
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$	63

```
[In] int((a+b*arccsc(c*x))/x^2,x,method=_RETURNVERBOSE)
[Out] -a/x+b*c*(-1/c/x*arccsc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^2
-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a + b\csc^{-1}(cx)}{x^2}\,dx = -\frac{b\,\mathrm{arccsc}\,(cx) + \sqrt{c^2x^2 - 1}b + a}{x}$$

```
[In] integrate((a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
[Out] -(b*arccsc(c*x) + sqrt(c^2*x^2 - 1)*b + a)/x
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{a + b\csc^{-1}(cx)}{x^2}\,dx = \begin{cases} -\frac{a}{x} - bc\sqrt{1 - \frac{1}{c^2x^2}} - \frac{b\,\mathrm{acsc}\,(cx)}{x} & \text{for } c \neq 0 \\ -\frac{a + \tilde{\infty}b}{x} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*acsc(c*x))/x**2,x)
[Out] Piecewise((-a/x - b*c*sqrt(1 - 1/(c**2*x**2)) - b*acsc(c*x)/x, Ne(c, 0)), (
-(a + zoo*b)/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{a + b\csc^{-1}(cx)}{x^2}\,dx = -\left(c\sqrt{-\frac{1}{c^2x^2} + 1} + \frac{\mathrm{arccsc}\,(cx)}{x}\right)b - \frac{a}{x}$$

```
[In] integrate((a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
[Out] -(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b - a/x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx = -\left(b\sqrt{-\frac{1}{c^2x^2}+1}+\frac{b\arcsin\left(\frac{1}{cx}\right)}{cx}+\frac{a}{cx}\right)c$$

```
[In] integrate((a+b*arccsc(c*x))/x^2,x, algorithm="giac")
[Out] -(b*sqrt(-1/(c^2*x^2) + 1) + b*arcsin(1/(c*x))/(c*x) + a/(c*x))*c
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx = -\frac{a}{x}-b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x}$$

```
[In] int((a + b*asin(1/(c*x)))/x^2,x)
[Out] - a/x - b*c*(1 - 1/(c^2*x^2))^(1/2) - (b*asin(1/(c*x)))/x
```

3.10 $\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx = -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2\csc^{-1}(cx) - \frac{a+b\csc^{-1}(cx)}{2x^2}$$

```
[Out] 1/4*b*c^2*arccsc(c*x)+1/2*(-a-b*arccsc(c*x))/x^2-1/4*b*c*(1-1/c^2/x^2)^(1/2
)/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5329, 342, 327, 222}

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx = -\frac{a+b\csc^{-1}(cx)}{2x^2} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2\csc^{-1}(cx)$$

```
[In] Int[(a + b*ArcCsc[c*x])/x^3,x]
```

```
[Out] -1/4*(b*c*Sqrt[1 - 1/(c^2*x^2)])/x + (b*c^2*ArcCsc[c*x])/4 - (a + b*ArcCsc[
c*x])/(2*x^2)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\csc^{-1}(cx)}{2x^2}-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^4}\,dx}{2c}\\
&= -\frac{a+b\csc^{-1}(cx)}{2x^2}+\frac{b\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2c}\\
&= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}-\frac{a+b\csc^{-1}(cx)}{2x^2}+\frac{1}{4}(bc)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2\csc^{-1}(cx)-\frac{a+b\csc^{-1}(cx)}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x}-\frac{b\csc^{-1}(cx)}{2x^2}+\frac{1}{4}bc^2\arcsin\left(\frac{1}{cx}\right)$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^3,x]
[Out] -1/2*a/x^2 - (b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcCsc[c*x])/(
2*x^2) + (b*c^2*ArcSin[1/(c*x)])/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.88

method	result	size
parts	$-\frac{a}{2x^2}+b\,c^2\left(-\frac{\operatorname{arccsc}(cx)}{2c^2x^2}+\frac{\sqrt{c^2x^2-1}\,\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2-\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3c^3}\right)$	96
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arccsc}(cx)}{2c^2x^2}-\frac{\sqrt{c^2x^2-1}\,\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2+\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)\right)$	99
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arccsc}(cx)}{2c^2x^2}-\frac{\sqrt{c^2x^2-1}\,\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2+\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)\right)$	99

```
[In] int((a+b*arccsc(c*x))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a/x^2+b*c^2*(-1/2/c^2/x^2*arccsc(c*x)+1/4*(c^2*x^2-1)^(1/2)*(arctan(1/
(c^2*x^2-1)^(1/2))*c^2*x^2-(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
^3/c^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx=\frac{(bc^2x^2-2\,b)\operatorname{arccsc}(cx)-\sqrt{c^2x^2-1}b-2\,a}{4\,x^2}$$

```
[In] integrate((a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
[Out] 1/4*((b*c^2*x^2 - 2*b)*arccsc(c*x) - sqrt(c^2*x^2 - 1)*b - 2*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.37

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{b\operatorname{acsc}(cx)}{2x^2}-\frac{b\left(\begin{cases}\frac{ic^3\operatorname{acosh}\left(\frac{1}{cx}\right)}{2}-\frac{ic^2}{2x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{2x^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|}>1\\ -\frac{c^3\operatorname{asin}\left(\frac{1}{cx}\right)}{2}+\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{2c}$$

```
[In] integrate((a+b*acsc(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) - b*acsc(c*x)/(2*x**2) - b*Piecewise((I*c**3*acosh(1/(c*x))/2 -
 I*c**2/(2*x*sqrt(-1 + 1/(c**2*x**2))) + I/(2*x**3*sqrt(-1 + 1/(c**2*x**2))
), 1/Abs(c**2*x**2) > 1), (-c**3*asin(1/(c*x))/2 + c**2*sqrt(1 - 1/(c**2*x*
*2))/(2*x), True))/(2*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx = \frac{1}{4}\,b\left(\frac{\frac{c^4x\sqrt{-\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1} - c^3\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c} - \frac{2\,\mathrm{arccsc}\left(cx\right)}{x^2}\right) - \frac{a}{2\,x^2}$$

```
[In] integrate((a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*
arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/2*a/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx = -\frac{1}{4}\left(2\,bc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right) + 2\,ac\left(\frac{1}{c^2x^2}-1\right) + bc\arcsin\left(\frac{1}{cx}\right) + \frac{b\sqrt{-\frac{1}{c^2x^2}+1}}{x}\right)c$$

```
[In] integrate((a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 2*a*c*(1/(c^2*x^2) - 1) + b
*c*arcsin(1/(c*x)) + b*sqrt(-1/(c^2*x^2) + 1)/x)*c
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx=-\frac{a}{2\,x^2}-\frac{b\,c^2\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\,\left(\frac{2}{c^2\,x^2}-1\right)}{4}-\frac{b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}}{4\,x}$$

```
[In] int((a + b*asin(1/(c*x)))/x^3,x)
[Out] - a/(2*x^2) - (b*c^2*asin(1/(c*x))*(2/(c^2*x^2) - 1))/4 - (b*c*(1 - 1/(c^2*
x^2))^(1/2))/(4*x)
```

3.11 $\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx = -\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}+\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{a+b\csc^{-1}(cx)}{3x^3}$$

```
[Out] 1/9*b*c^3*(1-1/c^2/x^2)^(3/2)+1/3*(-a-b*arccsc(c*x))/x^3-1/3*b*c^3*(1-1/c^2
/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5329, 272, 45}

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx = -\frac{a+b\csc^{-1}(cx)}{3x^3}+\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/x^4,x]
```

```
[Out] -1/3*(b*c^3*Sqrt[1 - 1/(c^2*x^2)]) + (b*c^3*(1 - 1/(c^2*x^2))^(3/2))/9 - (a
 + b*ArcCsc[c*x])/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\csc^{-1}(cx)}{3x^3}-\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^5}\,dx}{3c}\\
&= -\frac{a+b\csc^{-1}(cx)}{3x^3}+\frac{b\text{Subst}\left(\int\frac{x}{\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{6c}\\
&= -\frac{a+b\csc^{-1}(cx)}{3x^3}+\frac{b\text{Subst}\left(\int\left(\frac{c^2}{\sqrt{1-\frac{x}{c^2}}}-c^2\sqrt{1-\frac{x}{c^2}}\right)dx,x,\frac{1}{x^2}\right)}{6c}\\
&= -\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}+\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{a+b\csc^{-1}(cx)}{3x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int\frac{a+b\csc^{-1}(cx)}{x^4}\,dx=-\frac{a}{3x^3}+b\left(-\frac{2c^3}{9}-\frac{c}{9x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\csc^{-1}(cx)}{3x^3}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^4,x]
[Out] -1/3*a/x^3 + b*((-2*c^3)/9 - c/(9*x^2))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b
*ArcCsc[c*x])/(3*x^3)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

method	result	size
parts	$-\frac{a}{3x^{3}}+b\,c^{3}\left(-\frac{\operatorname{arccsc}(cx)}{3c^{3}x^{3}}-\frac{(c^{2}x^{2}-1)\,(2c^{2}x^{2}+1)}{9\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{4}x^{4}}\right)$	71
derivativedivides	$c^{3}\left(-\frac{a}{3c^{3}x^{3}}+b\left(-\frac{\operatorname{arccsc}(cx)}{3c^{3}x^{3}}-\frac{(c^{2}x^{2}-1)\,(2c^{2}x^{2}+1)}{9\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{4}x^{4}}\right)\right)$	75
default	$c^{3}\left(-\frac{a}{3c^{3}x^{3}}+b\left(-\frac{\operatorname{arccsc}(cx)}{3c^{3}x^{3}}-\frac{(c^{2}x^{2}-1)\,(2c^{2}x^{2}+1)}{9\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{4}x^{4}}\right)\right)$	75

```
[In] int((a+b*arccsc(c*x))/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((
c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{a+b\csc^{-1}(cx)}{x^{4}}\,dx=-\frac{3\,b\operatorname{arccsc}\left(cx\right)+\left(2\,bc^{2}x^{2}+b\right)\sqrt{c^{2}x^{2}-1}+3\,a}{9\,x^{3}}$$

```
[In] integrate((a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
[Out] -1/9*(3*b*arccsc(c*x) + (2*b*c^2*x^2 + b)*sqrt(c^2*x^2 - 1) + 3*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.87

$$\int \frac{a+b\csc^{-1}(cx)}{x^{4}}\,dx=-\frac{a}{3x^{3}}-\frac{b\operatorname{acsc}\left(cx\right)}{3x^{3}}-\frac{b\left(\begin{cases}\frac{2c^{3}\sqrt{c^{2}x^{2}-1}}{3x}+\frac{c\sqrt{c^{2}x^{2}-1}}{3x^{3}} & \text{for}\ \left|c^{2}x^{2}\right|>1\\ \frac{2ic^{3}\sqrt{-c^{2}x^{2}+1}}{3x}+\frac{ic\sqrt{-c^{2}x^{2}+1}}{3x^{3}} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate((a+b*acsc(c*x))/x**4,x)
[Out] -a/(3*x**3) - b*acsc(c*x)/(3*x**3) - b*Piecewise((2*c**3*sqrt(c**2*x**2 - 1
)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sq
rt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx = \frac{1}{9}\,b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{3\,\operatorname{arccsc}\left(cx\right)}{x^3}\right) - \frac{a}{3\,x^3}$$

```
[In] integrate((a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
[Out] 1/9*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*
arccsc(c*x)/x^3) - 1/3*a/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.45

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx = \frac{1}{9}\left(bc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,bc^2\sqrt{-\frac{1}{c^2x^2}+1} - \frac{3\,bc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x} - \frac{3\,bc\arcsin\left(\frac{1}{cx}\right)}{x} - \frac{3\,a}{cx^3}\right)c$$

```
[In] integrate((a+b*arccsc(c*x))/x^4,x, algorithm="giac")
[Out] 1/9*(b*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 3*b*c^2*sqrt(-1/(c^2*x^2) + 1) - 3*b*
c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 3*b*c*arcsin(1/(c*x))/x - 3*a/(c*x^
3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/x^4,x)
[Out] int((a + b*asin(1/(c*x)))/x^4, x)
```

3.12 $\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx=-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}-\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}+\frac{3}{32}bc^4\csc^{-1}(cx)-\frac{a+b\csc^{-1}(cx)}{4x^4}$$

```
[Out] 3/32*b*c^4*arccsc(c*x)+1/4*(-a-b*arccsc(c*x))/x^4-1/16*b*c*(1-1/c^2/x^2)^(1
/2)/x^3-3/32*b*c^3*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5329, 342, 327, 222}

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx=-\frac{a+b\csc^{-1}(cx)}{4x^4}+\frac{3}{32}bc^4\csc^{-1}(cx)-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}-\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}$$

```
[In] Int[(a + b*ArcCsc[c*x])/x^5,x]
```

```
[Out] -1/16*(b*c*Sqrt[1 - 1/(c^2*x^2)])/x^3 - (3*b*c^3*Sqrt[1 - 1/(c^2*x^2)])/(32
*x) + (3*b*c^4*ArcCsc[c*x])/32 - (a + b*ArcCsc[c*x])/(4*x^4)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\csc^{-1}(cx)}{4x^4} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^6}\,dx}{4c}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4x^4} + \frac{b\text{Subst}\left(\int \frac{x^4}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4c}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} - \frac{a+b\csc^{-1}(cx)}{4x^4} + \frac{1}{16}(3bc)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x} - \frac{a+b\csc^{-1}(cx)}{4x^4} + \frac{1}{32}\left(3bc^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x} + \frac{3}{32}bc^4\csc^{-1}(cx) - \frac{a+b\csc^{-1}(cx)}{4x^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx = -\frac{a}{4x^4}+b\left(-\frac{c}{16x^3}-\frac{3c^3}{32x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{4x^4}+\frac{3}{32}bc^4\arcsin\left(\frac{1}{cx}\right)$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^5,x]
```

```
[Out] -1/4*a/x^4 + b*(-1/16*c/x^3 - (3*c^3)/(32*x))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)
] - (b*ArcCsc[c*x])/(4*x^4) + (3*b*c^4*ArcSin[1/(c*x)])/32
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(67) = 134.

Time = 0.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.82

method	result	siz
parts	$-\frac{a}{4x^4}-\frac{b\,\mathrm{arccsc}(cx)}{4x^4}+\frac{3b\,c^3\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{3bc(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3}-\frac{b(c^2x^2-1)}{16c\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^5}$	13
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arccsc}(cx)}{4c^4x^4}+\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}-\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}-\frac{b(c^2x^2-1)}{16\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}\right)$	15
default	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arccsc}(cx)}{4c^4x^4}+\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}-\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}-\frac{b(c^2x^2-1)}{16\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}\right)$	15

```
[In] int((a+b*arccsc(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arccsc(c*x)+3/32*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))-3/32*b*c*(c^2*x^2-1)/((c^2*x^2
-1)/c^2/x^2)^(1/2)/x^3-1/16*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx = \frac{(3\,bc^4x^4-8\,b)\,\mathrm{arccsc}\,(cx)-(3\,bc^2x^2+2\,b)\sqrt{c^2x^2-1}-8\,a}{32\,x^4}$$

```
[In] integrate((a+b*arccsc(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*((3*b*c^4*x^4 - 8*b)*arccsc(c*x) - (3*b*c^2*x^2 + 2*b)*sqrt(c^2*x^2 -
1) - 8*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.55

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx$$
$$= -\frac{a}{4x^4}-\frac{b\,\mathrm{acsc}\,(cx)}{4x^4} - \frac{b\left(\begin{cases}\frac{3ic^5\,\mathrm{acosh}\,\left(\frac{1}{cx}\right)}{8}-\frac{3ic^4}{8x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{ic^2}{8x^3\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{4x^5\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|}>1\\ -\frac{3c^5\,\mathrm{asin}\,\left(\frac{1}{cx}\right)}{8}+\frac{3c^4}{8x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{c^2}{8x^3\sqrt{1-\frac{1}{c^2x^2}}}-\frac{1}{4x^5\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise}\end{cases}\right)}{4c}$$

```
[In] integrate((a+b*acsc(c*x))/x**5,x)
```

```
[Out] -a/(4*x**4) - b*acsc(c*x)/(4*x**4) - b*Piecewise((3*I*c**5*acosh(1/(c*x))/8
 - 3*I*c**4/(8*x*sqrt(-1 + 1/(c**2*x**2))) + I*c**2/(8*x**3*sqrt(-1 + 1/(c*
*2*x**2))) + I/(4*x**5*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-
3*c**5*asin(1/(c*x))/8 + 3*c**4/(8*x*sqrt(1 - 1/(c**2*x**2))) - c**2/(8*x**
3*sqrt(1 - 1/(c**2*x**2))) - 1/(4*x**5*sqrt(1 - 1/(c**2*x**2))), True))/(4*
c)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.64

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx = -\frac{1}{32}\,b\left(\frac{3\,c^5\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)+\frac{3\,c^8x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+5\,c^6x\sqrt{-\frac{1}{c^2x^2}+1}}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c}+\frac{8\,\mathrm{arccsc}\,(cx)}{x^4}\right) - \frac{a}{4\,x^4}$$

```
[In] integrate((a+b*arccsc(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] -1/32*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*x^
2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^
2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/4*a/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.54

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx = -\frac{1}{32}\left(8\,bc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)+16\,bc^3\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)+5\,bc^3\arcsin\left(\frac{1}{cx}\right)-\frac{2\,bc^2\left(-\frac{1}{c^2}\right.}{x}\right.$$

```
[In] integrate((a+b*arccsc(c*x))/x^5,x, algorithm="giac")
```

```
[Out] -1/32*(8*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 16*b*c^3*(1/(c^2*x^2)
- 1)*arcsin(1/(c*x)) + 5*b*c^3*arcsin(1/(c*x)) - 2*b*c^2*(-1/(c^2*x^2) + 1)
^(3/2)/x + 5*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 8*a/(c*x^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^5}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/x^5,x)
[Out] int((a + b*asin(1/(c*x)))/x^5, x)
```

3.13 $\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 82

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx=-\frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}}+\frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2}-\frac{a+b\csc^{-1}(cx)}{5x^5}$$

```
[Out] 2/15*b*c^5*(1-1/c^2/x^2)^(3/2)-1/25*b*c^5*(1-1/c^2/x^2)^(5/2)+1/5*(-a-b*arc
csc(c*x))/x^5-1/5*b*c^5*(1-1/c^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5329, 272, 45}

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx=-\frac{a+b\csc^{-1}(cx)}{5x^5}-\frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2}+\frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/x^6,x]
```

```
[Out] -1/5*(b*c^5*Sqrt[1 - 1/(c^2*x^2)]) + (2*b*c^5*(1 - 1/(c^2*x^2))^(3/2))/15 -
 (b*c^5*(1 - 1/(c^2*x^2))^(5/2))/25 - (a + b*ArcCsc[c*x])/(5*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\csc^{-1}(cx)}{5x^5} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^7}\,dx}{5c} \\
&= -\frac{a+b\csc^{-1}(cx)}{5x^5} + \frac{b\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{10c} \\
&= -\frac{a+b\csc^{-1}(cx)}{5x^5} + \frac{b\text{Subst}\left(\int \left(\frac{c^4}{\sqrt{1-\frac{x}{c^2}}} - 2c^4\sqrt{1-\frac{x}{c^2}} + c^4\left(1-\frac{x}{c^2}\right)^{3/2}\right)\,dx, x, \frac{1}{x^2}\right)}{10c} \\
&= -\frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}} + \frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2} - \frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2} - \frac{a+b\csc^{-1}(cx)}{5x^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = -\frac{a}{5x^5} + b\left(-\frac{8c^5}{75} - \frac{c}{25x^4} - \frac{4c^3}{75x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{5x^5}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^6,x]
[Out] -1/5*a/x^5 + b*((-8*c^5)/75 - c/(25*x^4) - (4*c^3)/(75*x^2))*Sqrt[(-1 + c^2
*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/(5*x^5)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{a}{5x^5}+b\,c^5\left(-\frac{\operatorname{arccsc}(cx)}{5c^5x^5}-\frac{(c^2x^2-1)(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)$	79
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arccsc}(cx)}{5c^5x^5}-\frac{(c^2x^2-1)(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$	83
default	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arccsc}(cx)}{5c^5x^5}-\frac{(c^2x^2-1)(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$	83

```
[In] int((a+b*arccsc(c*x))/x^6,x,method=_RETURNVERBOSE)
[Out] -1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arccsc(c*x)-1/75*(c^2*x^2-1)*(8*c^4*x^4+4*c^
2*x^2+3)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.61

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = -\frac{15\,b\operatorname{arccsc}(cx)+(8\,bc^4x^4+4\,bc^2x^2+3\,b)\sqrt{c^2x^2-1}+15\,a}{75\,x^5}$$

```
[In] integrate((a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
[Out] -1/75*(15*b*arccsc(c*x) + (8*b*c^4*x^4 + 4*b*c^2*x^2 + 3*b)*sqrt(c^2*x^2 -
1) + 15*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.93

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{b\operatorname{acsc}(cx)}{5x^5} - \frac{b\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$

```
[In] integrate((a+b*acsc(c*x))/x**6,x)
```

```
[Out] -a/(5*x**5) - b*acsc(c*x)/(5*x**5) - b*Piecewise((8*c**5*sqrt(c**2*x**2 - 1
)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*
x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**
3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True)
)/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = -\frac{1}{75}\,b\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\mathrm{arccsc}\left(cx\right)}{x^5}\right) - \frac{a}{5\,x^5}$$

```
[In] integrate((a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/75*b*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2)
+ 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) - 1/5*a/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(68) = 136.

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.82

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = -\frac{1}{75}\left(3\,bc^4\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}-10\,bc^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{15\,bc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+15\,bc^4\sqrt{}\right.$$

```
[In] integrate((a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/75*(3*b*c^4*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 10*b*c^4*(-1/(c
^2*x^2) + 1)^(3/2) + 15*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 15*b*
c^4*sqrt(-1/(c^2*x^2) + 1) + 30*b*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x +
 15*b*c^3*arcsin(1/(c*x))/x + 15*a/(c*x^5))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = \int \frac{a+b\operatorname{asin}\left(\frac{1}{cx}\right)}{x^6}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/x^6,x)
[Out] int((a + b*asin(1/(c*x)))/x^6, x)
```

3.14 $\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx = -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} + \frac{5}{96}bc^6\csc^{-1}(cx) - \frac{a+b\csc^{-1}(cx)}{6x^6}$$

```
[Out] 5/96*b*c^6*arccsc(c*x)+1/6*(-a-b*arccsc(c*x))/x^6-1/36*b*c*(1-1/c^2/x^2)^(1
/2)/x^5-5/144*b*c^3*(1-1/c^2/x^2)^(1/2)/x^3-5/96*b*c^5*(1-1/c^2/x^2)^(1/2)/
x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5329, 342, 327, 222}

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx = -\frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{5}{96}bc^6\csc^{-1}(cx) - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} - \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}$$

```
[In] Int[(a + b*ArcCsc[c*x])/x^7,x]
```

```
[Out] -1/36*(b*c*Sqrt[1 - 1/(c^2*x^2)])/x^5 - (5*b*c^3*Sqrt[1 - 1/(c^2*x^2)])/(14
4*x^3) - (5*b*c^5*Sqrt[1 - 1/(c^2*x^2)])/(96*x) + (5*b*c^6*ArcCsc[c*x])/96
- (a + b*ArcCsc[c*x])/(6*x^6)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\csc^{-1}(cx)}{6x^6} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^8}\,dx}{6c} \\
&= -\frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{b\text{Subst}\left(\int \frac{x^6}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6c} \\
&= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{1}{36}(5bc)\text{Subst}\left(\int \frac{x^4}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{1}{48}\left(5bc^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} \\
&\quad - \frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{1}{96}\left(5bc^5\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} + \frac{5}{96}bc^6\csc^{-1}(cx) - \frac{a+b\csc^{-1}(cx)}{6x^6}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6} + b\left(-\frac{c}{36x^5} - \frac{5c^3}{144x^3} - \frac{5c^5}{96x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{6x^6} + \frac{5}{96}bc^6\arcsin\left(\frac{1}{cx}\right)$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^7,x]
```

```
[Out] -1/6*a/x^6 + b*(-1/36*c/x^5 - (5*c^3)/(144*x^3) - (5*c^5)/(96*x))*Sqrt[(-1
+ c^2*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/(6*x^6) + (5*b*c^6*ArcSin[1/(c*x)])
/96
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.72

method	result
parts	$-\frac{a}{6x^6} - \frac{b\,\mathrm{arccsc}(cx)}{6x^6} + \frac{5b\,c^5\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x} - \frac{5b\,c^3(c^2x^2-1)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3} - \frac{5bc(c^2x^2-1)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^5} - \frac{b(c^2x^2-}{36c\sqrt{\frac{c^2x^2}{c^2x}}}$
derivativedivides	$c^6\left(-\frac{a}{6c^6x^6} - \frac{b\,\mathrm{arccsc}(cx)}{6c^6x^6} + \frac{5b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx} - \frac{5b(c^2x^2-1)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3} - \frac{5b(c^2x^2-1)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5} - \frac{}{3}\right.$
default	$c^6\left(-\frac{a}{6c^6x^6} - \frac{b\,\mathrm{arccsc}(cx)}{6c^6x^6} + \frac{5b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx} - \frac{5b(c^2x^2-1)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3} - \frac{5b(c^2x^2-1)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5} - \frac{}{3}\right.$

```
[In] int((a+b*arccsc(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arccsc(c*x)+5/96*b*c^5*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))-5/96*b*c^3*(c^2*x^2-1)/((c^2*x
^2-1)/c^2/x^2)^(1/2)/x^3-5/144*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x^5-1/36*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{a + b\csc^{-1}(cx)}{x^7}\,dx = \frac{3\,(5\,bc^6x^6 - 16\,b)\,\text{arccsc}\,(cx) - (15\,bc^4x^4 + 10\,bc^2x^2 + 8\,b)\sqrt{c^2x^2 - 1} - 48\,a}{288\,x^6}$$

```
[In] integrate((a+b*arccsc(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/288*(3*(5*b*c^6*x^6 - 16*b)*arccsc(c*x) - (15*b*c^4*x^4 + 10*b*c^2*x^2 +
8*b)*sqrt(c^2*x^2 - 1) - 48*a)/x^6
```

Sympy [A] (verification not implemented)

Time = 8.63 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.41

$$\int \frac{a + b\csc^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6} - \frac{b\,\text{acsc}\,(cx)}{6x^6} - \frac{b\left(\begin{cases} \frac{5ic^7\,\text{acosh}\,\left(\frac{1}{cx}\right)}{16} - \frac{5ic^6}{16x\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{5ic^4}{48x^3\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{ic^2}{24x^5\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{i}{6x^7\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|} > 1 \\ -\frac{5c^7\,\text{asin}\,\left(\frac{1}{cx}\right)}{16} + \frac{5c^6}{16x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{5c^4}{48x^3\sqrt{1-\frac{1}{c^2x^2}}} - \frac{c^2}{24x^5\sqrt{1-\frac{1}{c^2x^2}}} - \frac{1}{6x^7\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise} \end{cases}\right)}{6c}$$

```
[In] integrate((a+b*acsc(c*x))/x**7,x)
```

```
[Out] -a/(6*x**6) - b*acsc(c*x)/(6*x**6) - b*Piecewise((5*I*c**7*acosh(1/(c*x))/1
6 - 5*I*c**6/(16*x*sqrt(-1 + 1/(c**2*x**2))) + 5*I*c**4/(48*x**3*sqrt(-1 +
1/(c**2*x**2))) + I*c**2/(24*x**5*sqrt(-1 + 1/(c**2*x**2))) + I/(6*x**7*sqr
t(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-5*c**7*asin(1/(c*x))/16 +
5*c**6/(16*x*sqrt(1 - 1/(c**2*x**2))) - 5*c**4/(48*x**3*sqrt(1 - 1/(c**2*x*
*2))) - c**2/(24*x**5*sqrt(1 - 1/(c**2*x**2))) - 1/(6*x**7*sqrt(1 - 1/(c**2
*x**2))), True))/(6*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.63

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx =$$

$$-\frac{1}{288}b\left(\frac{15\,c^7\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)-\frac{15\,c^{12}x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+40\,c^{10}x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\,c^8x\sqrt{-\frac{1}{c^2x^2}+1}}{c^6x^6\left(\frac{1}{c^2x^2}-1\right)^3-3\,c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}}{c}+\frac{48\,\mathrm{arccsc}}{x^6}\right.$$

$$-\frac{a}{6\,x^6}$$

```
[In] integrate((a+b*arccsc(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/288*b*((15*c^7*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) - (15*c^12*x^5*(-1/(c^
2*x^2) + 1)^(5/2) + 40*c^10*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 33*c^8*x*sqrt(-1
/(c^2*x^2) + 1))/(c^6*x^6*(1/(c^2*x^2) - 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) - 1)
^2 + 3*c^2*x^2*(1/(c^2*x^2) - 1) - 1))/c + 48*arccsc(c*x)/x^6) - 1/6*a/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(85) = 170.

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.72

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx =$$

$$-\frac{1}{288}\left(48\,bc^5\left(\frac{1}{c^2x^2}-1\right)^3\arcsin\left(\frac{1}{cx}\right)+144\,bc^5\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)+144\,bc^5\left(\frac{1}{c^2x^2}-1\right)\mathrm{arcsi}\right.$$

```
[In] integrate((a+b*arccsc(c*x))/x^7,x, algorithm="giac")
```

```
[Out] -1/288*(48*b*c^5*(1/(c^2*x^2) - 1)^3*arcsin(1/(c*x)) + 144*b*c^5*(1/(c^2*x^
2) - 1)^2*arcsin(1/(c*x)) + 144*b*c^5*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 3
3*b*c^5*arcsin(1/(c*x)) + 8*b*c^4*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1
)/x - 26*b*c^4*(-1/(c^2*x^2) + 1)^(3/2)/x + 33*b*c^4*sqrt(-1/(c^2*x^2) + 1)
/x + 48*a/(c*x^6))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{cx}\right)}{x^7}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/x^7,x)
[Out] int((a + b*asin(1/(c*x)))/x^7, x)
```

3.15 $\int x^3(a+b\csc^{-1}(cx))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 107

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^2\,dx=\frac{b^2x^2}{12c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))}{3c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(a+b\csc^{-1}(cx))}{6c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{3c^4}$$

```
[Out] 1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arccsc(c*x))^2+1/3*b^2*ln(x)/c^4+1/3*b*x*(a+b
*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/c^3+1/6*b*x^3*(a+b*arccsc(c*x))*(1-1/c^2/
x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5331, 4495, 4270, 4269, 3556}

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^2\,dx=\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{6c}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{3c^3}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{3c^4}+\frac{b^2x^2}{12c^2}$$

```
[In] Int[x^3*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (b^2*x^2)/(12*c^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*ArcCsc[c*x]))/(3*c^3
) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^3*(a + b*ArcCsc[c*x]))/(6*c) + (x^4*(a + b*A
rcCsc[c*x])^2)/4 + (b^2*Log[x])/(3*c^4)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int (a+bx)^2 \cot(x)\csc^4(x)\,dx, x, \csc^{-1}(cx)\right)}{c^4}$$

$$= \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2 - \frac{b\text{Subst}\left(\int (a+bx)\csc^4(x)\,dx, x, \csc^{-1}(cx)\right)}{2c^4}$$

$$= \frac{b^2x^2}{12c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(a+b\csc^{-1}(cx))}{6c} + \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2 - \frac{b\text{Subst}\left(\int (a+bx)\csc^2(x)\,dx, x, \csc^{-1}(cx)\right)}{3c^4}$$

$$= \frac{b^2x^2}{12c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\csc^{-1}(cx))}{3c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\csc^{-1}(cx))}{6c} + \frac{1}{4}x^4(a+b\csc^{-1}(cx))^2 - \frac{b^2\text{Subst}\left(\int \cot(x)\,dx, x, \csc^{-1}(cx)\right)}{3c^4}$$

$$= \frac{b^2x^2}{12c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\csc^{-1}(cx))}{3c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\csc^{-1}(cx))}{6c} + \frac{1}{4}x^4(a+b\csc^{-1}(cx))^2 + \frac{b^2\log(x)}{3c^4}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.16

$$\int x^3(a+b\csc^{-1}(cx))^2\,dx = \frac{cx\left(b^2cx+3a^2c^3x^3+2ab\sqrt{1-\frac{1}{c^2x^2}}(2+c^2x^2)\right)+2bcx\left(3ac^3x^3+b\sqrt{1-\frac{1}{c^2x^2}}(2+c^2x^2)\right)\csc^{-1}(cx)+3b^2c}{12c^4}$$

```
[In] Integrate[x^3*(a + b*ArcCsc[c*x])^2,x]
[Out] (c*x*(b^2*c*x + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))
+ 2*b*c*x*(3*a*c^3*x^3 + b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))*ArcCsc[c*x]
 + 3*b^2*c^4*x^4*ArcCsc[c*x]^2 + 4*b^2*Log[x])/(12*c^4)
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.59

method	result
parts	$\frac{a^2x^4}{4} + \frac{b^2\left(\frac{\text{arccsc}(cx)^2c^4x^4}{4} + \frac{\text{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6} + \frac{c^2x^2}{12} + \frac{\text{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3} - \frac{\ln\left(\frac{1}{cx}\right)}{3}\right)}{c^4} + \frac{2ab\left(\frac{c^4x^4\,\text{arccsc}(c}{4}\right.}{}$
derivativedivides	$\frac{\frac{a^2c^4x^4}{4} + b^2\left(\frac{\text{arccsc}(cx)^2c^4x^4}{4} + \frac{\text{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6} + \frac{c^2x^2}{12} + \frac{\text{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3} - \frac{\ln\left(\frac{1}{cx}\right)}{3}\right) + 2ab\left(\frac{c^4x^4\,\text{arccsc}(cx)}{4} + \right.}{c^4}$
default	$\frac{\frac{a^2c^4x^4}{4} + b^2\left(\frac{\text{arccsc}(cx)^2c^4x^4}{4} + \frac{\text{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6} + \frac{c^2x^2}{12} + \frac{\text{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3} - \frac{\ln\left(\frac{1}{cx}\right)}{3}\right) + 2ab\left(\frac{c^4x^4\,\text{arccsc}(cx)}{4} + \right.}{c^4}$

```
[In] int(x^3*(a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+b^2/c^4*(1/4*arccsc(c*x)^2*c^4*x^4+1/6*arccsc(c*x)*((c^2*x^2-1)
/c^2/x^2)^(1/2)*c^3*x^3+1/12*c^2*x^2+1/3*arccsc(c*x)*c*x*((c^2*x^2-1)/c^2/x
^2)^(1/2)-1/3*ln(1/c/x))+2*a*b/c^4*(1/4*c^4*x^4*arccsc(c*x)+1/12*(c^2*x^2-1
)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.36

$$\int x^3 \left(a + b\csc^{-1}(cx)\right)^2 \, dx = \frac{3\,b^2c^4x^4 \operatorname{arccsc}\left(cx\right)^2 + 3\,a^2c^4x^4 - 12\,abc^4 \arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + b^2c^2x^2 + 4\,b^2 \log\left(x\right) + 6\left(abc^4x^4 - a\right.}{12\,c^4}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4*arccsc(c*x)^2 + 3*a^2*c^4*x^4 - 12*a*b*c^4*arctan(-c*x
 + sqrt(c^2*x^2 - 1)) + b^2*c^2*x^2 + 4*b^2*log(x) + 6*(a*b*c^4*x^4 - a*b*c^
4)*arccsc(c*x) + 2*(a*b*c^2*x^2 + 2*a*b + (b^2*c^2*x^2 + 2*b^2)*arccsc(c*x)
)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [F]

$$\int x^3 \left(a + b\csc^{-1}(cx)\right)^2 \, dx = \int x^3 (a + b\,\mathrm{acsc}\,(cx))^2 \, dx$$

```
[In] integrate(x**3*(a+b*acsc(c*x))**2,x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(93) = 186.

Time = 0.43 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.84

$$\int x^3 \left(a + b\csc^{-1}(cx)\right)^2 \, dx = \frac{1}{4}\,b^2x^4 \operatorname{arccsc}\left(cx\right)^2 + \frac{1}{4}\,a^2x^4 + \frac{1}{6}\left(3\,x^4 \operatorname{arccsc}\left(cx\right) + \frac{c^2x^3\left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2} + 1}}{c^3}\right)ab + \frac{\left(2\,c^4x^4 \arctan\left(1, \sqrt{cx+1}\sqrt{cx-1}\right) + 2\,c^2x^2 \arctan\left(1, \sqrt{cx+1}\sqrt{cx-1}\right) + \left(c^2x^2 + 2\,\log\left(x^2\right)\right)\sqrt{cx+1}\right.}{12\,\sqrt{cx+1}\sqrt{cx-1}c^4}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arccsc(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arccsc(c*x) + (c^2*x^3
*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a*b + 1/12*(2*
c^4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*c^2*x^2*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1)) + (c^2*x^2 + 2*log(x^2))*sqrt(c*x + 1)*sqrt(c*x -
1) - 4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x
 - 1)*c^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(93) = 186$.

Time = 0.43 (sec) , antiderivative size = 811, normalized size of antiderivative = 7.58

$$\int x^3 \left(a + b \csc^{-1}(cx)\right)^2 dx$$
$$= \frac{1}{192} \left(\frac{3 b^2 x^4 \left(\sqrt{-\frac{1}{c^2 x^2} + 1} + 1\right)^4 \arcsin\left(\frac{1}{cx}\right)^2}{c} + \frac{6 abx^4 \left(\sqrt{-\frac{1}{c^2 x^2} + 1} + 1\right)^4 \arcsin\left(\frac{1}{cx}\right)}{c} + \frac{3 a^2 x^4 \left(\sqrt{-\frac{1}{c^2 x^2}\right.}{c}\right.$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*b^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))^2/c + 6*a*b
*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a^2*x^4*(sqrt(-1/
(c^2*x^2) + 1) + 1)^4/c + 4*b^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1
/(c*x))/c^2 + 4*a*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 12*b^2*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))^2/c^3 + 24*a*b*x^2*(sqrt(-1/(c^
2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 12*a^2*x^2*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^2/c^3 + 4*b^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 36*b^2*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^4 + 36*a*b*x*(sqrt(-1/(c^2*x^2) +
 1) + 1)/c^4 + 18*b^2*arcsin(1/(c*x))^2/c^5 + 36*a*b*arcsin(1/(c*x))/c^5 -
128*b^2*log(2)/c^5 + 64*b^2*log(2*sqrt(-1/(c^2*x^2) + 1) + 2)/c^5 - 64*b^2*
log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^5 - 64*b^2*log(1/(abs(c)*abs(x)))/c^5 + 1
8*a^2/c^5 + 8*b^2/c^5 - 36*b^2*arcsin(1/(c*x))/(c^6*x*(sqrt(-1/(c^2*x^2) +
1) + 1)) - 36*a*b/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 12*b^2*arcsin(1/(c
*x))^2/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 24*a*b*arcsin(1/(c*x))/(c
^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 12*a^2/(c^7*x^2*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^2) + 4*b^2/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 4*b^2*arcs
in(1/(c*x))/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 4*a*b/(c^8*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 3*b^2*arcsin(1/(c*x))^2/(c^9*x^4*(sqrt(-1/(c^2
*x^2) + 1) + 1)^4) + 6*a*b*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^4) + 3*a^2/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \csc^{-1}(cx)\right)^2 \, dx = \int x^3 \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

```
[In] int(x^3*(a + b*asin(1/(c*x)))^2,x)
[Out] int(x^3*(a + b*asin(1/(c*x)))^2, x)
```

3.16 $\int x^2(a+b\csc^{-1}(cx))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 139

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\csc^{-1}(cx))}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2b(a+b\csc^{-1}(cx))\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{3c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{3c^3}+\frac{ib^2\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{3c^3}$$

```
[Out] 1/3*b^2*x/c^2+1/3*x^3*(a+b*arccsc(c*x))^2+2/3*b*(a+b*arccsc(c*x))*arctanh(I
/c/x+(1-1/c^2/x^2)^(1/2))/c^3-1/3*I*b^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2
))/c^3+1/3*I*b^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+1/3*b*x^2*(a+b*ar
ccsc(c*x))*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5331, 4495, 4270, 4268, 2317, 2438}

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx=\frac{2b\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{3c^3}+\frac{bx^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2-\frac{ib^2\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{3c^3}+\frac{ib^2\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{3c^3}+\frac{b^2x}{3c^2}$$

```
[In] Int[x^2*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcCsc[c*x]))/(3*c) +
 (x^3*(a + b*ArcCsc[c*x])^2)/3 + (2*b*(a + b*ArcCsc[c*x])*ArcTanh[E^(I*ArcC
sc[c*x])])/(3*c^3) - ((I/3)*b^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + ((I/3
)*b^2*PolyLog[2, E^(I*ArcCsc[c*x])])/c^3
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
```

```
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^2\cot(x)\csc^3(x)\,dx, x, \csc^{-1}(cx)\right)}{c^3}\\
&= \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2 - \frac{(2b)\text{Subst}\left(\int (a+bx)\csc^3(x)\,dx, x, \csc^{-1}(cx)\right)}{3c^3}\\
&= \frac{b^2x}{3c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2\left(a+b\csc^{-1}(cx)\right)}{3c} + \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2\\
&\quad - \frac{b\text{Subst}\left(\int (a+bx)\csc(x)\,dx, x, \csc^{-1}(cx)\right)}{3c^3}\\
&= \frac{b^2x}{3c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2\left(a+b\csc^{-1}(cx)\right)}{3c} + \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2\\
&\quad + \frac{2b\left(a+b\csc^{-1}(cx)\right)\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{3c^3}\\
&\quad + \frac{b^2\text{Subst}\left(\int \log\left(1-e^{ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{3c^3}\\
&\quad - \frac{b^2\text{Subst}\left(\int \log\left(1+e^{ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{3c^3}
\end{aligned}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\csc^{-1}(cx))}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2b(a+b\csc^{-1}(cx))\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{3c^3}-\frac{(ib^2)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{3c^3}+\frac{(ib^2)\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{3c^3}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\csc^{-1}(cx))}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2b(a+b\csc^{-1}(cx))\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{3c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{3c^3}+\frac{ib^2\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{3c^3}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.53

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx$$

$$=\frac{1}{3}\left(a^2x^3+2abx^3\csc^{-1}(cx)+\frac{ab\left(-cx+c^3x^3-\sqrt{-1+c^2x^2}\log\left(-cx+\sqrt{-1+c^2x^2}\right)\right)}{c^4\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{ib^2\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c^3}+\frac{b^2\left(cx+c^3x^3\csc^{-1}(cx)^2+\csc^{-1}(cx)\left(c^2\sqrt{1-\frac{1}{c^2x^2}}x^2-\log\left(1-e^{i\csc^{-1}(cx)}\right)+\log\left(1+e^{i\csc^{-1}(cx)}\right)\right)+i\right.}{c^3}\right.$$

```
[In] Integrate[x^2*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (a^2*x^3 + 2*a*b*x^3*ArcCsc[c*x] + (a*b*(-(c*x) + c^3*x^3 - Sqrt[-1 + c^2*x
^2]*Log[-(c*x) + Sqrt[-1 + c^2*x^2]]))/(c^4*Sqrt[1 - 1/(c^2*x^2)]*x) - (I*b
^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + (b^2*(c*x + c^3*x^3*ArcCsc[c*x]^2
+ ArcCsc[c*x]*(c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2 - Log[1 - E^(I*ArcCsc[c*x])] +
 Log[1 + E^(I*ArcCsc[c*x])]) + I*PolyLog[2, E^(I*ArcCsc[c*x])]))/c^3)/3
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.93

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{\left(c^2x^2\operatorname{arccsc}(cx)^2+\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}-\frac{\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{3}+\frac{i\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{3} \ldots\right)}{c^3}$ …
derivativedivides	$\frac{\frac{c^3x^3a^2}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arccsc}(cx)^2+\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}-\frac{\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{3}+\frac{i\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{3} \ldots\right)}{\ldots}$ …
default	$\frac{\frac{c^3x^3a^2}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arccsc}(cx)^2+\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}-\frac{\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{3}+\frac{i\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{3} \ldots\right)}{\ldots}$ …

```
[In] int(x^2*(a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*x^3+b^2/c^3*(1/3*(c^2*x^2*arccsc(c*x)^2+arccsc(c*x)*c*x*((c^2*x^2-1
)/c^2/x^2)^(1/2)+1)*c*x-1/3*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+1/3
*I*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+1/3*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2
/x^2)^(1/2))-1/3*I*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))+2*a*b/c^3*(1/3*c^
3*x^3*arccsc(c*x)+1/6*(c^2*x^2-1)^(1/2)*(c*x*(c^2*x^2-1)^(1/2)+ln(c*x+(c^2*
x^2-1)^(1/2)))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

Fricas [F]

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx=\int(b\operatorname{arccsc}(cx)+a)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arccsc(c*x)^2 + 2*a*b*x^2*arccsc(c*x) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx=\int x^2(a+b\operatorname{acsc}(cx))^2\,dx$$

```
[In] integrate(x**2*(a+b*acsc(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))**2, x)
```

Maxima [F]

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx=\int\left(b\operatorname{arccsc}\left(cx\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/6*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c
^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^
2*x^2) + 1) - 1)/c^2)/c)*a*b + 1/12*(4*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*
x - 1))^2 - x^3*log(c^2*x^2)^2 - 2*c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x +
 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 + 36*c^2*integrate(1/3*x^4*log(c^2*x
^2)/(c^2*x^2 - 1), x)*log(c) - 72*c^2*integrate(1/3*x^4*log(x)/(c^2*x^2 - 1
), x)*log(c) + 36*c^2*integrate(1/3*x^4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1),
x) - 36*c^2*integrate(1/3*x^4*log(x)^2/(c^2*x^2 - 1), x) + 12*c^2*integrate
(1/3*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x) + 6*(2*x/c^2 - log(c*x + 1)/c^3 + l
og(c*x - 1)/c^3)*log(c)^2 - 36*integrate(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1)
, x)*log(c) + 72*integrate(1/3*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 24*int
egrate(1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x
 - 1)))/(c^2*x^2 - 1), x) - 36*integrate(1/3*x^2*log(c^2*x^2)*log(x)/(c^2*x
^2 - 1), x) + 36*integrate(1/3*x^2*log(x)^2/(c^2*x^2 - 1), x) - 12*integrat
e(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2
```

Giac [F]

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx=\int\left(b\operatorname{arccsc}\left(cx\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx=\int x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

```
[In] int(x^2*(a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int(x^2*(a + b*asin(1/(c*x)))^2, x)
```

3.17 $\int x(a+b\csc^{-1}(cx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 55

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

```
[Out] 1/2*x^2*(a+b*arccsc(c*x))^2+b^2*ln(x)/c^2+b*x*(a+b*arccsc(c*x))*(1-1/c^2/x^
2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5331, 4495, 4269, 3556}

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx=\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

```
[In] Int[x*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*ArcCsc[c*x]))/c + (x^2*(a + b*ArcCsc[c*x]
)^2)/2 + (b^2*Log[x])/c^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^2\cot(x)\csc^2(x)\,dx,x,\csc^{-1}(cx)\right)}{c^2}\\
&= \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2-\frac{b\text{Subst}\left(\int (a+bx)\csc^2(x)\,dx,x,\csc^{-1}(cx)\right)}{c^2}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\csc^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2-\frac{b^2\text{Subst}\left(\int\cot(x)\,dx,x,\csc^{-1}(cx)\right)}{c^2}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\csc^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx = \frac{acx\left(2b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)+2bcx\left(b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)\csc^{-1}(cx)+b^2c^2x^2\csc^{-1}(cx)^2+2b^2\log(cx)}{2c^2}$$

```
[In] Integrate[x*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (a*c*x*(2*b*Sqrt[1 - 1/(c^2*x^2)] + a*c*x) + 2*b*c*x*(b*Sqrt[1 - 1/(c^2*x^2
)] + a*c*x)*ArcCsc[c*x] + b^2*c^2*x^2*ArcCsc[c*x]^2 + 2*b^2*Log[c*x])/(2*c^
2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(51) = 102$.

Time = 1.01 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.22

method	result	size
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\frac{c^2x^2\operatorname{arccsc}(cx)^2}{2}+\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}-\ln\left(\frac{1}{cx}\right)\right)}{c^2}+\frac{2ab\left(\frac{c^2x^2\operatorname{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	122
derivativedivides	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2\operatorname{arccsc}(cx)^2}{2}+\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}-\ln\left(\frac{1}{cx}\right)\right)+2ab\left(\frac{c^2x^2\operatorname{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	123
default	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2\operatorname{arccsc}(cx)^2}{2}+\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}-\ln\left(\frac{1}{cx}\right)\right)+2ab\left(\frac{c^2x^2\operatorname{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	123

```
[In] int(x*(a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^2+b^2/c^2*(1/2*c^2*x^2*arccsc(c*x)^2+arccsc(c*x)*c*x*((c^2*x^2-1)
/c^2/x^2)^(1/2)-ln(1/c/x))+2*a*b/c^2*(1/2*c^2*x^2*arccsc(c*x)+1/2/((c^2*x^2
-1)/c^2/x^2)^(1/2)/c/x*(c^2*x^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.02

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx$$
$$=\frac{b^2c^2x^2\operatorname{arccsc}(cx)^2+a^2c^2x^2-4\,abc^2\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2\,b^2\log(x)+2\,(abc^2x^2-abc^2)\operatorname{arccsc}(}{2\,c^2}$$

```
[In] integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*arccsc(c*x)^2 + a^2*c^2*x^2 - 4*a*b*c^2*arctan(-c*x + sqrt
(c^2*x^2 - 1)) + 2*b^2*log(x) + 2*(a*b*c^2*x^2 - a*b*c^2)*arccsc(c*x) + 2*s
qrt(c^2*x^2 - 1)*(b^2*arccsc(c*x) + a*b))/c^2
```

Sympy [F]

$$\int x(a+b\csc^{-1}(cx))^2\,dx=\int x(a+b\,\mathrm{acsc}\,(cx))^2\,dx$$

```
[In] integrate(x*(a+b*acsc(c*x))**2,x)
[Out] Integral(x*(a + b*acsc(c*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

$$\int x(a+b\csc^{-1}(cx))^2\,dx=\frac{1}{2}b^2x^2\,\mathrm{arccsc}\,(cx)^2+\frac{1}{2}a^2x^2+\left(x^2\,\mathrm{arccsc}\,(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)ab+\left(\frac{x\sqrt{-\frac{1}{c^2x^2}+1}\,\mathrm{arccsc}\,(cx)}{c}+\frac{\log(x)}{c^2}\right)b^2$$

```
[In] integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
[Out] 1/2*b^2*x^2*arccsc(c*x)^2 + 1/2*a^2*x^2 + (x^2*arccsc(c*x) + x*sqrt(-1/(c^2
*x^2) + 1)/c)*a*b + (x*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x)/c + log(x)/c^2)*b
^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(51) = 102.

Time = 0.35 (sec) , antiderivative size = 427, normalized size of antiderivative = 7.76

$$\int x(a+b\csc^{-1}(cx))^2\,dx=\frac{1}{8}\left(\frac{b^2x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)^2}{c}+\frac{2\,abx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{a^2x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+\right.}{c}\right.$$

```
[In] integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/8*(b^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))^2/c + 2*a*b*x^2
*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + a^2*x^2*(sqrt(-1/(c^2*x
^2) + 1) + 1)^2/c + 4*b^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^
2 + 4*a*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 2*b^2*arcsin(1/(c*x))^2/c^3
+ 4*a*b*arcsin(1/(c*x))/c^3 - 16*b^2*log(2)/c^3 + 8*b^2*log(2*sqrt(-1/(c^2*
x^2) + 1) + 2)/c^3 - 8*b^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 - 8*b^2*log(
1/(abs(c)*abs(x)))/c^3 + 2*a^2/c^3 - 4*b^2*arcsin(1/(c*x))/(c^4*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) - 4*a*b/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + b^2*arc
sin(1/(c*x))^2/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 2*a*b*arcsin(1/(c
*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a^2/(c^5*x^2*(sqrt(-1/(c^2*
x^2) + 1) + 1)^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx=\int x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

```
[In] int(x*(a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int(x*(a + b*asin(1/(c*x)))^2, x)
```

3.18 $\int (a+b\csc^{-1}(cx))^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 84

$$\int (a+b\csc^{-1}(cx))^2\,dx = x(a+b\csc^{-1}(cx))^2+\frac{4b(a+b\csc^{-1}(cx))\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{2ib^2\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c}+\frac{2ib^2\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a+b*arccsc(c*x))^2+4*b*(a+b*arccsc(c*x))*arctanh(I/c/x+(1-1/c^2/x^2)^(1/
2))/c-2*I*b^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c+2*I*b^2*polylog(2,I/c
/x+(1-1/c^2/x^2)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5325, 4495, 4268, 2317, 2438}

$$\int (a+b\csc^{-1}(cx))^2\,dx = \frac{4b\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{c}+x(a+b\csc^{-1}(cx))^2 - \frac{2ib^2\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c}+\frac{2ib^2\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c}$$

```
[In] Int[(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] x*(a + b*ArcCsc[c*x])^2 + (4*b*(a + b*ArcCsc[c*x])*ArcTanh[E^(I*ArcCsc[c*x]
)])/c - ((2*I)*b^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c + ((2*I)*b^2*PolyLog[2
, E^(I*ArcCsc[c*x])])/c
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5325

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Sub
st[Int[(a + b*x)^n*Csc[x]*Cot[x], x], x, ArcCsc[c*x]], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^2\cot(x)\csc(x)\,dx, x, \csc^{-1}(cx)\right)}{c}\\
&= x\left(a+b\csc^{-1}(cx)\right)^2 - \frac{(2b)\text{Subst}\left(\int (a+bx)\csc(x)\,dx, x, \csc^{-1}(cx)\right)}{c}\\
&= x\left(a+b\csc^{-1}(cx)\right)^2 + \frac{4b\left(a+b\csc^{-1}(cx)\right)\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c}\\
&\quad + \frac{(2b^2)\,\text{Subst}\left(\int \log\left(1-e^{ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{c}\\
&\quad - \frac{(2b^2)\,\text{Subst}\left(\int \log\left(1+e^{ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{c}
\end{aligned}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^2 + \frac{4b(a+b\csc^{-1}(cx))\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{(2ib^2)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{c} + \frac{(2ib^2)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{c}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^2 + \frac{4b(a+b\csc^{-1}(cx))\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{2ib^2\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c} + \frac{2ib^2\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.75

$$\int \left(a+b\csc^{-1}(cx)\right)^2\,dx$$

$$= \frac{a^2cx + 2abcx\csc^{-1}(cx) + b^2cx\csc^{-1}(cx)^2 - 2b^2\csc^{-1}(cx)\log\left(1-e^{i\csc^{-1}(cx)}\right) + 2b^2\csc^{-1}(cx)\log\left(1+e^{i\csc ...}\right.}{}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (a^2*c*x + 2*a*b*c*x*ArcCsc[c*x] + b^2*c*x*ArcCsc[c*x]^2 - 2*b^2*ArcCsc[c*x
]*Log[1 - E^(I*ArcCsc[c*x])] + 2*b^2*ArcCsc[c*x]*Log[1 + E^(I*ArcCsc[c*x])]
 + 2*a*b*Log[Cos[ArcCsc[c*x]/2]] - 2*a*b*Log[Sin[ArcCsc[c*x]/2]] - (2*I)*b^
2*PolyLog[2, -E^(I*ArcCsc[c*x])] + (2*I)*b^2*PolyLog[2, E^(I*ArcCsc[c*x])])
/c
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.10

method	result
derivativedivides	$\frac{a^2cx+b^2\left(\operatorname{arccsc}(cx)^2cx-2\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+2\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-2i\operatorname{dilog}\left(1+\frac{i}{cx}+\sqrt{1-\cdots}\right.\right.}{c}$
default	$\frac{a^2cx+b^2\left(\operatorname{arccsc}(cx)^2cx-2\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+2\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-2i\operatorname{dilog}\left(1+\frac{i}{cx}+\sqrt{1-\cdots}\right.\right.}{c}$
parts	$a^2x+\frac{b^2\left(\operatorname{arccsc}(cx)^2cx-2\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+2\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-2i\operatorname{dilog}\left(1+\frac{i}{cx}+\sqrt{\cdots}\right.\right.}{c}$

```
[In] int((a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2*c*x+b^2*(arccsc(c*x)^2*c*x-2*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^
(1/2))+2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-2*I*dilog(1+I/c/x+(1-1
/c^2/x^2)^(1/2))+2*I*dilog(1-I/c/x-(1-1/c^2/x^2)^(1/2)))+2*a*b*(arccsc(c*x)
*c*x+ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))))
```

Fricas [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^2\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2, x)
```

Sympy [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^2\,dx=\int \left(a+b\,\mathrm{acsc}\,(cx)\right)^2\,dx$$

```
[In] integrate((a+b*acsc(c*x))**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2, x)
```

Maxima [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^2\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 4*c^
2*integrate(x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 8*c^2*integrate(x^2
*log(x)/(c^2*x^2 - 1), x)*log(c) - 4*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1))^2 - 4*c^2*integrate(x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 4*c^2*i
ntegrate(x^2*log(x)^2/(c^2*x^2 - 1), x) - 4*c^2*integrate(x^2*log(c^2*x^2)/
(c^2*x^2 - 1), x) + x*log(c^2*x^2)^2 + 2*(log(c*x + 1)/c - log(c*x - 1)/c)*
log(c)^2 + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 8*integrate(
log(x)/(c^2*x^2 - 1), x)*log(c) - 8*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*a
rctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x) + 4*integrate(log(
c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 4*integrate(log(x)^2/(c^2*x^2 - 1), x)
+ 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2 + a^2*x + (2*c*x*arccsc(c
*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*a
*b/c
```

Giac [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^2\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arccsc(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\csc^{-1}(cx)\right)^2\,dx=\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

```
[In] int((a + b*asin(1/(c*x)))^2,x)
[Out] int((a + b*asin(1/(c*x)))^2, x)
```

3.19 $\int \frac{(a+b\csc^{-1}(cx))^2}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\begin{aligned}\int \frac{(a+b\csc^{-1}(cx))^2}{x}\,dx &= \frac{i(a+b\csc^{-1}(cx))^3}{3b} - (a+b\csc^{-1}(cx))^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\ &\quad + ib(a+b\csc^{-1}(cx))\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\\ &\quad - \frac{1}{2}b^2\operatorname{PolyLog}\left(3,e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

```
[Out] 1/3*I*(a+b*arccsc(c*x))^3/b-(a+b*arccsc(c*x))^2*ln(1-(I/c/x+(1-1/c^2/x^2)^(
1/2))^2)+I*b*(a+b*arccsc(c*x))*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-1/2
*b^2*polylog(3,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5331, 3798, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{(a+b\csc^{-1}(cx))^2}{x}\,dx &= ib\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))\\ &\quad + \frac{i(a+b\csc^{-1}(cx))^3}{3b} - \log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))^2\\ &\quad - \frac{1}{2}b^2\operatorname{PolyLog}\left(3,e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

```
[In] Int[(a + b*ArcCsc[c*x])^2/x,x]
```

```
[Out] ((I/3)*(a + b*ArcCsc[c*x])^3)/b - (a + b*ArcCsc[c*x])^2*Log[1 - E^((2*I)*Ar
cCsc[c*x])] + I*b*(a + b*ArcCsc[c*x])*PolyLog[2, E^((2*I)*ArcCsc[c*x])] - (
b^2*PolyLog[3, E^((2*I)*ArcCsc[c*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int (a+bx)^2\cot(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} + 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^2}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\
&\quad + (2b)\text{Subst}\left(\int (a+bx)\log\left(1-e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\
&\quad + ib\left(a+b\csc^{-1}(cx)\right)\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\\
&\quad - \left(ib^2\right)\text{Subst}\left(\int \text{PolyLog}\left(2, e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\
&\quad + ib\left(a+b\csc^{-1}(cx)\right)\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\\
&\quad - \frac{1}{2}b^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\
&\quad + ib\left(a+b\csc^{-1}(cx)\right)\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right) - \frac{1}{2}b^2\,\text{PolyLog}\left(3, e^{2i\csc^{-1}(cx)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.51

$$\begin{aligned}
\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}\,dx &= -2ab\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + a^2\log(cx)\\
&\quad + iab\left(\csc^{-1}(cx)^2 + \text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\right)\\
&\quad + \frac{1}{24}ib^2\left(\pi^3 - 8\csc^{-1}(cx)^3 + 24i\csc^{-1}(cx)^2\log\left(1-e^{-2i\csc^{-1}(cx)}\right)\right.\\
&\qquad - 24\csc^{-1}(cx)\,\text{PolyLog}\left(2, e^{-2i\csc^{-1}(cx)}\right)\\
&\qquad \left. + 12i\,\text{PolyLog}\left(3, e^{-2i\csc^{-1}(cx)}\right)\right)
\end{aligned}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x,x]
```

```
[Out] -2*a*b*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + a^2*Log[c*x] + I*a*b*(A
rcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]) + (I/24)*b^2*(Pi^3 - 8*Ar
cCsc[c*x]^3 + (24*I)*ArcCsc[c*x]^2*Log[1 - E^((-2*I)*ArcCsc[c*x])] - 24*Arc
Csc[c*x]*PolyLog[2, E^((-2*I)*ArcCsc[c*x])] + (12*I)*PolyLog[3, E^((-2*I)*A
rcCsc[c*x])])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(127) = 254$.

Time = 1.09 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.71

method	result
parts	$a^2 \ln(x) + b^2 \left(\frac{i \operatorname{arccsc}(cx)^3}{3} - \operatorname{arccsc}(cx)^2 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 2i \operatorname{arccsc}(cx) \operatorname{polylog}\right.$
derivativedivides	$a^2 \ln(cx) + b^2 \left(\frac{i \operatorname{arccsc}(cx)^3}{3} - \operatorname{arccsc}(cx)^2 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 2i \operatorname{arccsc}(cx) \operatorname{polylo}\right.$
default	$a^2 \ln(cx) + b^2 \left(\frac{i \operatorname{arccsc}(cx)^3}{3} - \operatorname{arccsc}(cx)^2 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 2i \operatorname{arccsc}(cx) \operatorname{polylo}\right.$

```
[In] int((a+b*arccsc(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*ln(x)+b^2*(1/3*I*arccsc(c*x)^3-arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2/x^2)^(
1/2))+2*I*arccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-2*polylog(3,I/c/
x+(1-1/c^2/x^2)^(1/2))-arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+2*I*ar
ccsc(c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-2*polylog(3,-I/c/x-(1-1/c^2
/x^2)^(1/2)))+2*a*b*(1/2*I*arccsc(c*x)^2-arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^
2)^(1/2))+I*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-arccsc(c*x)*ln(1+I/c/x+(1-
1/c^2/x^2)^(1/2))+I*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{(a + b \csc^{-1}(cx))^2}{x} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)^2}{x} \, dx$$

```
[In] integrate((a+b*arccsc(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x}\,dx = \int \frac{(a + b\,\mathrm{acsc}\,(cx))^2}{x}\,dx$$

[In] integrate((a+b*acsc(c*x))**2/x,x)

[Out] Integral((a + b*acsc(c*x))**2/x, x)

Maxima [F]

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)^2}{x}\,dx$$

[In] integrate((a+b*arccsc(c*x))^2/x,x, algorithm="maxima")

[Out] -1/2*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 + b^2*c^2*integrate(x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 2*b^2*c^2*integrate(x^2*log(x)/(c^2*x^3 - x), x)*log(c) + 2*b^2*c^2*integrate(x^2*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) - b^2*c^2*integrate(x^2*log(x)^2/(c^2*x^3 - x), x) + 2*a*b*c^2*integrate(x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^3 - x), x) + 1/2*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 + b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2*log(x) - 1/4*b^2*log(c^2*x^2)^2*log(x) - b^2*integrate(log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 2*b^2*integrate(log(x)/(c^2*x^3 - x), x)*log(c) + 2*b^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x) - 2*b^2*integrate(log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + b^2*integrate(log(x)^2/(c^2*x^3 - x), x) - 2*a*b*integrate(arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^3 - x), x) + a^2*log(x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate((a+b*arccsc(c*x))^2/x,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:INPUT:sage2OUTPUT:ln of unsigned or minus infinity Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))^2/x,x)
[Out] int((a + b*asin(1/(c*x)))^2/x, x)
```

3.20 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx = \frac{2b^2}{x} - 2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right) - \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}$$

```
[Out] 2*b^2/x-(a+b*arccsc(c*x))^2/x-2*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5331, 3377, 2717}

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx = -2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right) - \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x} + \frac{2b^2}{x}$$

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^2,x]
```

```
[Out] (2*b^2)/x - 2*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]) - (a + b*ArcCsc
[c*x])^2/x
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int (a+bx)^2\cos(x)\,dx, x, \csc^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\csc^{-1}(cx)\right)^2}{x} + (2bc)\text{Subst}\left(\int (a+bx)\sin(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= -2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right) - \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}\\
&\quad + \left(2b^2c\right)\text{Subst}\left(\int \cos(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{2b^2}{x} - 2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right) - \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{a^2-2b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x+2b\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}x\right)\csc^{-1}(cx)+b^2\csc^{-1}(cx)^2}{x}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^2,x]
[Out] -((a^2 - 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 2*b*(a + b*c*Sqrt[1 - 1/
(c^2*x^2)]*x)*ArcCsc[c*x] + b^2*ArcCsc[c*x]^2)/x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(48) = 96$.

Time = 0.76 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.30

method	result
parts	$-\frac{a^2}{x}+b^2c\left(-\frac{\operatorname{arccsc}(cx)^2}{cx}+\frac{2}{cx}-2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2abc\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right.$
derivativedivides	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{cx}+\frac{2}{cx}-2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2ab\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x}\right.\right.$
default	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{cx}+\frac{2}{cx}-2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2ab\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x}\right.\right.$

```
[In] int((a+b*arccsc(c*x))^2/x^2,x,method=_RETURNVERBOSE)
[Out] -a^2/x+b^2*c*(-1/c/x*arccsc(c*x)^2+2/c/x-2*arccsc(c*x)*((c^2*x^2-1)/c^2/x^2
)^(1/2))+2*a*b*c*(-1/c/x*arccsc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*
(c^2*x^2-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx = -\frac{b^2\operatorname{arccsc}(cx)^2+2\,ab\operatorname{arccsc}(cx)+a^2-2\,b^2+2\,\sqrt{c^2x^2-1}\left(b^2\operatorname{arccsc}(cx)+ab\right)}{x}$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="fricas")
[Out] -(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2 - 2*b^2 + 2*sqrt(c^2*x^2 - 1)
*(b^2*arccsc(c*x) + a*b))/x
```

Sympy [F]

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{\left(a+b\operatorname{acsc}(cx)\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*acsc(c*x))**2/x**2,x)
[Out] Integral((a + b*acsc(c*x))**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^2}\,dx = -2\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)ab - 2\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}(cx)-\frac{1}{x}\right)b^2 - \frac{b^2\operatorname{arccsc}(cx)^2}{x} - \frac{a^2}{x}$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="maxima")
[Out] -2*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*a*b - 2*(c*sqrt(-1/(c^2*x^2)
+ 1)*arccsc(c*x) - 1/x)*b^2 - b^2*arccsc(c*x)^2/x - a^2/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(48) = 96.

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.08

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^2}\,dx = -\left(2\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)+2\,ab\sqrt{-\frac{1}{c^2x^2}+1}+\frac{b^2\arcsin\left(\frac{1}{cx}\right)^2}{cx}+\frac{2\,ab\arcsin\left(\frac{1}{cx}\right)}{cx}+\frac{a^2}{cx}-\frac{2\,b^2}{cx}\right)c$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="giac")
[Out] -(2*b^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) + 2*a*b*sqrt(-1/(c^2*x^2) +
1) + b^2*arcsin(1/(c*x))^2/(c*x) + 2*a*b*arcsin(1/(c*x))/(c*x) + a^2/(c*x)
- 2*b^2/(c*x))*c
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.76

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^2}\,dx = -\frac{a^2}{x} - \frac{b^2\left(\operatorname{asin}\left(\frac{1}{c\,x}\right)^2-2\right)}{x} - 2\,b^2\,c\operatorname{asin}\left(\frac{1}{c\,x}\right)\sqrt{1-\frac{1}{c^2\,x^2}} - 2\,a\,b\,c\left(\sqrt{1-\frac{1}{c^2\,x^2}}+\frac{\operatorname{asin}\left(\frac{1}{c\,x}\right)}{c\,x}\right)$$

```
[In] int((a + b*asin(1/(c*x)))^2/x^2,x)
[Out] - a^2/x - (b^2*(asin(1/(c*x))^2 - 2))/x - 2*b^2*c*asin(1/(c*x))*(1 - 1/(c^2
*x^2))^(1/2) - 2*a*b*c*((1 - 1/(c^2*x^2))^(1/2) + asin(1/(c*x))/(c*x))
```

3.21 $\int \frac{(a+b\csc^{-1}(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^3}\,dx = \frac{b^2}{4x^2} + \frac{1}{2}abc^2\csc^{-1}(cx) + \frac{1}{4}b^2c^2\csc^{-1}(cx)^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{2x} - \frac{(a+b\csc^{-1}(cx))^2}{2x^2}$$

```
[Out] 1/4*b^2/x^2+1/2*a*b*c^2*arccsc(c*x)+1/4*b^2*c^2*arccsc(c*x)^2-1/2*(a+b*arcc
sc(c*x))^2/x^2-1/2*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5331, 4489, 3391}

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^3}\,dx = -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{2x} + \frac{1}{2}abc^2\csc^{-1}(cx) - \frac{(a+b\csc^{-1}(cx))^2}{2x^2} + \frac{1}{4}b^2c^2\csc^{-1}(cx)^2 + \frac{b^2}{4x^2}$$

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^3,x]
```

```
[Out] b^2/(4*x^2) + (a*b*c^2*ArcCsc[c*x])/2 + (b^2*c^2*ArcCsc[c*x]^2)/4 - (b*c*Sq
rt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(2*x) - (a + b*ArcCsc[c*x])^2/(2*x
^2)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^2\text{Subst}\left(\int (a+bx)^2\cos(x)\sin(x)\,dx, x, \csc^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\csc^{-1}(cx)\right)^2}{2x^2}+\left(bc^2\right)\text{Subst}\left(\int (a+bx)\sin^2(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{b^2}{4x^2}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{2x}-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{2x^2}\\
&\quad+\frac{1}{2}\left(bc^2\right)\text{Subst}\left(\int (a+bx)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{b^2}{4x^2}+\frac{1}{2}abc^2\csc^{-1}(cx)+\frac{1}{4}b^2c^2\csc^{-1}(cx)^2\\
&\quad-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{2x}-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^3}\,dx$$
$$= \frac{-2a^2 + b^2 - 2abc\sqrt{1-\frac{1}{c^2x^2}}x - 2b\left(2a + bc\sqrt{1-\frac{1}{c^2x^2}}x\right)\csc^{-1}(cx) + b^2(-2 + c^2x^2)\csc^{-1}(cx)^2 + 2abc^2x^2}{4x^2}$$

[In] Integrate[(a + b*ArcCsc[c*x])^2/x^3,x]

[Out] (-2*a^2 + b^2 - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 2*b*(2*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[c*x] + b^2*(-2 + c^2*x^2)*ArcCsc[c*x]^2 + 2*a*b*c^2*x^2*ArcSin[1/(c*x)])/(4*x^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(76) = 152.

Time = 0.59 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.09

method	result
parts	$-\frac{a^2}{2x^2} + b^2c^2\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^2}{2c^2x^2} - \frac{\operatorname{arccsc}(cx)\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx} + \frac{\operatorname{arccsc}(cx)^2}{4} + \frac{1}{4c^2x^2}\right) + 2$
derivativedivides	$c^2\left(-\frac{a^2}{2c^2x^2} + b^2\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^2}{2c^2x^2} - \frac{\operatorname{arccsc}(cx)\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx} + \frac{\operatorname{arccsc}(cx)^2}{4} + \frac{1}{4c^2x^2}\right)\right.$
default	$c^2\left(-\frac{a^2}{2c^2x^2} + b^2\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^2}{2c^2x^2} - \frac{\operatorname{arccsc}(cx)\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx} + \frac{\operatorname{arccsc}(cx)^2}{4} + \frac{1}{4c^2x^2}\right)\right.$

[In] int((a+b*arccsc(c*x))^2/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*a^2/x^2+b^2*c^2*(1/2*(c^2*x^2-1)/c^2/x^2*arccsc(c*x)^2-1/2*arccsc(c*x)*(arccsc(c*x)*c*x+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x+1/4*arccsc(c*x)^2+1/4/c^2/x^2)+2*a*b*c^2*(-1/2/c^2/x^2*arccsc(c*x)-1/4*(c^2*x^2-1)^(1/2)*(-arctan(1/(c^2*x^2-1)^(1/2))*c^2*x^2+(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^3/x^3)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^3}\,dx = \frac{(b^2c^2x^2-2\,b^2)\operatorname{arccsc}(cx)^2-2\,a^2+b^2+2\,(abc^2x^2-2\,ab)\operatorname{arccsc}(cx)-2\,\sqrt{c^2x^2-1}(b^2\operatorname{arccsc}(cx)+ab)}{4\,x^2}$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="fricas")
[Out] 1/4*((b^2*c^2*x^2 - 2*b^2)*arccsc(c*x)^2 - 2*a^2 + b^2 + 2*(a*b*c^2*x^2 - 2
*a*b)*arccsc(c*x) - 2*sqrt(c^2*x^2 - 1)*(b^2*arccsc(c*x) + a*b))/x^2
```

Sympy [F]

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^3}\,dx = \int \frac{(a+b\operatorname{acsc}(cx))^2}{x^3}\,dx$$

```
[In] integrate((a+b*acsc(c*x))**2/x**3,x)
[Out] Integral((a + b*acsc(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^3}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="maxima")
[Out] 1/2*a*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^
3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/8*(4*(c^2*
(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 - 4*c^2*integrate(1/2*x^2
*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 8*c^2*integrate(1/2*x^2*log(x)/(
c^2*x^5 - x^3), x)*log(c) - 4*c^2*integrate(1/2*x^2*log(c^2*x^2)*log(x)/(c^
2*x^5 - x^3), x) + 4*c^2*integrate(1/2*x^2*log(x)^2/(c^2*x^5 - x^3), x) + 2
*c^2*integrate(1/2*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x) - (c^2*log(c*x + 1)
 + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*log(c)^2 + 4*integrate(1/2*log(
c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 8*integrate(1/2*log(x)/(c^2*x^5 - x^3
), x)*log(c) + 4*integrate(1/2*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c
*x + 1)*sqrt(c*x - 1)))/(c^2*x^5 - x^3), x) + 4*integrate(1/2*log(c^2*x^2)*
log(x)/(c^2*x^5 - x^3), x) - 4*integrate(1/2*log(x)^2/(c^2*x^5 - x^3), x) -
 2*integrate(1/2*log(c^2*x^2)/(c^2*x^5 - x^3), x))*x^2 + 4*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1))^2 - log(c^2*x^2)^2)*b^2/x^2 - 1/2*a^2/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(76) = 152$.

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.85

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^3}\,dx =$$

$$-\frac{1}{8}\left(4\,b^2c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^2+8\,abc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)+2\,b^2c\arcsin\left(\frac{1}{cx}\right)^2+4\,a^2c\left(\frac{1}{c^2x^2}\right.\right.$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^2 + 8*a*b*c*(1/(c^2*x^2) -
1)*arcsin(1/(c*x)) + 2*b^2*c*arcsin(1/(c*x))^2 + 4*a^2*c*(1/(c^2*x^2) - 1)
- 2*b^2*c*(1/(c^2*x^2) - 1) + 4*a*b*c*arcsin(1/(c*x)) - b^2*c + 4*b^2*sqrt(
-1/(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 4*a*b*sqrt(-1/(c^2*x^2) + 1)/x)*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^3}\,dx = \int \frac{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x^3}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))^2/x^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2/x^3, x)
```

3.22 $\int \frac{(a+b\csc^{-1}(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^4}\,dx = \frac{2b^2}{27x^3}+\frac{4b^2c^2}{9x}-\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx)) - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{9x^2}-\frac{(a+b\csc^{-1}(cx))^2}{3x^3}$$

```
[Out] 2/27*b^2/x^3+4/9*b^2*c^2/x-1/3*(a+b*arccsc(c*x))^2/x^3-4/9*b*c^3*(a+b*arccs
c(c*x))*(1-1/c^2/x^2)^(1/2)-2/9*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/x
^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5331, 4489, 3391, 3377, 2717}

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^4}\,dx = -\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{9x^2} - \frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx)) - \frac{(a+b\csc^{-1}(cx))^2}{3x^3}+\frac{4b^2c^2}{9x}+\frac{2b^2}{27x^3}$$

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^4,x]
```

```
[Out] (2*b^2)/(27*x^3) + (4*b^2*c^2)/(9*x) - (4*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a +
b*ArcCsc[c*x]))/9 - (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(9*x^
2) - (a + b*ArcCsc[c*x])^2/(3*x^3)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(c^3\text{Subst}\left(\int (a+bx)^2\cos(x)\sin^2(x)\,dx,x,\csc^{-1}(cx)\right)\right)\\ &= -\frac{\left(a+b\csc^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(2bc^3\right)\text{Subst}\left(\int (a+bx)\sin^3(x)\,dx,x,\csc^{-1}(cx)\right)\end{aligned}$$

$$= \frac{2b^2}{27x^3} - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{9x^2} - \frac{(a+b\csc^{-1}(cx))^2}{3x^3} + \frac{1}{9}(4bc^3)\,\text{Subst}\left(\int (a+bx)\sin(x)\,dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{2b^2}{27x^3} - \frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx)) - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{9x^2} - \frac{(a+b\csc^{-1}(cx))^2}{3x^3} + \frac{1}{9}(4b^2c^3)\,\text{Subst}\left(\int \cos(x)\,dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{2b^2}{27x^3} + \frac{4b^2c^2}{9x} - \frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx)) - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{9x^2} - \frac{(a+b\csc^{-1}(cx))^2}{3x^3}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^4}\,dx = -\frac{9a^2 + 6abc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2) - 2b^2(1+6c^2x^2) + 6b\left(3a+bc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2)\right)\csc^{-1}(cx) + 9b}{27x^3}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^4,x]
[Out] -1/27*(9*a^2 + 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) - 2*b^2*(1 +
 6*c^2*x^2) + 6*b*(3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*ArcCs
c[c*x] + 9*b^2*ArcCsc[c*x]^2)/x^3
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.50

method	result
parts	$-\frac{a^2}{3x^3} + b^2c^3\left(-\frac{\operatorname{arccsc}(cx)^2}{3c^3x^3} - \frac{2\operatorname{arccsc}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2} + \frac{2}{27c^3x^3} + \frac{4}{9cx}\right) + 2ab\,c^3\left(-\frac{\operatorname{arccsc}(cx)}{3c^3x^3}\right.$
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3} + b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{3c^3x^3} - \frac{2\operatorname{arccsc}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2} + \frac{2}{27c^3x^3} + \frac{4}{9cx}\right) + 2ab\left(-\frac{\operatorname{arccsc}(cx)}{3c^3x^3}\right.\right.$
default	$c^3\left(-\frac{a^2}{3c^3x^3} + b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{3c^3x^3} - \frac{2\operatorname{arccsc}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2} + \frac{2}{27c^3x^3} + \frac{4}{9cx}\right) + 2ab\left(-\frac{\operatorname{arccsc}(cx)}{3c^3x^3}\right.\right.$

```
[In] int((a+b*arccsc(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^2/x^3+b^2*c^3*(-1/3/c^3/x^3*arccsc(c*x)^2-2/9*arccsc(c*x)*(2*c^2*x^2
+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/x)+2*a*b*c^3*(-1
/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/c^4/x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^4}\,dx$$
$$= \frac{12\,b^2c^2x^2 - 9\,b^2\,\mathrm{arccsc}\left(cx\right)^2 - 18\,ab\,\mathrm{arccsc}\left(cx\right) - 9\,a^2 + 2\,b^2 - 6\left(2\,abc^2x^2 + ab + \left(2\,b^2c^2x^2 + b^2\right)\mathrm{arccsc}\left(c\right.\right.}{27\,x^3}$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(12*b^2*c^2*x^2 - 9*b^2*arccsc(c*x)^2 - 18*a*b*arccsc(c*x) - 9*a^2 + 2
*b^2 - 6*(2*a*b*c^2*x^2 + a*b + (2*b^2*c^2*x^2 + b^2)*arccsc(c*x))*sqrt(c^2
*x^2 - 1))/x^3
```

Sympy [F]

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^4}\,dx = \int \frac{\left(a+b\,\mathrm{acsc}\left(cx\right)\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*acsc(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(88) = 176$.

Time = 0.46 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.93

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^4}\,dx$$
$$= \frac{2}{9}\,ab\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{3\,\mathrm{arccsc}\left(cx\right)}{x^3}\right) - \frac{b^2\,\mathrm{arccsc}\left(cx\right)^2}{3\,x^3} - \frac{a^2}{3\,x^3}$$
$$- \frac{2\left(6\,c^5x^4\arctan\left(1, \sqrt{cx+1}\sqrt{cx-1}\right) - 3\,c^3x^2\arctan\left(1, \sqrt{cx+1}\sqrt{cx-1}\right) - \left(6\,c^3x^2+c\right)\sqrt{cx+1}\sqrt{cx\right.}{27\,\sqrt{cx+1}\sqrt{cx-1}cx^3}$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*a*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c -
3*arccsc(c*x)/x^3) - 1/3*b^2*arccsc(c*x)^2/x^3 - 1/3*a^2/x^3 - 2/27*(6*c^5*
x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*c^3*x^2*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)) - (6*c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c*x - 1) - 3*c*ar
ctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c*x
^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(88) = 176$.

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.20

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^4}\,dx$$
$$=\frac{1}{27}\left(6\,b^2c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)+6\,abc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-18\,b^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)-\frac{9\,b^2}{}\right.$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] 1/27*(6*b^2*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x)) + 6*a*b*c^2*(-1/(c
^2*x^2) + 1)^(3/2) - 18*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) - 9*
b^2*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^2/x - 18*a*b*c^2*sqrt(-1/(c^2*x^2)
+ 1) - 18*a*b*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 9*b^2*c*arcsin(1/(c*x
))^2/x + 2*b^2*c*(1/(c^2*x^2) - 1)/x - 18*a*b*c*arcsin(1/(c*x))/x + 14*b^2*
c/x - 9*a^2/(c*x^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^4}\,dx=\int \frac{{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}^2}{x^4}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))^2/x^4,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2/x^4, x)
```

3.23 $\int \frac{(a+b\csc^{-1}(cx))^2}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 134

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^5}\,dx = \frac{b^2}{32x^4}+\frac{3b^2c^2}{32x^2}+\frac{3}{16}abc^4\csc^{-1}(cx) + \frac{3}{32}b^2c^4\csc^{-1}(cx)^2-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{8x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{16x}-\frac{(a+b\csc^{-1}(cx))^2}{4x^4}$$

```
[Out] 1/32*b^2/x^4+3/32*b^2*c^2/x^2+3/16*a*b*c^4*arccsc(c*x)+3/32*b^2*c^4*arccsc(
c*x)^2-1/4*(a+b*arccsc(c*x))^2/x^4-1/8*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^
(1/2)/x^3-3/16*b*c^3*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5331, 4489, 3391}

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^5}\,dx = \frac{3}{16}abc^4\csc^{-1}(cx)-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{8x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{16x}-\frac{(a+b\csc^{-1}(cx))^2}{4x^4} + \frac{3}{32}b^2c^4\csc^{-1}(cx)^2+\frac{3b^2c^2}{32x^2}+\frac{b^2}{32x^4}$$

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^5,x]
```

```
[Out] b^2/(32*x^4) + (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcCsc[c*x])/16 + (3*b^2*c
^4*ArcCsc[c*x]^2)/32 - (b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(8*x
^3) - (3*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(16*x) - (a + b*A
rcCsc[c*x])^2/(4*x^4)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^4\text{Subst}\left(\int (a+bx)^2\cos(x)\sin^3(x)\,dx, x, \csc^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4x^4}+\frac{1}{2}\left(bc^4\right)\text{Subst}\left(\int (a+bx)\sin^4(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{b^2}{32x^4}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{8x^3}-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4x^4}\\
&\quad+\frac{1}{8}\left(3bc^4\right)\text{Subst}\left(\int (a+bx)\sin^2(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{b^2}{32x^4}+\frac{3b^2c^2}{32x^2}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{8x^3}-\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{16x}\\
&\quad-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4x^4}+\frac{1}{16}\left(3bc^4\right)\text{Subst}\left(\int (a+bx)\,dx, x, \csc^{-1}(cx)\right)
\end{aligned}$$

$$= \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{16}abc^4\csc^{-1}(cx) + \frac{3}{32}b^2c^4\csc^{-1}(cx)^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{8x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{16x} - \frac{(a+b\csc^{-1}(cx))^2}{4x^4}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^5}\,dx = \frac{-8a^2+b^2-4abc\sqrt{1-\frac{1}{c^2x^2}}x+3b^2c^2x^2-6abc^3\sqrt{1-\frac{1}{c^2x^2}}x^3-2b\left(8a+bc\sqrt{1-\frac{1}{c^2x^2}}x(2+3c^2x^2)\right)\csc^{-1}}{32x^4}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^5,x]
```

```
[Out] (-8*a^2 + b^2 - 4*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 - 6*a*b*c^3
*Sqrt[1 - 1/(c^2*x^2)]*x^3 - 2*b*(8*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*
c^2*x^2))*ArcCsc[c*x] + b^2*(-8 + 3*c^4*x^4)*ArcCsc[c*x]^2 + 6*a*b*c^4*x^4*
ArcSin[1/(c*x)])/(32*x^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(116) = 232$.

Time = 1.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.98

method	result
parts	$-\frac{a^2}{4x^4}+b^2c^4\left(-\frac{\operatorname{arccsc}(cx)^2}{4c^4x^4}+\frac{\operatorname{arccsc}(cx)\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arccsc}(cx)^2}{32}+\right.$
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{4c^4x^4}+\frac{\operatorname{arccsc}(cx)\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arccsc}(cx}{32}\right.\right.$
default	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{4c^4x^4}+\frac{\operatorname{arccsc}(cx)\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arccsc}(cx}{32}\right.\right.$

```
[In] int((a+b*arccsc(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^2/x^4+b^2*c^4*(-1/4/c^4/x^4*arccsc(c*x)^2+1/16*arccsc(c*x)*(3*c^3*x^
3*arccsc(c*x)-3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-2*((c^2*x^2-1)/c^2/x^2)
^(1/2))/c^3/x^3-3/32*arccsc(c*x)^2+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-1/2*a*b/x
^4*arccsc(c*x)+3/16*a*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

```
*arctan(1/(c^2*x^2-1)^(1/2))-3/16*a*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(
1/2)/x^3-1/8*a*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^5}\,dx$$
$$= \frac{3\,b^2c^2x^2 + (3\,b^2c^4x^4 - 8\,b^2)\operatorname{arccsc}(cx)^2 - 8\,a^2 + b^2 + 2\,(3\,abc^4x^4 - 8\,ab)\operatorname{arccsc}(cx) - 2\,(3\,abc^2x^2 + 2\,ab + (}{32\,x^4}$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*arccsc(c*x)^2 - 8*a^2 + b^2 +
 2*(3*a*b*c^4*x^4 - 8*a*b)*arccsc(c*x) - 2*(3*a*b*c^2*x^2 + 2*a*b + (3*b^2*
c^2*x^2 + 2*b^2)*arccsc(c*x))*sqrt(c^2*x^2 - 1))/x^4
```

Sympy [F]

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^5}\,dx = \int \frac{(a + b\operatorname{acsc}(cx))^2}{x^5}\,dx$$

```
[In] integrate((a+b*acsc(c*x))**2/x**5,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x**5, x)
```

Maxima [F]

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^5}\,dx = \int \frac{(b\operatorname{arccsc}(cx) + a)^2}{x^5}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*a*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*
x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) - 1
)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/16*(4*(2
*(c^2*log(c*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*c^2*log(c)^2
- 16*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 32*c^2
*integrate(1/4*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) - 16*c^2*integrate(1/4
*x^2*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 16*c^2*integrate(1/4*x^2*log
```

```
(x)^2/(c^2*x^7 - x^5), x) + 4*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^7 -
 x^5), x) - (2*c^4*log(c*x + 1) + 2*c^4*log(c*x - 1) - 4*c^4*log(x) + (2*c^
2*x^2 + 1)/x^4)*log(c)^2 + 16*integrate(1/4*log(c^2*x^2)/(c^2*x^7 - x^5), x
)*log(c) - 32*integrate(1/4*log(x)/(c^2*x^7 - x^5), x)*log(c) + 8*integrate
(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c
^2*x^7 - x^5), x) + 16*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x
) - 16*integrate(1/4*log(x)^2/(c^2*x^7 - x^5), x) - 4*integrate(1/4*log(c^2
*x^2)/(c^2*x^7 - x^5), x))*x^4 + 4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^
2 - log(c^2*x^2)^2)*b^2/x^4 - 1/4*a^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(116) = 232$.

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.27

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^5}\,dx =$$

$$-\frac{1}{256}\left(64\,b^2c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)^2+128\,abc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)+128\,b^2c^3\left(\frac{1}{c^2x^2}-1\right)a\right.$$

```
[In] integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] -1/256*(64*b^2*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^2 + 128*a*b*c^3*(1/(
c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 128*b^2*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c
*x))^2 - 8*b^2*c^3*(1/(c^2*x^2) - 1)^2 + 256*a*b*c^3*(1/(c^2*x^2) - 1)*arcs
in(1/(c*x)) + 40*b^2*c^3*arcsin(1/(c*x))^2 - 40*b^2*c^3*(1/(c^2*x^2) - 1) +
 80*a*b*c^3*arcsin(1/(c*x)) - 32*b^2*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/
(c*x))/x - 17*b^2*c^3 - 32*a*b*c^2*(-1/(c^2*x^2) + 1)^(3/2)/x + 80*b^2*c^2*
sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 80*a*b*c^2*sqrt(-1/(c^2*x^2) + 1
)/x + 64*a^2/(c*x^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^5}\,dx = \int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x^5}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))^2/x^5,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2/x^5, x)
```

3.24 $\int x^3(a+b\csc^{-1}(cx))^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 207

$$\begin{aligned}\int x^3(a+b\csc^{-1}(cx))^3\,dx &= \frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2}\\&\quad+\frac{ib(a+b\csc^{-1}(cx))^2}{2c^4}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))^2}{2c^3}\\&\quad+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(a+b\csc^{-1}(cx))^2}{4c}+\frac{1}{4}x^4(a+b\csc^{-1}(cx))^3\\&\quad-\frac{b^2(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{c^4}\\&\quad+\frac{ib^3\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2c^4}\end{aligned}$$

```
[Out] 1/4*b^2*x^2*(a+b*arccsc(c*x))/c^2+1/2*I*b*(a+b*arccsc(c*x))^2/c^4+1/4*x^4*(
a+b*arccsc(c*x))^3-b^2*(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2
)/c^4+1/2*I*b^3*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/c^4+1/4*b^3*x*(1-1
/c^2/x^2)^(1/2)/c^3+1/2*b*x*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c^3+1/4
*b*x^3*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5331, 4495, 4271, 3852, 8, 4269, 3798, 2221, 2317, 2438}

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^3\,dx=-\frac{b^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^4}+\frac{b^2x^2\left(a+b\csc^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^4}+\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4c}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{2c^3}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^3+\frac{ib^3\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2c^4}+\frac{b^3x\sqrt{1-\frac{1}{c^2x^2}}}{4c^3}$$

```
[In] Int[x^3*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (b^3*Sqrt[1 - 1/(c^2*x^2)]*x)/(4*c^3) + (b^2*x^2*(a + b*ArcCsc[c*x]))/(4*c^
2) + ((I/2)*b*(a + b*ArcCsc[c*x])^2)/c^4 + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(a +
b*ArcCsc[c*x])^2)/(2*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^3*(a + b*ArcCsc[c*x]
)^2)/(4*c) + (x^4*(a + b*ArcCsc[c*x])^3)/4 - (b^2*(a + b*ArcCsc[c*x])*Log[1
 - E^((2*I)*ArcCsc[c*x])])/c^4 + ((I/2)*b^3*PolyLog[2, E^((2*I)*ArcCsc[c*x]
)])/c^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
```

```
0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int (a+bx)^3\cot(x)\csc^4(x)\,dx,x,\csc^{-1}(cx)\right)}{c^4}$$
$$= \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^3 - \frac{(3b)\text{Subst}\left(\int (a+bx)^2\csc^4(x)\,dx,x,\csc^{-1}(cx)\right)}{4c^4}$$
$$= \frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\csc^{-1}(cx))^2}{4c} + \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^3 - \frac{b\text{Subst}\left(\int (a+bx)^2\csc^2(x)\,dx,x,\csc^{-1}(cx)\right)}{2c^4} - \frac{b^3\text{Subst}\left(\int \csc^2(x)\,dx,x,\csc^{-1}(cx)\right)}{4c^4}$$
$$= \frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\csc^{-1}(cx))^2}{2c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\csc^{-1}(cx))^2}{4c} + \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^3 - \frac{b^2\text{Subst}\left(\int (a+bx)\cot(x)\,dx,x,\csc^{-1}(cx)\right)}{c^4} + \frac{b^3\text{Subst}\left(\int 1\,dx,x,c\sqrt{1-\frac{1}{c^2x^2}}x\right)}{4c^4}$$
$$= \frac{b^3\sqrt{1-\frac{1}{c^2x^2}}x}{4c^3} + \frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2} + \frac{ib(a+b\csc^{-1}(cx))^2}{2c^4} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\csc^{-1}(cx))^2}{2c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\csc^{-1}(cx))^2}{4c} + \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^3 + \frac{(2ib^2)\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx,x,\csc^{-1}(cx)\right)}{c^4}$$
$$= \frac{b^3\sqrt{1-\frac{1}{c^2x^2}}x}{4c^3} + \frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2} + \frac{ib(a+b\csc^{-1}(cx))^2}{2c^4} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\csc^{-1}(cx))^2}{2c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\csc^{-1}(cx))^2}{4c} + \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^3 - \frac{b^2(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{c^4} + \frac{b^3\text{Subst}\left(\int \log\left(1-e^{2ix}\right)\,dx,x,\csc^{-1}(cx)\right)}{c^4}$$

$$= \frac{b^3\sqrt{1-\frac{1}{c^2x^2}}x}{4c^3} + \frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2} + \frac{ib(a+b\csc^{-1}(cx))^2}{2c^4} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\csc^{-1}(cx))^2}{2c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\csc^{-1}(cx))^2}{4c} + \frac{1}{4}x^4(a+b\csc^{-1}(cx))^3 - \frac{b^2(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{c^4} - \frac{(ib^3)\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)}{2c^4}$$

$$= \frac{b^3\sqrt{1-\frac{1}{c^2x^2}}x}{4c^3} + \frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2} + \frac{ib(a+b\csc^{-1}(cx))^2}{2c^4} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\csc^{-1}(cx))^2}{2c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\csc^{-1}(cx))^2}{4c} + \frac{1}{4}x^4(a+b\csc^{-1}(cx))^3 - \frac{b^2(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{c^4} + \frac{ib^3\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)}{2c^4}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.38

$$\int x^3(a+b\csc^{-1}(cx))^3\,dx$$
$$= \frac{2a^2bc\sqrt{1-\frac{1}{c^2x^2}}x + b^3c\sqrt{1-\frac{1}{c^2x^2}}x + ab^2c^2x^2 + a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}x^3 + a^3c^4x^4 + b^2\left(3ac^4x^4 + b\left(2i + 2c\sqrt{1-\right.\right.}{}$$

```
[In] Integrate[x^3*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (2*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*b^2*
c^2*x^2 + a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 + b^2*(3*a*c^4*
x^4 + b*(2*I + 2*c*Sqrt[1 - 1/(c^2*x^2)]*x + c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3)
)*ArcCsc[c*x]^2 + b^3*c^4*x^4*ArcCsc[c*x]^3 + b*ArcCsc[c*x]*(c*x*(b^2*c*x +
 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2)) - 4*b^2*Log[1 -
 E^((2*I)*ArcCsc[c*x])]) - 4*a*b^2*Log[1/(c*x)] + (2*I)*b^3*PolyLog[2, E^((
2*I)*ArcCsc[c*x])])/(4*c^4)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arccsc}(cx)^3c^4x^4}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arccsc}(cx)^2}{2}+\frac{c^2x^2\operatorname{arccsc}(cx)}{4}+\frac{xc\sqrt{\cdots}}{4}\cdots\right.}{\cdots}$
default	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arccsc}(cx)^3c^4x^4}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arccsc}(cx)^2}{2}+\frac{c^2x^2\operatorname{arccsc}(cx)}{4}+\frac{xc\sqrt{\cdots}}{4}\cdots\right.}{\cdots}$
parts	$\frac{a^3x^4}{4}+\frac{b^3\left(\frac{\operatorname{arccsc}(cx)^3c^4x^4}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arccsc}(cx)^2}{2}+\frac{c^2x^2\operatorname{arccsc}(cx)}{4}+\frac{xc\sqrt{\cdots}}{4}\cdots\right.}{\cdots}$

```
[In] int(x^3*(a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*a^3*c^4*x^4+b^3*(1/4*arccsc(c*x)^3*c^4*x^4+1/4*arccsc(c*x)^2*((c
^2*x^2-1)/c^2/x^2)^(1/2)*c^3*x^3+1/2*arccsc(c*x)^2*((c^2*x^2-1)/c^2/x^2)^(1
/2)*c*x+1/2*I*arccsc(c*x)^2+1/4*c^2*x^2*arccsc(c*x)+1/4*x*c*((c^2*x^2-1)/c^
2/x^2)^(1/2)-1/4*I-arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))-arccsc(c*x)*
ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+I*po
lylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))+3*a*b^2*(1/4*arccsc(c*x)^2*c^4*x^4+1/6
*arccsc(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*x^3+1/12*c^2*x^2+1/3*arccsc(c*
x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/3*ln(1/c/x))+3*a^2*b*(1/4*c^4*x^4*arcc
sc(c*x)+1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))
```

Fricas [F]

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int(b\operatorname{arccsc}(cx)+a)^3x^3\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arccsc(c*x)^3 + 3*a*b^2*x^3*arccsc(c*x)^2 + 3*a^2*b*x^3*ar
ccsc(c*x) + a^3*x^3, x)
```

Sympy [F]

$$\int x^3 \left(a + b \csc^{-1}(cx)\right)^3 \, dx = \int x^3 (a + b \operatorname{acsc}(cx))^3 \, dx$$

```
[In] integrate(x**3*(a+b*acsc(c*x))**3,x)
[Out] Integral(x**3*(a + b*acsc(c*x))**3, x)
```

Maxima [F]

$$\int x^3 \left(a + b \csc^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arccsc}(cx) + a)^3 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
[Out] 3/4*a*b^2*x^4*arccsc(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arccsc(c*x) + (c^2*x
^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a^2*b + 1/16
*(4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x^4*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 16*integrate(3/16*(16*c^2*x^5*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 16*x^3*arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))*log(c)^2 + 16*(c^2*x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1)) - x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2 - (4*x^3*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x^3*log(c^2*x^2)^2)*sqrt(c*x + 1)*s
qrt(c*x - 1) - 4*((4*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c) + c
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^5 - (4*arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))*log(c) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^3 + 4
*(c^2*x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - x^3*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)))*log(x))*log(c^2*x^2) + 32*(c^2*x^5*arctan2(1, sqrt(c*
x + 1)*sqrt(c*x - 1))*log(c) - x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c))*log(x))/(c^2*x^2 - 1), x))*b^3 + 1/4*(2*c^4*x^4*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)) + 2*c^2*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
(c^2*x^2 + 2*log(x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) - 4*arctan2(1, sqrt(c*x
+ 1)*sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c^4)
```

Giac [F]

$$\int x^3 \left(a+b\csc^{-1}(cx)\right)^3 \, dx=\int \left(b\operatorname{arccsc}\left(cx\right)+a\right)^3 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)^3*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\csc^{-1}(cx)\right)^3 \, dx=\int x^3 \left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

```
[In] int(x^3*(a + b*asin(1/(c*x)))^3,x)
[Out] int(x^3*(a + b*asin(1/(c*x)))^3, x)
```

3.25 $\int x^2(a+b\csc^{-1}(cx))^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 220

$$\begin{aligned}\int x^2\left(a+b\csc^{-1}(cx)\right)^3\,dx &= \frac{b^2x(a+b\csc^{-1}(cx))}{c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\csc^{-1}(cx))^2}{2c}\\ &+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^3\\ &+\frac{b(a+b\csc^{-1}(cx))^2\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c^3}\\ &+\frac{b^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3}\\ &-\frac{ib^2(a+b\csc^{-1}(cx))\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c^3}\\ &+\frac{ib^2(a+b\csc^{-1}(cx))\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c^3}\\ &+\frac{b^3\operatorname{PolyLog}\left(3,-e^{i\csc^{-1}(cx)}\right)}{c^3}-\frac{b^3\operatorname{PolyLog}\left(3,e^{i\csc^{-1}(cx)}\right)}{c^3}\end{aligned}$$

```
[Out] b^2*x*(a+b*arccsc(c*x))/c^2+1/3*x^3*(a+b*arccsc(c*x))^3+b*(a+b*arccsc(c*x))
^2*arctanh(I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+b^3*arctanh((1-1/c^2/x^2)^(1/2))/
c^3-I*b^2*(a+b*arccsc(c*x))*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c^3+I*b^2
*(a+b*arccsc(c*x))*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+b^3*polylog(3,-
I/c/x-(1-1/c^2/x^2)^(1/2))/c^3-b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3
+1/2*b*x^2*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5331, 4495, 4271, 3855, 4268, 2611, 2320, 6724}

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^3\,dx=\frac{b\text{arctanh}\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2}{c^3}-\frac{ib^2\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^3}+\frac{ib^2\,\text{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^3}+\frac{b^2x(a+b\csc^{-1}(cx))}{c^2}+\frac{bx^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^3+\frac{b^3\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3}+\frac{b^3\,\text{PolyLog}\left(3,-e^{i\csc^{-1}(cx)}\right)}{c^3}-\frac{b^3\,\text{PolyLog}\left(3,e^{i\csc^{-1}(cx)}\right)}{c^3}$$

```
[In] Int[x^2*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (b^2*x*(a + b*ArcCsc[c*x]))/c^2 + (b*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcCs
c[c*x])^2)/(2*c) + (x^3*(a + b*ArcCsc[c*x])^3)/3 + (b*(a + b*ArcCsc[c*x])^2
*ArcTanh[E^(I*ArcCsc[c*x])])/c^3 + (b^3*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c^3
 - (I*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + (I*b^2*
(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*x])])/c^3 + (b^3*PolyLog[3, -E
^(I*ArcCsc[c*x])])/c^3 - (b^3*PolyLog[3, E^(I*ArcCsc[c*x])])/c^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int (a+bx)^3 \cot(x) \csc^3(x)\, dx, x, \csc^{-1}(cx)\right)}{c^3}$$

$$= \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^3 - \frac{b\text{Subst}\left(\int (a+bx)^2\csc^3(x)\,dx, x, \csc^{-1}(cx)\right)}{c^3}$$

$$= \frac{b^2x(a+b\csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^3 - \frac{b\text{Subst}\left(\int (a+bx)^2\csc(x)\,dx, x, \csc^{-1}(cx)\right)}{2c^3} - \frac{b^3\text{Subst}\left(\int \csc(x)\,dx, x, \csc^{-1}(cx)\right)}{c^3}$$

$$= \frac{b^2x(a+b\csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^3 + \frac{b(a+b\csc^{-1}(cx))^2\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c^3} + \frac{b^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3} + \frac{b^2\text{Subst}\left(\int (a+bx)\log\left(1-e^{ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{c^3} - \frac{b^2\text{Subst}\left(\int (a+bx)\log\left(1+e^{ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{c^3}$$

$$= \frac{b^2x(a+b\csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^3 + \frac{b(a+b\csc^{-1}(cx))^2\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c^3} + \frac{b^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3} - \frac{ib^2(a+b\csc^{-1}(cx))\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c^3} + \frac{ib^2(a+b\csc^{-1}(cx))\,\text{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c^3} + \frac{(ib^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{c^3} - \frac{(ib^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2,e^{ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{c^3}$$

$$= \frac{b^2x(a+b\csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\csc^{-1}(cx))^2}{2c}$$
$$+ \frac{1}{3}x^3(a+b\csc^{-1}(cx))^3 + \frac{b(a+b\csc^{-1}(cx))^2\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c^3}$$
$$+ \frac{b^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3} - \frac{ib^2(a+b\csc^{-1}(cx))\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c^3}$$
$$+ \frac{ib^2(a+b\csc^{-1}(cx))\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c^3}$$
$$+ \frac{b^3\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{c^3}$$
$$- \frac{b^3\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{c^3}$$

$$= \frac{b^2x(a+b\csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\csc^{-1}(cx))^2}{2c}$$
$$+ \frac{1}{3}x^3(a+b\csc^{-1}(cx))^3 + \frac{b(a+b\csc^{-1}(cx))^2\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c^3}$$
$$+ \frac{b^3\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3} - \frac{ib^2(a+b\csc^{-1}(cx))\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c^3}$$
$$+ \frac{ib^2(a+b\csc^{-1}(cx))\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c^3}$$
$$+ \frac{b^3\operatorname{PolyLog}\left(3,-e^{i\csc^{-1}(cx)}\right)}{c^3} - \frac{b^3\operatorname{PolyLog}\left(3,e^{i\csc^{-1}(cx)}\right)}{c^3}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 580 vs. $2(220) = 440$.

Time = 7.39 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.64

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^3\,dx$$

$$= \frac{a^3x^3}{3}+\frac{a^2bx^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+a^2bx^3\csc^{-1}(cx)+\frac{a^2b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{2c^3}$$
$$+\frac{ab^2\left(-8i\,\mathrm{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)+2c^3x^3\left(2+4\csc^{-1}(cx)^2-2\cos\left(2\csc^{-1}(cx)\right)-\frac{3\csc^{-1}(cx)\log\left(1-e^{i\csc^{-1}(cx)}\right)}{cx}\right.\right.}{}$$
$$+\frac{b^3\left(24\csc^{-1}(cx)\cot\left(\frac{1}{2}\csc^{-1}(cx)\right)+4\csc^{-1}(cx)^3\cot\left(\frac{1}{2}\csc^{-1}(cx)\right)+6\csc^{-1}(cx)^2\csc^2\left(\frac{1}{2}\csc^{-1}(cx)\right)+\right.}{}$$

```
[In] Integrate[x^2*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (a^3*x^3)/3 + (a^2*b*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + a^2*b*x^3*
ArcCsc[c*x] + (a^2*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(2*c^3) +
 (a*b^2*((-8*I)*PolyLog[2, -E^(I*ArcCsc[c*x])] + 2*c^3*x^3*(2 + 4*ArcCsc[c*
x]^2 - 2*Cos[2*ArcCsc[c*x]] - (3*ArcCsc[c*x]*Log[1 - E^(I*ArcCsc[c*x])])/(c
*x) + (3*ArcCsc[c*x]*Log[1 + E^(I*ArcCsc[c*x])])/(c*x) + ((4*I)*PolyLog[2,
E^(I*ArcCsc[c*x])])/(c^3*x^3) + 2*ArcCsc[c*x]*Sin[2*ArcCsc[c*x]] + ArcCsc[c
*x]*Log[1 - E^(I*ArcCsc[c*x])]*Sin[3*ArcCsc[c*x]] - ArcCsc[c*x]*Log[1 + E^(
I*ArcCsc[c*x])]*Sin[3*ArcCsc[c*x]])))/(8*c^3) + (b^3*(24*ArcCsc[c*x]*Cot[Ar
cCsc[c*x]/2] + 4*ArcCsc[c*x]^3*Cot[ArcCsc[c*x]/2] + 6*ArcCsc[c*x]^2*Csc[Arc
Csc[c*x]/2]^2 + (ArcCsc[c*x]^3*Csc[ArcCsc[c*x]/2]^4)/(c*x) - 24*ArcCsc[c*x]
^2*Log[1 - E^(I*ArcCsc[c*x])] + 24*ArcCsc[c*x]^2*Log[1 + E^(I*ArcCsc[c*x])]
 - 48*Log[Tan[ArcCsc[c*x]/2]] - (48*I)*ArcCsc[c*x]*PolyLog[2, -E^(I*ArcCsc[
c*x])] + (48*I)*ArcCsc[c*x]*PolyLog[2, E^(I*ArcCsc[c*x])] + 48*PolyLog[3, -
E^(I*ArcCsc[c*x])] - 48*PolyLog[3, E^(I*ArcCsc[c*x])] - 6*ArcCsc[c*x]^2*Sec
[ArcCsc[c*x]/2]^2 + 16*c^3*x^3*ArcCsc[c*x]^3*Sin[ArcCsc[c*x]/2]^4 + 24*ArcC
sc[c*x]*Tan[ArcCsc[c*x]/2] + 4*ArcCsc[c*x]^3*Tan[ArcCsc[c*x]/2]))/(48*c^3)
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.43

method	result
derivativedivides	$\frac{c^3x^3a^3}{3}+b^3\left(\frac{\operatorname{arccsc}(cx)\left(2c^2x^2\operatorname{arccsc}(cx)^2+3\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}-\frac{\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{2}+i\operatorname{arccsc}(cx)\,\mathrm{po}\right.$
default	$\frac{c^3x^3a^3}{3}+b^3\left(\frac{\operatorname{arccsc}(cx)\left(2c^2x^2\operatorname{arccsc}(cx)^2+3\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}-\frac{\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{2}+i\operatorname{arccsc}(cx)\,\mathrm{po}\right.$
parts	$\frac{a^3x^3}{3}+\,b^3\left(\frac{\operatorname{arccsc}(cx)\left(2c^2x^2\operatorname{arccsc}(cx)^2+3\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}-\frac{\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{2}+i\operatorname{arccsc}(cx)\,\mathrm{po}\right.$

```
[In] int(x^2*(a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a^3+b^3*(1/6*arccsc(c*x)*(2*c^2*x^2*arccsc(c*x)^2+3*arcc
sc(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)+6)*c*x-1/2*arccsc(c*x)^2*ln(1-I/c/x
-(1-1/c^2/x^2)^(1/2))+I*arccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-po
lylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))+1/2*arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^2/x^
2)^(1/2))-I*arccsc(c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+polylog(3,-I/
c/x-(1-1/c^2/x^2)^(1/2))+2*arctanh(I/c/x+(1-1/c^2/x^2)^(1/2)))+3*a*b^2*(1/3
*(c^2*x^2*arccsc(c*x)^2+arccsc(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)+1)*c*x-
1/3*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+1/3*I*polylog(2,I/c/x+(1-1/
c^2/x^2)^(1/2))+1/3*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/3*I*polyl
og(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))+3*a^2*b*(1/3*c^3*x^3*arccsc(c*x)+1/6*(c^2
*x^2-1)^(1/2)*(c*x*(c^2*x^2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1
)/c^2/x^2)^(1/2)/c/x))
```

Fricas [F]

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int\left(b\operatorname{arccsc}\left(cx\right)+a\right)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arccsc(c*x)^3 + 3*a*b^2*x^2*arccsc(c*x)^2 + 3*a^2*b*x^2*ar
ccsc(c*x) + a^3*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int x^2(a+b\operatorname{acsc}(cx))^3\,dx$$

```
[In] integrate(x**2*(a+b*acsc(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))**3, x)
```

Maxima [F]

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int (b\operatorname{arccsc}(cx)+a)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 1/4*b^3*x^3*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 1/2*a*b^2*c^2*(2*(c^2*x^3
 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 - 12*b^3*c^
2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1),
x)*log(c)^2 + 12*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x
 - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^
4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) +
 12*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*
a*b^2*c^2*integrate(1/4*x^4*log(x)/(c^2*x^2 - 1), x)*log(c) + 1/3*a^3*x^3 +
 12*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c
^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*b^3*c^2*integrate(1/4*x^4*arctan(1/(s
qrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*inte
grate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^2 - 1), x) +
 4*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^
2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)^2/(c^
2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)*log(x)/(c^2*x^
2 - 1), x) - 12*a*b^2*c^2*integrate(1/4*x^4*log(x)^2/(c^2*x^2 - 1), x) + 3/
2*a*b^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 + 12*b^3*i
ntegrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x)*
log(c)^2 - 12*b^3*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))
*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*b^3*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) - 12*a*b^2*i
ntegrate(1/4*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*a*b^2*integrate
(1/4*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 1/4*(4*x^3*arccsc(c*x) + (2*sqrt
(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) +
1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*a^2*b + 4*b^3*integra
te(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1
```

```
)))^2/(c^2*x^2 - 1), x) - b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2
*log(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*b^3*integrate(1/4*x^2*arctan(1/(sqrt
(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*b^3*in
tegrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^2 -
 1), x) - 12*a*b^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))
)^2/(c^2*x^2 - 1), x) - 4*b^3*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqr
t(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x) + 3*a*b^2*integrate(1/4*x^2*log
(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*a*b^2*integrate(1/4*x^2*log(c^2*x^2)*log
(x)/(c^2*x^2 - 1), x) + 12*a*b^2*integrate(1/4*x^2*log(x)^2/(c^2*x^2 - 1),
x)
```

Giac [F]

$$\int x^2 \left(a+b \csc^{-1}(cx)\right)^3 \, dx=\int (b \operatorname{arccsc}(cx)+a)^3 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)^3*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b \csc^{-1}(cx)\right)^3 \, dx=\int x^2 \left(a+b \operatorname{asin}\left(\frac{1}{c x}\right)\right)^3 \, dx$$

```
[In] int(x^2*(a + b*asin(1/(c*x)))^3,x)
[Out] int(x^2*(a + b*asin(1/(c*x)))^3, x)
```

3.26 $\int x(a+b\csc^{-1}(cx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 126

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\frac{3ib(a+b\csc^{-1}(cx))^2}{2c^2}+\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3-\frac{3b^2(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{c^2}+\frac{3ib^3\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2c^2}$$

```
[Out] 3/2*I*b*(a+b*arccsc(c*x))^2/c^2+1/2*x^2*(a+b*arccsc(c*x))^3-3*b^2*(a+b*arcc
sc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/c^2+3/2*I*b^3*polylog(2,(I/c/x
+(1-1/c^2/x^2)^(1/2))^2)/c^2+3/2*b*x*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2
)/c
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used

= {5331, 4495, 4269, 3798, 2221, 2317, 2438}

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=-\frac{3b^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c^2}+\frac{3bx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{3ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3+\frac{3ib^3\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2c^2}$$

```
[In] Int[x*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (((3*I)/2)*b*(a + b*ArcCsc[c*x])^2)/c^2 + (3*b*Sqrt[1 - 1/(c^2*x^2)]*x*(a +
 b*ArcCsc[c*x])^2)/(2*c) + (x^2*(a + b*ArcCsc[c*x])^3)/2 - (3*b^2*(a + b*Ar
cCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/c^2 + (((3*I)/2)*b^3*PolyLog[2,
E^((2*I)*ArcCsc[c*x])])/c^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^3\cot(x)\csc^2(x)\,dx, x, \csc^{-1}(cx)\right)}{c^2}\\
&= \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3 - \frac{(3b)\text{Subst}\left(\int (a+bx)^2\csc^2(x)\,dx, x, \csc^{-1}(cx)\right)}{2c^2}\\
&= \frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^2}{2c} + \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3\\
&\quad - \frac{(3b^2)\,\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)}{c^2}\\
&= \frac{3ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^2} + \frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^2}{2c}\\
&\quad + \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3 + \frac{(6ib^2)\,\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)}{c^2}\\
&= \frac{3ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^2} + \frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^2}{2c}\\
&\quad + \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3 - \frac{3b^2\left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{c^2}\\
&\quad + \frac{(3b^3)\,\text{Subst}\left(\int \log\left(1-e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{c^2}\\
&= \frac{3ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^2} + \frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^2}{2c} + \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3\\
&\quad - \frac{3b^2\left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{c^2} - \frac{(3ib^3)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)}{2c^2}
\end{aligned}$$

$$= \frac{3ib(a+b\csc^{-1}(cx))^2}{2c^2} + \frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))^2}{2c} + \frac{1}{2}x^2(a+b\csc^{-1}(cx))^3 - \frac{3b^2(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{c^2} + \frac{3ib^3\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.44

$$\int x(a+b\csc^{-1}(cx))^3\,dx$$

$$= \frac{3b^2\left(ac^2x^2+b\left(i+c\sqrt{1-\frac{1}{c^2x^2}}\,x\right)\right)\csc^{-1}(cx)^2+b^3c^2x^2\csc^{-1}(cx)^3+3b\csc^{-1}(cx)\left(acx\left(2b\sqrt{1-\frac{1}{c^2x^2}}+acx\right.\right.}{2}$$

```
[In] Integrate[x*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (3*b^2*(a*c^2*x^2 + b*(I + c*Sqrt[1 - 1/(c^2*x^2)]*x))*ArcCsc[c*x]^2 + b^3*
c^2*x^2*ArcCsc[c*x]^3 + 3*b*ArcCsc[c*x]*(a*c*x*(2*b*Sqrt[1 - 1/(c^2*x^2)] +
 a*c*x) - 2*b^2*Log[1 - E^((2*I)*ArcCsc[c*x])]) + a*(a*c*x*(3*b*Sqrt[1 - 1/
(c^2*x^2)] + a*c*x) - 6*b^2*Log[1/(c*x)]) + (3*I)*b^3*PolyLog[2, E^((2*I)*A
rcCsc[c*x])])/(2*c^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(142) = 284.

Time = 1.59 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.40

method	result
derivativedivides	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{\operatorname{arccsc}(cx)^2\left(c^2x^2\operatorname{arccsc}(cx)+3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}-3\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-3\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\right.\right.}{}$
default	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{\operatorname{arccsc}(cx)^2\left(c^2x^2\operatorname{arccsc}(cx)+3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}-3\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-3\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\right.\right.}{}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\frac{\operatorname{arccsc}(cx)^2\left(c^2x^2\operatorname{arccsc}(cx)+3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}-3\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-3\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\right.\right.}{c^2}$

```
[In] int(x*(a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

[Out] 1/c^2*(1/2*c^2*x^2*a^3+b^3*(1/2*arccsc(c*x)^2*(c^2*x^2*arccsc(c*x)+3*x*c*((
c^2*x^2-1)/c^2/x^2)^(1/2)-3*I)-3*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2)
)-3*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+3*I*arccsc(c*x)^2+3*I*polyl
og(2,I/c/x+(1-1/c^2/x^2)^(1/2))+3*I*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))+
3*a*b^2*(1/2*c^2*x^2*arccsc(c*x)^2+arccsc(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1
/2)-ln(1/c/x))+3*a^2*b*(1/2*c^2*x^2*arccsc(c*x)+1/2/((c^2*x^2-1)/c^2/x^2)^(
1/2)/c/x*(c^2*x^2-1)))

Fricas [F]

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int(b\operatorname{arccsc}(cx)+a)^3x\,dx$$

[In] integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="fricas")

[Out] integral(b^3*x*arccsc(c*x)^3 + 3*a*b^2*x*arccsc(c*x)^2 + 3*a^2*b*x*arccsc(c
*x) + a^3*x, x)

Sympy [F]

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int x\left(a+b\operatorname{acsc}(cx)\right)^3\,dx$$

[In] integrate(x*(a+b*acsc(c*x))**3,x)

[Out] Integral(x*(a + b*acsc(c*x))**3, x)

Maxima [F]

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int(b\operatorname{arccsc}(cx)+a)^3x\,dx$$

[In] integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="maxima")

[Out] 3/2*a*b^2*x^2*arccsc(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arccsc(c*x) + x*sqrt(-
1/(c^2*x^2) + 1)/c)*a^2*b + 3*(x*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x)/c + log
(x)/c^2)*a*b^2 + 1/8*(4*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 8*integrate(3/8
*(8*c^2*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 8*x*arctan2(
1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 + 8*(c^2*x^3*arctan2(1, sqrt(c*x +
 1)*sqrt(c*x - 1)) - x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2 -
(4*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x*log(c^2*x^2)^2)*sqrt(c*x

```
 + 1)*sqrt(c*x - 1) - 4*((2*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log
(c) + c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^3 - (2*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1))*log(c) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*
x + 2*(c^2*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - x*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1)))*log(x))*log(c^2*x^2) + 16*(c^2*x^3*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1))*log(c) - x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
*log(c))*log(x))/(c^2*x^2 - 1), x))*b^3
```

Giac [F]

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int (b\operatorname{arccsc}(cx)+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

```
[In] int(x*(a + b*asin(1/(c*x)))^3,x)
[Out] int(x*(a + b*asin(1/(c*x)))^3, x)
```

3.27 $\int \left(a+b\csc^{-1}(cx)\right)^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 144

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx = x\left(a+b\csc^{-1}(cx)\right)^3+\frac{6b\left(a+b\csc^{-1}(cx)\right)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6ib^2\left(a+b\csc^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c} + \frac{6ib^2\left(a+b\csc^{-1}(cx)\right)\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c} + \frac{6b^3\operatorname{PolyLog}\left(3,-e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6b^3\operatorname{PolyLog}\left(3,e^{i\csc^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a+b*arccsc(c*x))^3+6*b*(a+b*arccsc(c*x))^2*arctanh(I/c/x+(1-1/c^2/x^2)^(
1/2))/c-6*I*b^2*(a+b*arccsc(c*x))*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c+6
*I*b^2*(a+b*arccsc(c*x))*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c+6*b^3*polyl
og(3,-I/c/x-(1-1/c^2/x^2)^(1/2))/c-6*b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2
))/c
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used

$= \{5325, 4495, 4268, 2611, 2320, 6724\}$

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx = \frac{6b\mathrm{arctanh}\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2}{c} - \frac{6ib^2\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c} + \frac{6ib^2\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c} + x\left(a+b\csc^{-1}(cx)\right)^3 + \frac{6b^3\operatorname{PolyLog}\left(3,-e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6b^3\operatorname{PolyLog}\left(3,e^{i\csc^{-1}(cx)}\right)}{c}$$

```
[In] Int[(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] x*(a + b*ArcCsc[c*x])^3 + (6*b*(a + b*ArcCsc[c*x])^2*ArcTanh[E^(I*ArcCsc[c*
x])])/c - ((6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E^(I*ArcCsc[c*x])])/c
+ ((6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*x])])/c + (6*b^3*
PolyLog[3, -E^(I*ArcCsc[c*x])])/c - (6*b^3*PolyLog[3, E^(I*ArcCsc[c*x])])/c
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5325

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Sub
st[Int[(a + b*x)^n*Csc[x]*Cot[x], x], x, ArcCsc[c*x]], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int (a+bx)^3\cot(x)\csc(x)\,dx,x,\csc^{-1}(cx)\right)}{c}\\
&= x\left(a+b\csc^{-1}(cx)\right)^3-\frac{(3b)\text{Subst}\left(\int (a+bx)^2\csc(x)\,dx,x,\csc^{-1}(cx)\right)}{c}\\
&= x\left(a+b\csc^{-1}(cx)\right)^3+\frac{6b\left(a+b\csc^{-1}(cx)\right)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c}\\
&\quad+\frac{(6b^2)\,\text{Subst}\left(\int (a+bx)\log\left(1-e^{ix}\right)\,dx,x,\csc^{-1}(cx)\right)}{c}\\
&\quad-\frac{(6b^2)\,\text{Subst}\left(\int (a+bx)\log\left(1+e^{ix}\right)\,dx,x,\csc^{-1}(cx)\right)}{c}\\
&= x\left(a+b\csc^{-1}(cx)\right)^3+\frac{6b\left(a+b\csc^{-1}(cx)\right)^2\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c}\\
&\quad-\frac{6ib^2\left(a+b\csc^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c}\\
&\quad+\frac{6ib^2\left(a+b\csc^{-1}(cx)\right)\text{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c}\\
&\quad+\frac{(6ib^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{ix}\right)\,dx,x,\csc^{-1}(cx)\right)}{c}\\
&\quad-\frac{(6ib^3)\,\text{Subst}\left(\int \text{PolyLog}\left(2,e^{ix}\right)\,dx,x,\csc^{-1}(cx)\right)}{c}
\end{aligned}$$

$$= x\left(a + b\csc^{-1}(cx)\right)^3 + \frac{6b(a + b\csc^{-1}(cx))^2 \operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6ib^2(a + b\csc^{-1}(cx)) \operatorname{PolyLog}\left(2, -e^{i\csc^{-1}(cx)}\right)}{c} + \frac{6ib^2(a + b\csc^{-1}(cx)) \operatorname{PolyLog}\left(2, e^{i\csc^{-1}(cx)}\right)}{c} + \frac{(6b^3) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x}\, dx, x, e^{i\csc^{-1}(cx)}\right)}{c} - \frac{(6b^3) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\, dx, x, e^{i\csc^{-1}(cx)}\right)}{c}$$

$$= x\left(a + b\csc^{-1}(cx)\right)^3 + \frac{6b(a + b\csc^{-1}(cx))^2 \operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6ib^2(a + b\csc^{-1}(cx)) \operatorname{PolyLog}\left(2, -e^{i\csc^{-1}(cx)}\right)}{c} + \frac{6ib^2(a + b\csc^{-1}(cx)) \operatorname{PolyLog}\left(2, e^{i\csc^{-1}(cx)}\right)}{c} + \frac{6b^3 \operatorname{PolyLog}\left(3, -e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6b^3 \operatorname{PolyLog}\left(3, e^{i\csc^{-1}(cx)}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.84

$$\int \left(a + b\csc^{-1}(cx)\right)^3 \, dx = \frac{a^3cx + 3a^2bcx\csc^{-1}(cx) + 3ab^2cx\csc^{-1}(cx)^2 + b^3cx\csc^{-1}(cx)^3 - 6ab^2\csc^{-1}(cx)\log\left(1 - e^{i\csc^{-1}(cx)}\right) - 3b^3}{}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (a^3*c*x + 3*a^2*b*c*x*ArcCsc[c*x] + 3*a*b^2*c*x*ArcCsc[c*x]^2 + b^3*c*x*Ar
cCsc[c*x]^3 - 6*a*b^2*ArcCsc[c*x]*Log[1 - E^(I*ArcCsc[c*x])] - 3*b^3*ArcCsc
[c*x]^2*Log[1 - E^(I*ArcCsc[c*x])] + 6*a*b^2*ArcCsc[c*x]*Log[1 + E^(I*ArcCs
c[c*x])] + 3*b^3*ArcCsc[c*x]^2*Log[1 + E^(I*ArcCsc[c*x])] + 3*a^2*b*Log[c*(
1 + Sqrt[1 - 1/(c^2*x^2)])*x] - (6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E
^(I*ArcCsc[c*x])] + (6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*
x])] + 6*b^3*PolyLog[3, -E^(I*ArcCsc[c*x])] - 6*b^3*PolyLog[3, E^(I*ArcCsc[
c*x])])/c
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.62

method	result
derivativedivides	$\frac{cx\,a^3+b^3\left(\operatorname{arccsc}(cx)^3 cx-3\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+6i\operatorname{arccsc}(cx)\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-6\operatorname{polylog}\left(3,\right.\right.}{}$
default	$\frac{cx\,a^3+b^3\left(\operatorname{arccsc}(cx)^3 cx-3\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+6i\operatorname{arccsc}(cx)\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-6\operatorname{polylog}\left(3,\right.\right.}{}$
parts	$a^3x+\frac{b^3\left(\operatorname{arccsc}(cx)^3 cx-3\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+6i\operatorname{arccsc}(cx)\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-6\operatorname{polylog}\left(\right.\right.}{}$

```
[In] int((a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*x*a^3+b^3*(arccsc(c*x)^3*c*x-3*arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2/x^2
)^(1/2))+6*I*arccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-6*polylog(3,I
/c/x+(1-1/c^2/x^2)^(1/2))+3*arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-6
*I*arccsc(c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+6*polylog(3,-I/c/x-(1-
1/c^2/x^2)^(1/2)))+3*a*b^2*(arccsc(c*x)^2*c*x-2*arccsc(c*x)*ln(1-I/c/x-(1-1
/c^2/x^2)^(1/2))+2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-2*I*dilog(1+
I/c/x+(1-1/c^2/x^2)^(1/2))+2*I*dilog(1-I/c/x-(1-1/c^2/x^2)^(1/2)))+3*a^2*b*
(arccsc(c*x)*c*x+ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))))
```

Fricas [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx=\int \left(b\operatorname{arccsc}\left(cx\right)+a\right)^3\,dx$$

```
[In] integrate((a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x) +
a^3, x)
```

Sympy [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx=\int \left(a+b\operatorname{acsc}\left(cx\right)\right)^3\,dx$$

```
[In] integrate((a+b*acsc(c*x))**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3, x)
```

Maxima [F]

$$\int \left(a + b\csc^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arccsc}(cx) + a)^3 \, dx$$

```
[In] integrate((a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*a*b^2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 1
2*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^
2 - 1), x)*log(c)^2 + b^3*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3/4
*b^3*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*b^3*c^2*
integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2
*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)
*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*a*b^2*c^2*integrate(1
/4*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*a*b^2*c^2*integrate(1/4*x
^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*b
^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(
c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*s
qrt(c*x - 1)))^2/(c^2*x^2 - 1), x) + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2
*integrate(1/4*x^2*log(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrat
e(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*a*b^2*c^2*integrate(1/
4*x^2*log(x)^2/(c^2*x^2 - 1), x) - 3/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)
/c)*log(c)^2 + 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))
/(c^2*x^2 - 1), x)*log(c)^2 - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*
sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*b^3*integrate(1/
4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) -
 12*a*b^2*integrate(1/4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*a*b^2*in
tegrate(1/4*log(x)/(c^2*x^2 - 1), x)*log(c) + a^3*x + 12*b^3*integrate(1/4*
sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*
x^2 - 1), x) - 3*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)
^2/(c^2*x^2 - 1), x) - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*
x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*b^3*integrate(1/4*arcta
n(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^2 - 1), x) - 12*a*b^2*in
tegrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^2 - 1), x) - 1
2*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c
^2*x^2 - 1), x) + 3*a*b^2*integrate(1/4*log(c^2*x^2)^2/(c^2*x^2 - 1), x) -
12*a*b^2*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*a*b^2*int
egrate(1/4*log(x)^2/(c^2*x^2 - 1), x) + 3/2*(2*c*x*arccsc(c*x) + log(sqrt(-
1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*a^2*b/c
```

Giac [F]

$$\int \left(a + b\csc^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arccsc}\,(cx) + a)^3 \, dx$$

```
[In] integrate((a+b*arccsc(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + b\csc^{-1}(cx)\right)^3 \, dx = \int \left(a + b\,\text{asin}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

```
[In] int((a + b*asin(1/(c*x)))^3,x)
[Out] int((a + b*asin(1/(c*x)))^3, x)
```

3.28 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 124

$$\begin{aligned}\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}\,dx &= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}-\left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\&\quad+\frac{3}{2}ib\left(a+b\csc^{-1}(cx)\right)^2\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\\&\quad-\frac{3}{2}b^2\left(a+b\csc^{-1}(cx)\right)\operatorname{PolyLog}\left(3,e^{2i\csc^{-1}(cx)}\right)\\&\quad-\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

```
[Out] 1/4*I*(a+b*arccsc(c*x))^4/b-(a+b*arccsc(c*x))^3*ln(1-(I/c/x+(1-1/c^2/x^2)^(
1/2))^2)+3/2*I*b*(a+b*arccsc(c*x))^2*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^
2)-3/2*b^2*(a+b*arccsc(c*x))*polylog(3,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-3/4*I
*b^3*polylog(4,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5331, 3798, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}\,dx &= -\frac{3}{2}b^2\operatorname{PolyLog}\left(3,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)\\&\quad+\frac{3}{2}ib\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2\\&\quad+\frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}-\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^3\\&\quad-\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

```
[In] Int[(a + b*ArcCsc[c*x])^3/x,x]
```

```
[Out] ((I/4)*(a + b*ArcCsc[c*x])^4)/b - (a + b*ArcCsc[c*x])^3*Log[1 - E^((2*I)*Ar
cCsc[c*x])] + ((3*I)/2)*b*(a + b*ArcCsc[c*x])^2*PolyLog[2, E^((2*I)*ArcCsc[
c*x])] - (3*b^2*(a + b*ArcCsc[c*x])*PolyLog[3, E^((2*I)*ArcCsc[c*x])])/2 -
((3*I)/4)*b^3*PolyLog[4, E^((2*I)*ArcCsc[c*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int (a+bx)^3\cot(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i(a+b\csc^{-1}(cx))^4}{4b} + 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^3}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i(a+b\csc^{-1}(cx))^4}{4b} - \left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\
&\quad + (3b)\text{Subst}\left(\int (a+bx)^2\log\left(1-e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i(a+b\csc^{-1}(cx))^4}{4b} - \left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\
&\quad + \frac{3}{2}ib\left(a+b\csc^{-1}(cx)\right)^2\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\\
&\quad - \left(3ib^2\right)\text{Subst}\left(\int (a+bx)\,\text{PolyLog}\left(2, e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i(a+b\csc^{-1}(cx))^4}{4b} - \left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\
&\quad + \frac{3}{2}ib\left(a+b\csc^{-1}(cx)\right)^2\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\\
&\quad - \frac{3}{2}b^2\left(a+b\csc^{-1}(cx)\right)\text{PolyLog}\left(3, e^{2i\csc^{-1}(cx)}\right)\\
&\quad + \frac{1}{2}\left(3b^3\right)\text{Subst}\left(\int \text{PolyLog}\left(3, e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{i(a+b\csc^{-1}(cx))^4}{4b} - (a+b\csc^{-1}(cx))^3 \log\left(1-e^{2i\csc^{-1}(cx)}\right) \\
&\quad + \frac{3}{2}ib(a+b\csc^{-1}(cx))^2 \operatorname{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right) \\
&\quad - \frac{3}{2}b^2(a+b\csc^{-1}(cx)) \operatorname{PolyLog}\left(3, e^{2i\csc^{-1}(cx)}\right) \\
&\quad - \frac{1}{4}(3ib^3)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3,x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right) \\
&= \frac{i(a+b\csc^{-1}(cx))^4}{4b} - (a+b\csc^{-1}(cx))^3 \log\left(1-e^{2i\csc^{-1}(cx)}\right) \\
&\quad + \frac{3}{2}ib(a+b\csc^{-1}(cx))^2 \operatorname{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right) \\
&\quad - \frac{3}{2}b^2(a+b\csc^{-1}(cx)) \operatorname{PolyLog}\left(3, e^{2i\csc^{-1}(cx)}\right) - \frac{3}{4}ib^3 \operatorname{PolyLog}\left(4, e^{2i\csc^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.95

$$
\begin{aligned}
\int \frac{(a+b\csc^{-1}(cx))^3}{x}\,dx &= a^3\log(cx) + \frac{3}{2}ia^2b\left(\csc^{-1}(cx)\left(\csc^{-1}(cx)+2i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\right)\right. \\
&\quad \left. + \operatorname{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\right) \\
&\quad + \frac{1}{8}iab^2\left(\pi^3 - 8\csc^{-1}(cx)^3 + 24i\csc^{-1}(cx)^2\log\left(1-e^{-2i\csc^{-1}(cx)}\right)\right. \\
&\quad - 24\csc^{-1}(cx)\operatorname{PolyLog}\left(2, e^{-2i\csc^{-1}(cx)}\right) \\
&\quad \left. + 12i\operatorname{PolyLog}\left(3, e^{-2i\csc^{-1}(cx)}\right)\right) \\
&\quad + \frac{1}{64}ib^3\left(\pi^4 - 16\csc^{-1}(cx)^4 + 64i\csc^{-1}(cx)^3\log\left(1-e^{-2i\csc^{-1}(cx)}\right)\right. \\
&\quad - 96\csc^{-1}(cx)^2\operatorname{PolyLog}\left(2, e^{-2i\csc^{-1}(cx)}\right) \\
&\quad + 96i\csc^{-1}(cx)\operatorname{PolyLog}\left(3, e^{-2i\csc^{-1}(cx)}\right) \\
&\quad \left. + 48\operatorname{PolyLog}\left(4, e^{-2i\csc^{-1}(cx)}\right)\right)
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x,x]
```

```
[Out] a^3*Log[c*x] + ((3*I)/2)*a^2*b*(ArcCsc[c*x]*(ArcCsc[c*x] + (2*I)*Log[1 - E^
((2*I)*ArcCsc[c*x])]) + PolyLog[2, E^((2*I)*ArcCsc[c*x])]) + (I/8)*a*b^2*(P
i^3 - 8*ArcCsc[c*x]^3 + (24*I)*ArcCsc[c*x]^2*Log[1 - E^((-2*I)*ArcCsc[c*x])
] - 24*ArcCsc[c*x]*PolyLog[2, E^((-2*I)*ArcCsc[c*x])] + (12*I)*PolyLog[3, E
^((-2*I)*ArcCsc[c*x])]) + (I/64)*b^3*(Pi^4 - 16*ArcCsc[c*x]^4 + (64*I)*ArcC
sc[c*x]^3*Log[1 - E^((-2*I)*ArcCsc[c*x])] - 96*ArcCsc[c*x]^2*PolyLog[2, E^(
(-2*I)*ArcCsc[c*x])] + (96*I)*ArcCsc[c*x]*PolyLog[3, E^((-2*I)*ArcCsc[c*x])
] + 48*PolyLog[4, E^((-2*I)*ArcCsc[c*x])])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 607 vs. 2(169) = 338.

Time = 1.20 (sec) , antiderivative size = 608, normalized size of antiderivative = 4.90

method	result
parts	$a^3 \ln(x) + b^3 \left(\frac{i \,\mathrm{arccsc}(cx)^4}{4} - \mathrm{arccsc}(cx)^3 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 3i \,\mathrm{arccsc}(cx)^2 \,\mathrm{polylo}\right.$
derivativedivides	$a^3 \ln(cx) + b^3 \left(\frac{i \,\mathrm{arccsc}(cx)^4}{4} - \mathrm{arccsc}(cx)^3 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 3i \,\mathrm{arccsc}(cx)^2 \,\mathrm{polyl}\right.$
default	$a^3 \ln(cx) + b^3 \left(\frac{i \,\mathrm{arccsc}(cx)^4}{4} - \mathrm{arccsc}(cx)^3 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 3i \,\mathrm{arccsc}(cx)^2 \,\mathrm{polyl}\right.$

```
[In] int((a+b*arccsc(c*x))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*ln(x)+b^3*(1/4*I*arccsc(c*x)^4-arccsc(c*x)^3*ln(1-I/c/x-(1-1/c^2/x^2)^(
1/2))+3*I*arccsc(c*x)^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-6*arccsc(c*x)*
polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))-6*I*polylog(4,I/c/x+(1-1/c^2/x^2)^(1/2
))-arccsc(c*x)^3*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+3*I*arccsc(c*x)^2*polylog(
2,-I/c/x-(1-1/c^2/x^2)^(1/2))-6*arccsc(c*x)*polylog(3,-I/c/x-(1-1/c^2/x^2)^
(1/2))-6*I*polylog(4,-I/c/x-(1-1/c^2/x^2)^(1/2)))+3*a*b^2*(1/3*I*arccsc(c*x
)^3-arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+2*I*arccsc(c*x)*polylog(2
,I/c/x+(1-1/c^2/x^2)^(1/2))-2*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))-arccsc(c
*x)^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+2*I*arccsc(c*x)*polylog(2,-I/c/x-(1-1
/c^2/x^2)^(1/2))-2*polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2)))+3*a^2*b*(1/2*I*ar
ccsc(c*x)^2-arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*polylog(2,I/c/x+(
1-1/c^2/x^2)^(1/2))-arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I*polylog(2
,-I/c/x-(1-1/c^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x}\, dx = \int \frac{(b\,\mathrm{arccsc}(cx) + a)^3}{x}\, dx$$

```
[In] integrate((a+b*arccsc(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x) +
 a^3)/x, x)
```

Sympy [F]

$$\int \frac{(a + b \csc^{-1}(cx))^3}{x}\,dx = \int \frac{(a + b \operatorname{acsc}(cx))^3}{x}\,dx$$

```
[In] integrate((a+b*acsc(c*x))**3/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x, x)
```

Maxima [F]

$$\int \frac{(a + b \csc^{-1}(cx))^3}{x}\,dx = \int \frac{(b \operatorname{arccsc}(cx) + a)^3}{x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] -3/2*a*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 - 12*b^3*c^2*
integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^3 - x), x)
*log(c)^2 + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x -
 1)))*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*
arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x)*log(c) + 1
2*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*a*
b^2*c^2*integrate(1/4*x^2*log(x)/(c^2*x^3 - x), x)*log(c) + b^3*arctan2(1,
sqrt(c*x + 1)*sqrt(c*x - 1))^3*log(x) - 3/4*b^3*arctan2(1, sqrt(c*x + 1)*sq
rt(c*x - 1))*log(c^2*x^2)^2*log(x) + 24*b^3*c^2*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) - 12*b
^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(
c^2*x^3 - x), x) + 12*a*b^2*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*s
qrt(c*x - 1)))^2/(c^2*x^3 - x), x) - 3*a*b^2*c^2*integrate(1/4*x^2*log(c^2*
x^2)^2/(c^2*x^3 - x), x) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)*log(
x)/(c^2*x^3 - x), x) - 12*a*b^2*c^2*integrate(1/4*x^2*log(x)^2/(c^2*x^3 - x
), x) + 12*a^2*b*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1
)))/(c^2*x^3 - x), x) + 3/2*a*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*
log(c)^2 + 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^
2*x^3 - x), x)*log(c)^2 - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt
(c*x - 1)))*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 24*b^3*integrate(1/4*ar
ctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x)*log(c) - 12*
a*b^2*integrate(1/4*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 24*a*b^2*integr
ate(1/4*log(x)/(c^2*x^3 - x), x)*log(c) + 12*b^3*integrate(1/4*sqrt(c*x + 1
)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2*log(x)/(c^2*x^3 -
 x), x) - 3*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2*lo
g(x)/(c^2*x^3 - x), x) - 24*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(
c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + 12*b^3*integrate(1/4*arc
```

```
tan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^3 - x), x) - 12*a*b^2*
integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^3 - x), x) +
 3*a*b^2*integrate(1/4*log(c^2*x^2)^2/(c^2*x^3 - x), x) - 12*a*b^2*integrat
e(1/4*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + 12*a*b^2*integrate(1/4*log(x)
^2/(c^2*x^3 - x), x) - 12*a^2*b*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(
c*x - 1)))/(c^2*x^3 - x), x) + a^3*log(x)
```

Giac [F]

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)^3}{x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))^3/x,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x}\,dx = \int \frac{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))^3/x,x)
[Out] int((a + b*asin(1/(c*x)))^3/x, x)
```

3.29 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 80

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^2}\,dx = 6b^3c\sqrt{1-\frac{1}{c^2x^2}}+\frac{6b^2\left(a+b\csc^{-1}(cx)\right)}{x} - 3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}$$

```
[Out] 6*b^2*(a+b*arccsc(c*x))/x-(a+b*arccsc(c*x))^3/x+6*b^3*c*(1-1/c^2/x^2)^(1/2)
-3*b*c*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5331, 3377, 2718}

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^2}\,dx = \frac{6b^2\left(a+b\csc^{-1}(cx)\right)}{x}-3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2 - \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}+6b^3c\sqrt{1-\frac{1}{c^2x^2}}$$

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^2,x]
```

```
[Out] 6*b^3*c*Sqrt[1 - 1/(c^2*x^2)] + (6*b^2*(a + b*ArcCsc[c*x]))/x - 3*b*c*Sqrt[
1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2 - (a + b*ArcCsc[c*x])^3/x
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int (a+bx)^3\cos(x)\,dx, x, \csc^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\csc^{-1}(cx)\right)^3}{x} + (3bc)\text{Subst}\left(\int (a+bx)^2\sin(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= -3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2 - \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}\\
&\quad + \left(6b^2c\right)\text{Subst}\left(\int (a+bx)\cos(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{6b^2\left(a+b\csc^{-1}(cx)\right)}{x} - 3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\\
&\quad - \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x} - \left(6b^3c\right)\text{Subst}\left(\int \sin(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= 6b^3c\sqrt{1-\frac{1}{c^2x^2}} + \frac{6b^2\left(a+b\csc^{-1}(cx)\right)}{x}\\
&\quad - 3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2 - \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.69

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^2}\,dx =$$
$$-\frac{a^3 - 6ab^2 + 3a^2bc\sqrt{1-\frac{1}{c^2x^2}}x - 6b^3c\sqrt{1-\frac{1}{c^2x^2}}x + 3b\left(a^2 - 2b^2 + 2abc\sqrt{1-\frac{1}{c^2x^2}}x\right)\csc^{-1}(cx) + 3b^2\left(a\right.}{x}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^2,x]
```

```
[Out] -((a^3 - 6*a*b^2 + 3*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 - 1/(
c^2*x^2)]*x + 3*b*(a^2 - 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[c*
x] + 3*b^2*(a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[c*x]^2 + b^3*ArcCsc[c*x
]^3)/x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(76) = 152.

Time = 0.89 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.46

method	result
parts	$-\frac{a^3}{x} + b^3c\left(-\frac{\operatorname{arccsc}(cx)^3}{cx} - 3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}} + 6\sqrt{\frac{c^2x^2-1}{c^2x^2}} + \frac{6\operatorname{arccsc}(cx)}{cx}\right) + 3a\,b^2c\left(-\right.$
derivativedivides	$c\left(-\frac{a^3}{cx} + b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{cx} - 3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}} + 6\sqrt{\frac{c^2x^2-1}{c^2x^2}} + \frac{6\operatorname{arccsc}(cx)}{cx}\right) + 3a\,b^2\left(-\right.\right.$
default	$c\left(-\frac{a^3}{cx} + b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{cx} - 3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}} + 6\sqrt{\frac{c^2x^2-1}{c^2x^2}} + \frac{6\operatorname{arccsc}(cx)}{cx}\right) + 3a\,b^2\left(-\right.\right.$

```
[In] int((a+b*arccsc(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3/x+b^3*c*(-1/c/x*arccsc(c*x)^3-3*arccsc(c*x)^2*((c^2*x^2-1)/c^2/x^2)^(1
/2)+6*((c^2*x^2-1)/c^2/x^2)^(1/2)+6/c/x*arccsc(c*x))+3*a*b^2*c*(-1/c/x*arcc
sc(c*x)^2+2/c/x-2*arccsc(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a^2*b*c*(-1/c/
x*arccsc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^2-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^2}\,dx = -\frac{b^3\operatorname{arccsc}(cx)^3+3\,ab^2\operatorname{arccsc}(cx)^2+a^3-6\,ab^2+3\,(a^2b-2\,b^3)\operatorname{arccsc}(cx)+3\,\left(b^3\operatorname{arccsc}(cx)^2+2\,ab^2\,\mathrm{ar}\right.}{x}$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + a^3 - 6*a*b^2 + 3*(a^2*b - 2*
b^3)*arccsc(c*x) + 3*(b^3*arccsc(c*x)^2 + 2*a*b^2*arccsc(c*x) + a^2*b - 2*b
^3)*sqrt(c^2*x^2 - 1))/x
```

Sympy [F]

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^2}\,dx = \int \frac{\left(a+b\operatorname{acsc}(cx)\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*acsc(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.84

$$\begin{aligned}\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^2}\,dx &= -\frac{b^3\operatorname{arccsc}(cx)^3}{x}-3\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)a^2b\\ &\quad -6\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}(cx)-\frac{1}{x}\right)ab^2\\ &\quad -3\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}(cx)^2-2\,c\sqrt{-\frac{1}{c^2x^2}+1}-\frac{2\operatorname{arccsc}(cx)}{x}\right)b^3\\ &\quad -\frac{3\,ab^2\operatorname{arccsc}(cx)^2}{x}-\frac{a^3}{x}\end{aligned}$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*arccsc(c*x)^3/x - 3*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*a^2*b -
 6*(c*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x) - 1/x)*a*b^2 - 3*(c*sqrt(-1/(c^2*x
^2) + 1)*arccsc(c*x)^2 - 2*c*sqrt(-1/(c^2*x^2) + 1) - 2*arccsc(c*x)/x)*b^3
- 3*a*b^2*arccsc(c*x)^2/x - a^3/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(76) = 152$.

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.44

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx = -\left(3\,b^3\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)^2+6\,ab^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)+\frac{b^3\arcsin\left(\frac{1}{cx}\right)^3}{cx}+3\,a^2b\sqrt{-\frac{1}{c^2x^2}+\right.$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] -(3*b^3*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))^2 + 6*a*b^2*sqrt(-1/(c^2*x^2
) + 1)*arcsin(1/(c*x)) + b^3*arcsin(1/(c*x))^3/(c*x) + 3*a^2*b*sqrt(-1/(c^2
*x^2) + 1) - 6*b^3*sqrt(-1/(c^2*x^2) + 1) + 3*a*b^2*arcsin(1/(c*x))^2/(c*x)
 + 3*a^2*b*arcsin(1/(c*x))/(c*x) - 6*b^3*arcsin(1/(c*x))/(c*x) + a^3/(c*x)
- 6*a*b^2/(c*x))*c
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.94

$$\begin{aligned}\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx &= \frac{b^3\left(6\,\mathrm{asin}\left(\frac{1}{c\,x}\right)-\mathrm{asin}\left(\frac{1}{c\,x}\right)^3\right)}{x}-\frac{a^3}{x}\\ &\quad-3\,a^2\,b\,c\left(\sqrt{1-\frac{1}{c^2\,x^2}}+\frac{\mathrm{asin}\left(\frac{1}{c\,x}\right)}{c\,x}\right)\\ &\quad-b^3\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}\left(3\,\mathrm{asin}\left(\frac{1}{c\,x}\right)^2-6\right)\\ &\quad-3\,a\,b^2\,c\left(2\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\sqrt{1-\frac{1}{c^2\,x^2}}+\frac{\mathrm{asin}\left(\frac{1}{c\,x}\right)^2-2}{c\,x}\right)\end{aligned}$$

```
[In] int((a + b*asin(1/(c*x)))^3/x^2,x)
```

```
[Out] (b^3*(6*asin(1/(c*x)) - asin(1/(c*x))^3))/x - a^3/x - 3*a^2*b*c*((1 - 1/(c^
2*x^2))^(1/2) + asin(1/(c*x))/(c*x)) - b^3*c*(1 - 1/(c^2*x^2))^(1/2)*(3*asi
n(1/(c*x))^2 - 6) - 3*a*b^2*c*(2*asin(1/(c*x))*(1 - 1/(c^2*x^2))^(1/2) + (a
sin(1/(c*x))^2 - 2)/(c*x))
```

3.30 $\int \frac{(a+b\csc^{-1}(cx))^3}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^3}\,dx = \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x} - \frac{3}{8}b^3c^2\csc^{-1}(cx) + \frac{3b^2(a+b\csc^{-1}(cx))}{4x^2} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{4x} + \frac{1}{4}c^2(a+b\csc^{-1}(cx))^3 - \frac{(a+b\csc^{-1}(cx))^3}{2x^2}$$

```
[Out] -3/8*b^3*c^2*arccsc(c*x)+3/4*b^2*(a+b*arccsc(c*x))/x^2+1/4*c^2*(a+b*arccsc(
c*x))^3-1/2*(a+b*arccsc(c*x))^3/x^2+3/8*b^3*c*(1-1/c^2/x^2)^(1/2)/x-3/4*b*c
*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5331, 4489, 3392, 32, 2715, 8}

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^3}\,dx = \frac{3b^2(a+b\csc^{-1}(cx))}{4x^2} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{4x} + \frac{1}{4}c^2(a+b\csc^{-1}(cx))^3 - \frac{(a+b\csc^{-1}(cx))^3}{2x^2} + \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x} - \frac{3}{8}b^3c^2\csc^{-1}(cx)$$

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^3,x]
```

```
[Out] (3*b^3*c*Sqrt[1 - 1/(c^2*x^2)])/(8*x) - (3*b^3*c^2*ArcCsc[c*x])/8 + (3*b^2*
(a + b*ArcCsc[c*x]))/(4*x^2) - (3*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c
*x])^2)/(4*x) + (c^2*(a + b*ArcCsc[c*x])^3)/4 - (a + b*ArcCsc[c*x])^3/(2*x^
2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c^2\text{Subst}\left(\int (a+bx)^3\cos(x)\sin(x)\,dx, x, \csc^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\csc^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}\left(3bc^2\right)\text{Subst}\left(\int (a+bx)^2\sin^2(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x}\\
&\quad -\frac{\left(a+b\csc^{-1}(cx)\right)^3}{2x^2}+\frac{1}{4}\left(3bc^2\right)\text{Subst}\left(\int (a+bx)^2\,dx, x, \csc^{-1}(cx)\right)\\
&\quad -\frac{1}{4}\left(3b^3c^2\right)\text{Subst}\left(\int \sin^2(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}+\frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x}+\frac{1}{4}c^2\left(a\right.\\
&\quad \left.+b\csc^{-1}(cx)\right)^3-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{2x^2}-\frac{1}{8}\left(3b^3c^2\right)\text{Subst}\left(\int 1\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}-\frac{3}{8}b^3c^2\csc^{-1}(cx)+\frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2}\\
&\quad -\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x}+\frac{1}{4}c^2\left(a+b\csc^{-1}(cx)\right)^3-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{2x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.49

$$
\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^3}\,dx = \frac{-4a^3+6ab^2-6a^2bc\sqrt{1-\frac{1}{c^2x^2}}x+3b^3c\sqrt{1-\frac{1}{c^2x^2}}x+6b\left(-2a^2+b^2-2abc\sqrt{1-\frac{1}{c^2x^2}}x\right)\csc^{-1}(cx)-6b^2\left(b\right.}{8x^2}
$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^3,x]
```

```
[Out] (-4*a^3 + 6*a*b^2 - 6*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^3*c*Sqrt[1 - 1/
(c^2*x^2)]*x + 6*b*(-2*a^2 + b^2 - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[
c*x] - 6*b^2*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*(2 - c^2*x^2))*ArcCsc[c*x]^2
+ 2*b^3*(-2 + c^2*x^2)*ArcCsc[c*x]^3 - 3*b*(-2*a^2 + b^2)*c^2*x^2*ArcSin[1/
(c*x)])/(8*x^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(109) = 218$.

Time = 1.07 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.54

method	result
derivativedivides	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^3}{2c^2x^2}-\frac{3\operatorname{arccsc}(cx)^2\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx}-\frac{3(c^2x^2-1)\operatorname{arccsc}(cx)}{4c^2x^2}\right.\right.$
default	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^3}{2c^2x^2}-\frac{3\operatorname{arccsc}(cx)^2\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx}-\frac{3(c^2x^2-1)\operatorname{arccsc}(cx)}{4c^2x^2}\right.\right.$
parts	$-\frac{a^3}{2x^2}+b^3c^2\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^3}{2c^2x^2}-\frac{3\operatorname{arccsc}(cx)^2\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx}-\frac{3(c^2x^2-1)\operatorname{arccsc}(cx)}{4c^2x^2}+\frac{3}{}\right.$

```
[In] int((a+b*arccsc(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a^3/c^2/x^2+b^3*(1/2*(c^2*x^2-1)/c^2/x^2*arccsc(c*x)^3-3/4*arccsc
(c*x)^2*(arccsc(c*x)*c*x+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x-3/4*(c^2*x^2-1)/c
^2/x^2*arccsc(c*x)+3/8/c/x*((c^2*x^2-1)/c^2/x^2)^(1/2)+3/8*arccsc(c*x)+1/2*
arccsc(c*x)^3)+3*a*b^2*(1/2*(c^2*x^2-1)/c^2/x^2*arccsc(c*x)^2-1/2*arccsc(c*
x)*(arccsc(c*x)*c*x+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x+1/4*arccsc(c*x)^2+1/4/
c^2/x^2)+3*a^2*b*(-1/2/c^2/x^2*arccsc(c*x)-1/4*(c^2*x^2-1)^(1/2)*(-arctan(1
/(c^2*x^2-1)^(1/2))*c^2*x^2+(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)/
c^3/x^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.20

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^3}\,dx$$
$$=\frac{2\,(b^3c^2x^2-2\,b^3)\operatorname{arccsc}\left(cx\right)^3-4\,a^3+6\,ab^2+6\,(ab^2c^2x^2-2\,ab^2)\operatorname{arccsc}\left(cx\right)^2+3\,((2\,a^2b-b^3)c^2x^2-4\,a^2b}{8\,x^2}$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(b^3*c^2*x^2 - 2*b^3)*arccsc(c*x)^3 - 4*a^3 + 6*a*b^2 + 6*(a*b^2*c^2
*x^2 - 2*a*b^2)*arccsc(c*x)^2 + 3*((2*a^2*b - b^3)*c^2*x^2 - 4*a^2*b + 2*b^
3)*arccsc(c*x) - 3*(2*b^3*arccsc(c*x)^2 + 4*a*b^2*arccsc(c*x) + 2*a^2*b - b
^3)*sqrt(c^2*x^2 - 1))/x^2
```

Sympy [F]

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^3}\,dx = \int \frac{(a + b\,\mathrm{acsc}\,(cx))^3}{x^3}\,dx$$

```
[In] integrate((a+b*acsc(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**3, x)
```

Maxima [F]

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^3}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*a^2*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/2*a^3/x^
2 - 1/8*(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan2(1,
 sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(a*b^2*c^2*(log(c*x + 1)
+ log(c*x - 1) - 2*log(x))*log(c)^2 + 16*b^3*c^2*integrate(1/8*x^2*arctan(1
/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^5 - x^3), x)*log(c)^2 - 16*b^3*c^2*i
ntegrate(1/8*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*
x^5 - x^3), x)*log(c) + 32*b^3*c^2*integrate(1/8*x^2*arctan(1/(sqrt(c*x + 1
)*sqrt(c*x - 1)))*log(x)/(c^2*x^5 - x^3), x)*log(c) - 16*a*b^2*c^2*integrat
e(1/8*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 32*a*b^2*c^2*integrate(
1/8*x^2*log(x)/(c^2*x^5 - x^3), x)*log(c) - 16*b^3*c^2*integrate(1/8*x^2*ar
ctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3),
x) + 16*b^3*c^2*integrate(1/8*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*l
og(x)^2/(c^2*x^5 - x^3), x) - 16*a*b^2*c^2*integrate(1/8*x^2*arctan(1/(sqrt
(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^5 - x^3), x) + 8*b^3*c^2*integrate(1/8*x
^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^5 - x^3), x)
 + 4*a*b^2*c^2*integrate(1/8*x^2*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) - 16*a*
b^2*c^2*integrate(1/8*x^2*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) + 16*a*b^
2*c^2*integrate(1/8*x^2*log(x)^2/(c^2*x^5 - x^3), x) - (c^2*log(c*x + 1) +
c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*a*b^2*log(c)^2 - 16*b^3*integrate(
1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^5 - x^3), x)*log(c)^2 +
16*b^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(
c^2*x^5 - x^3), x)*log(c) - 32*b^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sq
rt(c*x - 1)))*log(x)/(c^2*x^5 - x^3), x)*log(c) + 16*a*b^2*integrate(1/8*lo
g(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 32*a*b^2*integrate(1/8*log(x)/(c^2*
x^5 - x^3), x)*log(c) + 8*b^3*integrate(1/8*sqrt(c*x + 1)*sqrt(c*x - 1)*arc
tan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^5 - x^3), x) - 2*b^3*integrat
```

```
e(1/8*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) + 16*b
^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x
)/(c^2*x^5 - x^3), x) - 16*b^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c
*x - 1)))*log(x)^2/(c^2*x^5 - x^3), x) + 16*a*b^2*integrate(1/8*arctan(1/(s
qrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^5 - x^3), x) - 8*b^3*integrate(1/8*ar
ctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^5 - x^3), x) - 4*
a*b^2*integrate(1/8*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) + 16*a*b^2*integrate
(1/8*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) - 16*a*b^2*integrate(1/8*log(x
)^2/(c^2*x^5 - x^3), x))*x^2)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(109) = 218$.

Time = 0.32 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.42

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^3}\,dx =$$
$$-\frac{1}{8}\left(4\,b^3c\left(\frac{1}{c^2x^2} - 1\right)\arcsin\left(\frac{1}{cx}\right)^3 + 12\,ab^2c\left(\frac{1}{c^2x^2} - 1\right)\arcsin\left(\frac{1}{cx}\right)^2 + 2\,b^3c\arcsin\left(\frac{1}{cx}\right)^3 + 12\,a^2b\right.$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^3 + 12*a*b^2*c*(1/(c^2*x^2)
 - 1)*arcsin(1/(c*x))^2 + 2*b^3*c*arcsin(1/(c*x))^3 + 12*a^2*b*c*(1/(c^2*x^
2) - 1)*arcsin(1/(c*x)) - 6*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 6*a*b
^2*c*arcsin(1/(c*x))^2 + 4*a^3*c*(1/(c^2*x^2) - 1) - 6*a*b^2*c*(1/(c^2*x^2)
 - 1) + 6*a^2*b*c*arcsin(1/(c*x)) - 3*b^3*c*arcsin(1/(c*x)) + 6*b^3*sqrt(-1
/(c^2*x^2) + 1)*arcsin(1/(c*x))^2/x - 3*a*b^2*c + 12*a*b^2*sqrt(-1/(c^2*x^2
) + 1)*arcsin(1/(c*x))/x + 6*a^2*b*sqrt(-1/(c^2*x^2) + 1)/x - 3*b^3*sqrt(-1
/(c^2*x^2) + 1)/x)*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^3}\,dx = \int \frac{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x^3}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))^3/x^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^3/x^3, x)
```

3.31 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 170

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^4}\,dx = \frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}}-\frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}+\frac{2b^2\left(a+b\csc^{-1}(cx)\right)}{9x^3}+\frac{4b^2c^2\left(a+b\csc^{-1}(cx)\right)}{3x}-\frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{3x^2}-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{3x^3}$$

```
[Out] -2/27*b^3*c^3*(1-1/c^2/x^2)^(3/2)+2/9*b^2*(a+b*arccsc(c*x))/x^3+4/3*b^2*c^2
*(a+b*arccsc(c*x))/x-1/3*(a+b*arccsc(c*x))^3/x^3+14/9*b^3*c^3*(1-1/c^2/x^2)
^(1/2)-2/3*b*c^3*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)-1/3*b*c*(a+b*arccs
c(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5331, 4489, 3392, 3377, 2718, 2713}

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^4}\,dx=\frac{4b^2c^2(a+b\csc^{-1}(cx))}{3x}+\frac{2b^2(a+b\csc^{-1}(cx))}{9x^3}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{3x^2}-\frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2-\frac{(a+b\csc^{-1}(cx))^3}{3x^3}-\frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}+\frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}}$$

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^4,x]
```

```
[Out] (14*b^3*c^3*Sqrt[1 - 1/(c^2*x^2)])/9 - (2*b^3*c^3*(1 - 1/(c^2*x^2))^(3/2))/
27 + (2*b^2*(a + b*ArcCsc[c*x]))/(9*x^3) + (4*b^2*c^2*(a + b*ArcCsc[c*x]))/
(3*x) - (2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2)/3 - (b*c*Sqrt
[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2)/(3*x^2) - (a + b*ArcCsc[c*x])^3/(3
*x^3)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^3\text{Subst}\left(\int (a+bx)^3\cos(x)\sin^2(x)\,dx, x, \csc^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\csc^{-1}(cx)\right)^3}{3x^3} + \left(bc^3\right)\text{Subst}\left(\int (a+bx)^2\sin^3(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{2b^2\left(a+b\csc^{-1}(cx)\right)}{9x^3} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{3x^2} - \frac{\left(a+b\csc^{-1}(cx)\right)^3}{3x^3}\\
&\quad + \frac{1}{3}\left(2bc^3\right)\text{Subst}\left(\int (a+bx)^2\sin(x)\,dx, x, \csc^{-1}(cx)\right)\\
&\quad - \frac{1}{9}\left(2b^3c^3\right)\text{Subst}\left(\int \sin^3(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{2b^2\left(a+b\csc^{-1}(cx)\right)}{9x^3} - \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\\
&\quad - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{3x^2} - \frac{\left(a+b\csc^{-1}(cx)\right)^3}{3x^3}\\
&\quad + \frac{1}{3}\left(4b^2c^3\right)\text{Subst}\left(\int (a+bx)\cos(x)\,dx, x, \csc^{-1}(cx)\right)\\
&\quad + \frac{1}{9}\left(2b^3c^3\right)\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)
\end{aligned}$$

$$= \frac{2}{9} b^3 c^3 \sqrt{1 - \frac{1}{c^2 x^2}} - \frac{2}{27} b^3 c^3 \left(1 - \frac{1}{c^2 x^2}\right)^{3/2} + \frac{2b^2 (a + b \csc^{-1}(cx))}{9x^3} + \frac{4b^2 c^2 (a + b \csc^{-1}(cx))}{3x} - \frac{2}{3} b c^3 \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \csc^{-1}(cx))^2 - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}} (a + b \csc^{-1}(cx))^2}{3x^2} - \frac{(a + b \csc^{-1}(cx))^3}{3x^3} - \frac{1}{3}(4b^3c^3) \operatorname{Subst}\left(\int \sin(x)\, dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{14}{9} b^3 c^3 \sqrt{1 - \frac{1}{c^2 x^2}} - \frac{2}{27} b^3 c^3 \left(1 - \frac{1}{c^2 x^2}\right)^{3/2} + \frac{2b^2 (a + b \csc^{-1}(cx))}{9x^3} + \frac{4b^2 c^2 (a + b \csc^{-1}(cx))}{3x} - \frac{2}{3} b c^3 \sqrt{1 - \frac{1}{c^2 x^2}} (a + b \csc^{-1}(cx))^2 - \frac{bc\sqrt{1 - \frac{1}{c^2x^2}} (a + b \csc^{-1}(cx))^2}{3x^2} - \frac{(a + b \csc^{-1}(cx))^3}{3x^3}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.20

$$\int \frac{(a + b \csc^{-1}(cx))^3}{x^4}\, dx = \frac{-9a^3 - 9a^2bc\sqrt{1 - \frac{1}{c^2x^2}}x(1 + 2c^2x^2) + 6ab^2(1 + 6c^2x^2) + 2b^3c\sqrt{1 - \frac{1}{c^2x^2}}x(1 + 20c^2x^2) + 3b\Big(-9a^2 - 6ab}{}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^4,x]
```

```
[Out] (-9*a^3 - 9*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 6*a*b^2*(1 +
6*c^2*x^2) + 2*b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 20*c^2*x^2) + 3*b*(-9*a^2
 - 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 2*b^2*(1 + 6*c^2*x^2))
*ArcCsc[c*x] - 9*b^2*(3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*Ar
cCsc[c*x]^2 - 9*b^3*ArcCsc[c*x]^3)/(27*x^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(148) = 296.

Time = 1.51 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.76

method	result
derivativedivides	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}+\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arccsc}(cx)}{3cx}+\frac{2\operatorname{arccsc}(}{9c^3x^3}\right.\right.$
default	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}+\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arccsc}(cx)}{3cx}+\frac{2\operatorname{arccsc}(}{9c^3x^3}\right.\right.$
parts	$-\frac{a^3}{3x^3}+b^3c^3\left(-\frac{\operatorname{arccsc}(cx)^3}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}+\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arccsc}(cx)}{3cx}+\frac{2\operatorname{arccsc}(cx)}{9c^3x^3}\right.$

```
[In] int((a+b*arccsc(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*arccsc(c*x)^3-1/3*arccsc(c*x)^2*(2*
c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+4/3*((c^2*x^2-1)/c^2/x^2)^(1
/2)+4/3/c/x*arccsc(c*x)+2/9/c^3/x^3*arccsc(c*x)+2/27*(2*c^2*x^2+1)/c^2/x^2*
((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a*b^2*(-1/3/c^3/x^3*arccsc(c*x)^2-2/9*arccsc
(c*x)*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/
x)+3*a^2*b*(-1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^
2-1)/c^2/x^2)^(1/2)/c^4/x^4))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^4}\,dx$$
$$=\frac{36\,ab^2c^2x^2-9\,b^3\operatorname{arccsc}(cx)^3-27\,ab^2\operatorname{arccsc}(cx)^2-9\,a^3+6\,ab^2+3\,(12\,b^3c^2x^2-9\,a^2b+2\,b^3)\operatorname{arccsc}(cx)-}{}$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(36*a*b^2*c^2*x^2 - 9*b^3*arccsc(c*x)^3 - 27*a*b^2*arccsc(c*x)^2 - 9*a
^3 + 6*a*b^2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b + 2*b^3)*arccsc(c*x) - (2*(9*a^2
*b - 20*b^3)*c^2*x^2 + 9*a^2*b - 2*b^3 + 9*(2*b^3*c^2*x^2 + b^3)*arccsc(c*x
)^2 + 18*(2*a*b^2*c^2*x^2 + a*b^2)*arccsc(c*x))*sqrt(c^2*x^2 - 1))/x^3
```

Sympy [F]

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^4} \, dx = \int \frac{(a + b\operatorname{acsc}(cx))^3}{x^4} \, dx$$

```
[In] integrate((a+b*acsc(c*x))**3/x**4,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**4, x)
```

Maxima [F]

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^4} \, dx = \int \frac{(b\operatorname{arccsc}(cx) + a)^3}{x^4} \, dx$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c
- 3*arccsc(c*x)/x^3) - a*b^2*arccsc(c*x)^2/x^3 + 1/12*(12*x^3*integrate(-1/
4*(12*c^2*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 12*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 + 12*(c^2*x^2*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2 +
sqrt(c*x + 1)*sqrt(c*x - 1)*(4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 -
log(c^2*x^2)^2) - 4*((3*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)
- c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^2 - 3*arctan2(1, sqrt(c*x
+ 1)*sqrt(c*x - 1))*log(c) + 3*(c^2*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
 1)) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x) + arctan2(1, sqrt(c*
x + 1)*sqrt(c*x - 1)))*log(c^2*x^2) + 24*(c^2*x^2*arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1))*log(c) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c))*log
(x))/(c^2*x^6 - x^4), x) - 4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 3*
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2)*b^3/x^3 - 1/3*a^3/x
^3 - 2/9*(6*c^5*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*c^3*x^2*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - (6*c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c
*x - 1) - 3*c*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)
*sqrt(c*x - 1)*c*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(148) = 296.

Time = 0.31 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.52

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^4}\,dx$$
$$= \frac{1}{27}\left(9\,b^3c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)^2 + 18\,ab^2c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right) - 27\,b^3c^2\sqrt{-\frac{1}{c^2x^2}+1}\,\mathrm{arcs}\right.$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] 1/27*(9*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))^2 + 18*a*b^2*c^2*(
-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x)) - 27*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)
*arcsin(1/(c*x))^2 - 9*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^3/x + 9*a^2*
b*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 2*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 54*a*
b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) - 27*a*b^2*c*(1/(c^2*x^2) -
1)*arcsin(1/(c*x))^2/x - 9*b^3*c*arcsin(1/(c*x))^3/x - 27*a^2*b*c^2*sqrt(-1
/(c^2*x^2) + 1) + 42*b^3*c^2*sqrt(-1/(c^2*x^2) + 1) - 27*a^2*b*c*(1/(c^2*x^
2) - 1)*arcsin(1/(c*x))/x + 6*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 2
7*a*b^2*c*arcsin(1/(c*x))^2/x + 6*a*b^2*c*(1/(c^2*x^2) - 1)/x - 27*a^2*b*c*
arcsin(1/(c*x))/x + 42*b^3*c*arcsin(1/(c*x))/x + 42*a*b^2*c/x - 9*a^3/(c*x^
3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^4}\,dx = \int \frac{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x^4}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))^3/x^4,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^3/x^4, x)
```

3.32 $\int \frac{(a+b\csc^{-1}(cx))^3}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 208

$$\begin{aligned}\int \frac{(a+b\csc^{-1}(cx))^3}{x^5}\,dx =& \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3}+\frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x}-\frac{45}{256}b^3c^4\csc^{-1}(cx)\\&+\frac{3b^2(a+b\csc^{-1}(cx))}{32x^4}+\frac{9b^2c^2(a+b\csc^{-1}(cx))}{32x^2}\\&-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2}{16x^3}\\&-\frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2}{32x}\\&+\frac{3}{32}c^4(a+b\csc^{-1}(cx))^3-\frac{(a+b\csc^{-1}(cx))^3}{4x^4}\end{aligned}$$

```
[Out] -45/256*b^3*c^4*arccsc(c*x)+3/32*b^2*(a+b*arccsc(c*x))/x^4+9/32*b^2*c^2*(a+
b*arccsc(c*x))/x^2+3/32*c^4*(a+b*arccsc(c*x))^3-1/4*(a+b*arccsc(c*x))^3/x^4
+3/128*b^3*c*(1-1/c^2/x^2)^(1/2)/x^3+45/256*b^3*c^3*(1-1/c^2/x^2)^(1/2)/x-3
/16*b*c*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^3-9/32*b*c^3*(a+b*arccsc(
c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5331, 4489, 3392, 32, 2715, 8}

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^5}\,dx = \frac{9b^2c^2(a+b\csc^{-1}(cx))}{32x^2}+\frac{3b^2(a+b\csc^{-1}(cx))}{32x^4} + \frac{3}{32}c^4(a+b\csc^{-1}(cx))^3-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{16x^3} - \frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{32x}-\frac{(a+b\csc^{-1}(cx))^3}{4x^4} - \frac{45}{256}b^3c^4\csc^{-1}(cx)+\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3}+\frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x}$$

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^5,x]
```

```
[Out] (3*b^3*c*Sqrt[1 - 1/(c^2*x^2)])/(128*x^3) + (45*b^3*c^3*Sqrt[1 - 1/(c^2*x^2
)])/(256*x) - (45*b^3*c^4*ArcCsc[c*x])/256 + (3*b^2*(a + b*ArcCsc[c*x]))/(3
2*x^4) + (9*b^2*c^2*(a + b*ArcCsc[c*x]))/(32*x^2) - (3*b*c*Sqrt[1 - 1/(c^2*
x^2)]*(a + b*ArcCsc[c*x])^2)/(16*x^3) - (9*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a +
 b*ArcCsc[c*x])^2)/(32*x) + (3*c^4*(a + b*ArcCsc[c*x])^3)/32 - (a + b*ArcCs
c[c*x])^3/(4*x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
```

```
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^4\text{Subst}\left(\int (a+bx)^3\cos(x)\sin^3(x)\,dx,x,\csc^{-1}(cx)\right)\right)\\
&= -\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4x^4}+\frac{1}{4}\left(3bc^4\right)\text{Subst}\left(\int (a+bx)^2\sin^4(x)\,dx,x,\csc^{-1}(cx)\right)\\
&= \frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{32x^4}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{16x^3}-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4x^4}\\
&\quad+\frac{1}{16}\left(9bc^4\right)\text{Subst}\left(\int (a+bx)^2\sin^2(x)\,dx,x,\csc^{-1}(cx)\right)\\
&\quad-\frac{1}{32}\left(3b^3c^4\right)\text{Subst}\left(\int \sin^4(x)\,dx,x,\csc^{-1}(cx)\right)\\
&= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3}+\frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{32x^4}+\frac{9b^2c^2\left(a+b\csc^{-1}(cx)\right)}{32x^2}\\
&\quad-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{16x^3}-\frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{32x}\\
&\quad-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4x^4}+\frac{1}{32}\left(9bc^4\right)\text{Subst}\left(\int (a+bx)^2\,dx,x,\csc^{-1}(cx)\right)\\
&\quad-\frac{1}{128}\left(9b^3c^4\right)\text{Subst}\left(\int \sin^2(x)\,dx,x,\csc^{-1}(cx)\right)\\
&\quad-\frac{1}{32}\left(9b^3c^4\right)\text{Subst}\left(\int \sin^2(x)\,dx,x,\csc^{-1}(cx)\right)
\end{aligned}$$

$$= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3} + \frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x} + \frac{3b^2(a+b\csc^{-1}(cx))}{32x^4} + \frac{9b^2c^2(a+b\csc^{-1}(cx))}{32x^2} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{16x^3} - \frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{32x} + \frac{3}{32}c^4(a+b\csc^{-1}(cx))^3 - \frac{(a+b\csc^{-1}(cx))^3}{4x^4} - \frac{1}{256}(9b^3c^4)\,\text{Subst}\left(\int 1\,dx, x, \csc^{-1}(cx)\right) - \frac{1}{64}(9b^3c^4)\,\text{Subst}\left(\int 1\,dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3} + \frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x} - \frac{45}{256}b^3c^4\csc^{-1}(cx) + \frac{3b^2(a+b\csc^{-1}(cx))}{32x^4} + \frac{9b^2c^2(a+b\csc^{-1}(cx))}{32x^2} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{16x^3} - \frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{32x} + \frac{3}{32}c^4(a+b\csc^{-1}(cx))^3 - \frac{(a+b\csc^{-1}(cx))^3}{4x^4}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^5}\,dx$$

$$= \frac{-64a^3 + 24ab^2 - 48a^2bc\sqrt{1-\frac{1}{c^2x^2}}x + 6b^3c\sqrt{1-\frac{1}{c^2x^2}}x + 72ab^2c^2x^2 - 72a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}x^3 + 45b^3c^3\sqrt{1-}}{}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^5,x]
```

```
[Out] (-64*a^3 + 24*a*b^2 - 48*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 6*b^3*c*Sqrt[1 -
 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 - 72*a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3
 + 45*b^3*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + 24*b*(-8*a^2 + b^2*(1 + 3*c^2*x^2
) - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2))*ArcCsc[c*x] - 24*b^2*(
b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2) + a*(8 - 3*c^4*x^4))*ArcCsc[c*x
]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcCsc[c*x]^3 + 9*b*(8*a^2 - 5*b^2)*c^4*x^4*Ar
cSin[1/(c*x)])/(256*x^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(182) = 364$.

Time = 1.66 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.30

method	result
parts	$-\frac{a^3}{4x^4}+b^3c^4\left(-\frac{\operatorname{arccsc}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arccsc}(cx)^2\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arccsc}(cx)}{32c^4x^4}\right.$
derivativedivides	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arccsc}(cx)^2\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arccsc}}{32c^4x}\right.\right.$
default	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arccsc}(cx)^2\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arccsc}}{32c^4x}\right.\right.$

```
[In] int((a+b*arccsc(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^3/x^4+b^3*c^4*(-1/4/c^4/x^4*arccsc(c*x)^3+3/32*arccsc(c*x)^2*(3*c^3*
x^3*arccsc(c*x)-3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-2*((c^2*x^2-1)/c^2/x^
2)^(1/2))/c^3/x^3+3/32*arccsc(c*x)/c^4/x^4+3/256*(3*c^2*x^2+2)/c^3/x^3*((c^
2*x^2-1)/c^2/x^2)^(1/2)+27/256*arccsc(c*x)-9/32*(c^2*x^2-1)/c^2/x^2*arccsc(
c*x)+9/64/c/x*((c^2*x^2-1)/c^2/x^2)^(1/2)-3/16*arccsc(c*x)^3)+3*a*b^2*c^4*(
-1/4/c^4/x^4*arccsc(c*x)^2+1/16*arccsc(c*x)*(3*c^3*x^3*arccsc(c*x)-3*c^2*x^
2*((c^2*x^2-1)/c^2/x^2)^(1/2)-2*((c^2*x^2-1)/c^2/x^2)^(1/2))/c^3/x^3-3/32*a
rccsc(c*x)^2+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-3/4*a^2*b/x^4*arccsc(c*x)+9/32*
a^2*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2
-1)^(1/2))-9/32*a^2*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^3-3/16*a^
2*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^5}\,dx = \frac{72\,ab^2c^2x^2+8\,(3\,b^3c^4x^4-8\,b^3)\operatorname{arccsc}(cx)^3-64\,a^3+24\,ab^2+24\,(3\,ab^2c^4x^4-8\,ab^2)\operatorname{arccsc}(cx)^2+3\,(3\,(8\cdots}{\cdots}$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*arccsc(c*x)^3 - 64*a^3
+ 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8*a*b^2)*arccsc(c*x)^2 + 3*(3*(8*a^2*b -
 5*b^3)*c^4*x^4 + 24*b^3*c^2*x^2 - 64*a^2*b + 8*b^3)*arccsc(c*x) - 3*(3*(8*
```

```
a^2*b - 5*b^3)*c^2*x^2 + 16*a^2*b - 2*b^3 + 8*(3*b^3*c^2*x^2 + 2*b^3)*arccs
c(c*x)^2 + 16*(3*a*b^2*c^2*x^2 + 2*a*b^2)*arccsc(c*x))*sqrt(c^2*x^2 - 1))/x
^4
```

Sympy [F]

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^5}\,dx = \int \frac{(a + b\operatorname{acsc}(cx))^3}{x^5}\,dx$$

```
[In] integrate((a+b*acsc(c*x))**3/x**5,x)
[Out] Integral((a + b*acsc(c*x))**3/x**5, x)
```

Maxima [F]

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^5}\,dx = \int \frac{(b\operatorname{arccsc}(cx) + a)^3}{x^5}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="maxima")
[Out] -3/32*a^2*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^
2*x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) -
 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/4*a^3/
x^4 - 1/16*(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(2*(c^2*log(c*x + 1) +
 c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*a*b^2*c^2*log(c)^2 + 64*b^3*c^2*i
ntegrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^7 - x^5),
x)*log(c)^2 - 64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*
x - 1)))*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 128*b^3*c^2*integrate(1/
16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^7 - x^5), x)*l
og(c) - 64*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*lo
g(c) + 128*a*b^2*c^2*integrate(1/16*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) -
 64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(
c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 64*b^3*c^2*integrate(1/16*x^2*arctan(
1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^7 - x^5), x) - 64*a*b^2*c^
2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^7 - x
^5), x) + 16*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x -
1)))*log(c^2*x^2)/(c^2*x^7 - x^5), x) + 16*a*b^2*c^2*integrate(1/16*x^2*log
(c^2*x^2)^2/(c^2*x^7 - x^5), x) - 64*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x
^2)*log(x)/(c^2*x^7 - x^5), x) + 64*a*b^2*c^2*integrate(1/16*x^2*log(x)^2/(
c^2*x^7 - x^5), x) - (2*c^4*log(c*x + 1) + 2*c^4*log(c*x - 1) - 4*c^4*log(x
) + (2*c^2*x^2 + 1)/x^4)*a*b^2*log(c)^2 - 64*b^3*integrate(1/16*arctan(1/(s
```

```
qrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^7 - x^5), x)*log(c)^2 + 64*b^3*integrat
e(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^7 - x^5)
, x)*log(c) - 128*b^3*integrate(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))
)*log(x)/(c^2*x^7 - x^5), x)*log(c) + 64*a*b^2*integrate(1/16*log(c^2*x^2)/
(c^2*x^7 - x^5), x)*log(c) - 128*a*b^2*integrate(1/16*log(x)/(c^2*x^7 - x^5
), x)*log(c) + 16*b^3*integrate(1/16*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(
sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^7 - x^5), x) - 4*b^3*integrate(1/16*
sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2/(c^2*x^7 - x^5), x) + 64*b^3*int
egrate(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^
2*x^7 - x^5), x) - 64*b^3*integrate(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x -
 1)))*log(x)^2/(c^2*x^7 - x^5), x) + 64*a*b^2*integrate(1/16*arctan(1/(sqrt
(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^7 - x^5), x) - 16*b^3*integrate(1/16*arc
tan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^7 - x^5), x) - 16*
a*b^2*integrate(1/16*log(c^2*x^2)^2/(c^2*x^7 - x^5), x) + 64*a*b^2*integrat
e(1/16*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) - 64*a*b^2*integrate(1/16*lo
g(x)^2/(c^2*x^7 - x^5), x))*x^4)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(182) = 364$.

Time = 0.32 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.77

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^5}\,dx =$$

$$-\frac{1}{256}\left(64\,b^3c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)^3+192\,ab^2c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)^2+128\,b^3c^3\left(\frac{1}{c^2x^2}-1\right)\right.$$

```
[In] integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] -1/256*(64*b^3*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^3 + 192*a*b^2*c^3*(1
/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^2 + 128*b^3*c^3*(1/(c^2*x^2) - 1)*arcsin(
1/(c*x))^3 + 192*a^2*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) - 24*b^3*c^3
*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 384*a*b^2*c^3*(1/(c^2*x^2) - 1)*arcs
in(1/(c*x))^2 + 40*b^3*c^3*arcsin(1/(c*x))^3 - 24*a*b^2*c^3*(1/(c^2*x^2) -
1)^2 + 384*a^2*b*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) - 120*b^3*c^3*(1/(c^
2*x^2) - 1)*arcsin(1/(c*x)) + 120*a*b^2*c^3*arcsin(1/(c*x))^2 - 48*b^3*c^2*
(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))^2/x - 120*a*b^2*c^3*(1/(c^2*x^2) -
 1) + 120*a^2*b*c^3*arcsin(1/(c*x)) - 51*b^3*c^3*arcsin(1/(c*x)) - 96*a*b^2
*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))/x + 120*b^3*c^2*sqrt(-1/(c^2*
x^2) + 1)*arcsin(1/(c*x))^2/x - 51*a*b^2*c^3 - 48*a^2*b*c^2*(-1/(c^2*x^2) +
 1)^(3/2)/x + 6*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2)/x + 240*a*b^2*c^2*sqrt(-1/
(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 120*a^2*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x -
 51*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 64*a^3/(c*x^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^5}\,dx = \int \frac{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x^5}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))^3/x^5,x)
[Out] int((a + b*asin(1/(c*x)))^3/x^5, x)
```

3.33 $\int \frac{x}{a+b\csc^{-1}(cx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx = \text{Int}\left(\frac{x}{a+b\csc^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x/(a+b*arccsc(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx = \int \frac{x}{a+b\csc^{-1}(cx)}\,dx$$

```
[In] Int[x/(a + b*ArcCsc[c*x]),x]
```

```
[Out] Defer[Int][x/(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{a+b\csc^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx=\int \frac{x}{a+b\csc^{-1}(cx)}\,dx$$

```
[In] Integrate[x/(a + b*ArcCsc[c*x]),x]
[Out] Integrate[x/(a + b*ArcCsc[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\ \mathrm{arccsc}\left(cx\right)}dx$$

```
[In] int(x/(a+b*arccsc(c*x)),x)
[Out] int(x/(a+b*arccsc(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx=\int \frac{x}{b\,\mathrm{arccsc}\left(cx\right)+a}\,dx$$

```
[In] integrate(x/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(x/(b*arccsc(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx=\int \frac{x}{a+b\,\mathrm{acsc}\left(cx\right)}\,dx$$

```
[In] integrate(x/(a+b*acsc(c*x)),x)
[Out] Integral(x/(a + b*acsc(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx = \int \frac{x}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

```
[In] integrate(x/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate(x/(b*arccsc(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 33.77 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx = \int \frac{x}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

```
[In] integrate(x/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate(x/(b*arccsc(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx = \int \frac{x}{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}\,dx$$

```
[In] int(x/(a + b*asin(1/(c*x))),x)
[Out] int(x/(a + b*asin(1/(c*x))), x)
```

3.34 $\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx = \text{Int}\left(\frac{1}{a+b\csc^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arccsc(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx = \int \frac{1}{a+b\csc^{-1}(cx)}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])^(-1),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])^(-1), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{a+b\csc^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx=\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])^(-1),x]
[Out] Integrate[(a + b*ArcCsc[c*x])^(-1), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\ \mathrm{arccsc}\left(cx\right)}dx$$

```
[In] int(1/(a+b*arccsc(c*x)),x)
[Out] int(1/(a+b*arccsc(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx=\int \frac{1}{b\,\mathrm{arccsc}\left(cx\right)+a}\,dx$$

```
[In] integrate(1/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*arccsc(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx=\int \frac{1}{a+b\,\mathrm{acsc}\left(cx\right)}\,dx$$

```
[In] integrate(1/(a+b*acsc(c*x)),x)
[Out] Integral(1/(a + b*acsc(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx=\int \frac{1}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

```
[In] integrate(1/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate(1/(b*arccsc(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 11.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx=\int \frac{1}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

```
[In] integrate(1/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate(1/(b*arccsc(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx=\int \frac{1}{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}\,dx$$

```
[In] int(1/(a + b*asin(1/(c*x))),x)
[Out] int(1/(a + b*asin(1/(c*x))), x)
```

3.35 $\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\csc^{-1}(cx)\right)},x\right)$$

`[Out] Unintegrable(1/x/(a+b*arccsc(c*x)),x)`

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*ArcCsc[c*x])),x]
[Out] Defer[Int][1/(x*(a + b*ArcCsc[c*x])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcCsc[c*x])),x]
[Out] Integrate[1/(x*(a + b*ArcCsc[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\ \mathrm{arccsc}\left(cx\right)\right)}dx$$

```
[In] int(1/x/(a+b*arccsc(c*x)),x)
[Out] int(1/x/(a+b*arccsc(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arccsc}\left(cx\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x*arccsc(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{acsc}\left(cx\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*acsc(c*x)),x)
[Out] Integral(1/(x*(a + b*acsc(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\,(a+b\csc^{-1}(cx))}\,dx = \int \frac{1}{(b\,\mathrm{arccsc}\,(cx)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsc(c*x) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\,(a+b\csc^{-1}(cx))}\,dx = \int \frac{1}{(b\,\mathrm{arccsc}\,(cx)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arccsc(c*x) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\,(a+b\csc^{-1}(cx))}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

```
[In] int(1/(x*(a + b*asin(1/(c*x)))),x)
[Out] int(1/(x*(a + b*asin(1/(c*x)))), x)
```

3.36 $\int \frac{1}{x^2(a+b\csc^{-1}(cx))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx=-\frac{c\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}-\frac{c\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}$$

```
[Out] -c*Ci(a/b+arccsc(c*x))*cos(a/b)/b-c*Si(a/b+arccsc(c*x))*sin(a/b)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5331, 3384, 3380, 3383}

$$\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx=-\frac{c\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}-\frac{c\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}$$

```
[In] Int[1/(x^2*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -((c*Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]])/b) - (c*Sin[a/b]*SinIntegral[
a/b + ArcCsc[c*x]])/b
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c\text{Subst}\left(\int \frac{\cos(x)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right) \\
&= -\left(\left(c\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right) \\
&\quad - \left(c\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \csc^{-1}(cx)\right) \\
&= -\frac{c\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b} - \frac{c\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx = -\frac{c\left(\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)\right)}{b}$$

```
[In] Integrate[1/(x^2*(a + b*ArcCsc[c*x])),x]
[Out] -((c*(Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]] + Sin[a/b]*SinIntegral[a/b +
ArcCsc[c*x]]))/b)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$c\left(-\frac{\operatorname{Si}\left(\frac{a}{b}+\operatorname{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{\operatorname{Ci}\left(\frac{a}{b}+\operatorname{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}\right)$	48
default	$c\left(-\frac{\operatorname{Si}\left(\frac{a}{b}+\operatorname{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{\operatorname{Ci}\left(\frac{a}{b}+\operatorname{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}\right)$	48

```
[In] int(1/x^2/(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
[Out] c*(-Si(a/b+arccsc(c*x))*sin(a/b)/b-Ci(a/b+arccsc(c*x))*cos(a/b)/b)
```

Fricas [F]

$$\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\operatorname{arccsc}(cx)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^2*arccsc(c*x) + a*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\left(a+b\operatorname{acsc}(cx)\right)}\,dx$$

```
[In] integrate(1/x**2/(a+b*acsc(c*x)),x)
[Out] Integral(1/(x**2*(a + b*acsc(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\operatorname{arccsc}(cx)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsc(c*x) + a)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^2 (a + b \csc^{-1}(cx))} dx = -c \left(\frac{\cos\left(\frac{a}{b}\right) \operatorname{Ci}\left(\frac{a}{b} + \arcsin\left(\frac{1}{cx}\right)\right)}{b} + \frac{\sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \arcsin\left(\frac{1}{cx}\right)\right)}{b} \right)$$

```
[In] integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] -c*(cos(a/b)*cos_integral(a/b + arcsin(1/(c*x)))/b + sin(a/b)*sin_integral(
a/b + arcsin(1/(c*x)))/b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + b \csc^{-1}(cx))} dx = \int \frac{1}{x^2 \left(a + b \operatorname{asin}\left(\frac{1}{cx}\right)\right)} dx$$

```
[In] int(1/(x^2*(a + b*asin(1/(c*x)))),x)
[Out] int(1/(x^2*(a + b*asin(1/(c*x)))), x)
```

3.37 $\int \frac{1}{x^3(a+b\csc^{-1}(cx))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{1}{x^3\left(a+b\csc^{-1}(cx)\right)}\,dx=\frac{c^2\,\text{CosIntegral}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}$$

```
[Out] -1/2*c^2*cos(2*a/b)*Si(2*a/b+2*arccsc(c*x))/b+1/2*c^2*Ci(2*a/b+2*arccsc(c*x
))*sin(2*a/b)/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5331, 4491, 12, 3384, 3380, 3383}

$$\int \frac{1}{x^3\left(a+b\csc^{-1}(cx)\right)}\,dx=\frac{c^2\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}-\frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}$$

```
[In] Int[1/(x^3*(a + b*ArcCsc[c*x])),x]
```

```
[Out] (c^2*CosIntegral[(2*a)/b + 2*ArcCsc[c*x]]*Sin[(2*a)/b])/(2*b) - (c^2*Cos[(2
*a)/b]*SinIntegral[(2*a)/b + 2*ArcCsc[c*x]])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c^2 \text{Subst}\left(\int \frac{\cos(x)\sin(x)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right) \\
&= -\left(c^2 \text{Subst}\left(\int \frac{\sin(2x)}{2(a+bx)}\,dx, x, \csc^{-1}(cx)\right)\right) \\
&= -\left(\frac{1}{2} c^2 \text{Subst}\left(\int \frac{\sin(2x)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right) \\
&= -\left(\frac{1}{2}\left(c^2 \cos\left(\frac{2a}{b}\right)\right) \text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right) \\
&\quad + \frac{1}{2}\left(c^2 \sin\left(\frac{2a}{b}\right)\right) \text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)
\end{aligned}
$$

$$= \frac{c^2 \operatorname{CosIntegral}\left(\frac{2a}{b} + 2\csc^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b} - \frac{c^2\cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2a}{b} + 2\csc^{-1}(cx)\right)}{2b}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a+b\csc^{-1}(cx)\right)}\,dx = -\frac{c^2\left(-\operatorname{CosIntegral}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)+\cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\right)}{2b}$$

```
[In] Integrate[1/(x^3*(a + b*ArcCsc[c*x])),x]
[Out] -1/2*(c^2*(-(CosIntegral[(2*a)/b + 2*ArcCsc[c*x]]*Sin[(2*a)/b]) + Cos[(2*a)
/b]*SinIntegral[(2*a)/b + 2*ArcCsc[c*x]]))/b
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$c^2\left(-\frac{\operatorname{Si}\left(\frac{2a}{b}+2\operatorname{arccsc}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b}+\frac{\operatorname{Ci}\left(\frac{2a}{b}+2\operatorname{arccsc}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$	58
default	$c^2\left(-\frac{\operatorname{Si}\left(\frac{2a}{b}+2\operatorname{arccsc}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b}+\frac{\operatorname{Ci}\left(\frac{2a}{b}+2\operatorname{arccsc}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$	58

```
[In] int(1/x^3/(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
[Out] c^2*(-1/2*Si(2*a/b+2*arccsc(c*x))*cos(2*a/b)/b+1/2*Ci(2*a/b+2*arccsc(c*x))*
sin(2*a/b)/b)
```

Fricas [F]

$$\int \frac{1}{x^3\left(a+b\csc^{-1}(cx)\right)}\,dx = \int \frac{1}{\left(b\operatorname{arccsc}(cx)+a\right)x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^3*arccsc(c*x) + a*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a+b \csc^{-1}(cx)\right)}\,dx = \int \frac{1}{x^3 \left(a+b\,\mathrm{acsc}\,(cx)\right)}\,dx$$

```
[In] integrate(1/x**3/(a+b*acsc(c*x)),x)
[Out] Integral(1/(x**3*(a + b*acsc(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a+b \csc^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\,\mathrm{arccsc}\,(cx)+a)x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsc(c*x) + a)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^3 \left(a+b \csc^{-1}(cx)\right)}\,dx$$
$$= \frac{1}{2}\left(\frac{2\,c\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{2\,a}{b}+2\,\arcsin\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b} - \frac{2\,c\cos\left(\frac{a}{b}\right)^2\mathrm{Si}\left(\frac{2\,a}{b}+2\,\arcsin\left(\frac{1}{cx}\right)\right)}{b} + \frac{c\,\mathrm{Si}\left(\frac{2\,a}{b}+2\,\arcsin\right.}{b}\right.$$

```
[In] integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] 1/2*(2*c*cos(a/b)*cos_integral(2*a/b + 2*arcsin(1/(c*x)))*sin(a/b)/b - 2*c*
cos(a/b)^2*sin_integral(2*a/b + 2*arcsin(1/(c*x)))/b + c*sin_integral(2*a/b
 + 2*arcsin(1/(c*x)))/b)*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+b \csc^{-1}(cx)\right)}\, dx = \int \frac{1}{x^3\, \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}\, dx$$

```
[In] int(1/(x^3*(a + b*asin(1/(c*x)))),x)
[Out] int(1/(x^3*(a + b*asin(1/(c*x)))), x)
```

3.38 $\int \frac{1}{x^4(a+b\csc^{-1}(cx))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx = -\frac{c^3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b} + \frac{c^3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b} - \frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b} + \frac{c^3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b}$$

```
[Out] -1/4*c^3*Ci(a/b+arccsc(c*x))*cos(a/b)/b+1/4*c^3*Ci(3*a/b+3*arccsc(c*x))*cos
(3*a/b)/b-1/4*c^3*Si(a/b+arccsc(c*x))*sin(a/b)/b+1/4*c^3*Si(3*a/b+3*arccsc(
c*x))*sin(3*a/b)/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5331, 4491, 3384, 3380, 3383}

$$\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx = -\frac{c^3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b} + \frac{c^3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b} - \frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b} + \frac{c^3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b}$$

```
[In] Int[1/(x^4*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -1/4*(c^3*Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]])/b + (c^3*Cos[(3*a)/b]*Co
sIntegral[(3*a)/b + 3*ArcCsc[c*x]])/(4*b) - (c^3*Sin[a/b]*SinIntegral[a/b +
 ArcCsc[c*x]])/(4*b) + (c^3*Sin[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcCsc[c*x
]])/(4*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(c^3\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right)\\
&= -\left(c^3\text{Subst}\left(\int \left(\frac{\cos(x)}{4(a+bx)}-\frac{\cos(3x)}{4(a+bx)}\right)\,dx, x, \csc^{-1}(cx)\right)\right)\\
&= -\left(\frac{1}{4}c^3\text{Subst}\left(\int \frac{\cos(x)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right)+\frac{1}{4}c^3\text{Subst}\left(\int \frac{\cos(3x)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)
\end{aligned}$$

$$= -\left(\frac{1}{4}\left(c^3\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)\right)$$
$$+\frac{1}{4}\left(c^3\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)$$
$$-\frac{1}{4}\left(c^3\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)$$
$$+\frac{1}{4}\left(c^3\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{3a}{b}+3x\right)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)$$
$$= -\frac{c^3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b}$$
$$+\frac{c^3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b}$$
$$-\frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b}+\frac{c^3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int\frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx = -\frac{c^3\left(\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)-\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\csc^{-1}(cx)\right)\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc\right.\right.}{4b}$$

```
[In] Integrate[1/(x^4*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -1/4*(c^3*(Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]] - Cos[(3*a)/b]*CosIntegr
al[3*(a/b + ArcCsc[c*x])] + Sin[a/b]*SinIntegral[a/b + ArcCsc[c*x]] - Sin[(
3*a)/b]*SinIntegral[3*(a/b + ArcCsc[c*x])]))/b
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

method	result
derivativedivides	$c^3\left(-\frac{\text{Si}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{4b}-\frac{\text{Ci}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b}+\frac{\text{Si}\left(\frac{3a}{b}+3\,\text{arccsc}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}+\frac{\text{Ci}\left(\frac{3a}{b}+3\,\text{arccsc}(\right.}{4b}\right.$
default	$c^3\left(-\frac{\text{Si}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{4b}-\frac{\text{Ci}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b}+\frac{\text{Si}\left(\frac{3a}{b}+3\,\text{arccsc}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}+\frac{\text{Ci}\left(\frac{3a}{b}+3\,\text{arccsc}(\right.}{4b}\right.$

```
[In] int(1/x^4/(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/4*Si(a/b+arccsc(c*x))*sin(a/b)/b-1/4*Ci(a/b+arccsc(c*x))*cos(a/b)/b
+1/4*Si(3*a/b+3*arccsc(c*x))*sin(3*a/b)/b+1/4*Ci(3*a/b+3*arccsc(c*x))*cos(3
*a/b)/b)
```

Fricas [F]

$$\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arccsc}\,(cx)+a)x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^4*arccsc(c*x) + a*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{x^4\left(a+b\,\mathrm{acsc}\,(cx)\right)}\,dx$$

```
[In] integrate(1/x**4/(a+b*acsc(c*x)),x)
[Out] Integral(1/(x**4*(a + b*acsc(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arccsc}\,(cx)+a)x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsc(c*x) + a)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.71

$$\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx$$
$$=\frac{1}{4}\left(\frac{4\,c^2\cos\left(\frac{a}{b}\right)^3\mathrm{Ci}\left(\frac{3\,a}{b}+3\,\arcsin\left(\frac{1}{cx}\right)\right)}{b}+\frac{4\,c^2\cos\left(\frac{a}{b}\right)^2\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{3\,a}{b}+3\,\arcsin\left(\frac{1}{cx}\right)\right)}{b}-\frac{3\,c^2\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{3\,a}{b}\right.}{b}\right.$$

```
[In] integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*(4*c^2*cos(a/b)^3*cos_integral(3*a/b + 3*arcsin(1/(c*x)))/b + 4*c^2*cos
(a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(1/(c*x)))/b - 3*c^2*cos(a/b)
*cos_integral(3*a/b + 3*arcsin(1/(c*x)))/b - c^2*cos(a/b)*cos_integral(a/b
+ arcsin(1/(c*x)))/b - c^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(1/(c*x)))
/b - c^2*sin(a/b)*sin_integral(a/b + arcsin(1/(c*x)))/b)*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + b \csc^{-1}(cx))} \, dx = \int \frac{1}{x^4 \, \left(a + b \operatorname{asin}\left(\frac{1}{c x}\right)\right)} \, dx$$

```
[In] int(1/(x^4*(a + b*asin(1/(c*x)))),x)
```

```
[Out] int(1/(x^4*(a + b*asin(1/(c*x)))), x)
```

3.39 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx = \text{Int}\left((dx)^m \left(a+b\csc^{-1}(cx)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arccsc(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx$$

```
[In] Int[(d*x)^m*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsc[c*x])^3, x]
```

Rubi steps

$$\text{integral} = \int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx$$

Mathematica [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx=\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.86 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\ \mathrm{arccsc}\left(cx\right)\right)^3 dx$$

```
[In] int((d*x)^m*(a+b*arccsc(c*x))^3,x)
[Out] int((d*x)^m*(a+b*arccsc(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx=\int (b\,\mathrm{arccsc}\left(cx\right)+a)^3(dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
[Out] integral((b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x) +
 a^3)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 21.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx=\int (dx)^m \left(a+b\,\mathrm{acsc}\left(cx\right)\right)^3 \, dx$$

```
[In] integrate((d*x)**m*(a+b*acsc(c*x))**3,x)
[Out] Integral((d*x)**m*(a + b*acsc(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 15.47 (sec) , antiderivative size = 1279, normalized size of antiderivative = 79.94

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^3(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/4*(4*b^3*d^m*x*x^m*arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))^3 - 3*b^3*d^m*x*x^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
))*log(c^2*x^2)^2 - 4*(m + 1)*integrate(-3/4*((a*b^2*d^m*m + a*b^2*d^m - (a
*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*log(c^2*x^2)^2 + 4*((b^3*arctan2(1
, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*d^m*m - ((b^3*arctan2(1, sqrt(c*x +
 1)*sqrt(c*x - 1)) + a*b^2)*c^2*d^m*m + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1)) + a*b^2)*c^2*d^m)*x^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1)) + a*b^2)*d^m)*x^m*log(x)^2 + 8*((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1)) + a*b^2)*d^m*m*log(c) - ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
 + a*b^2)*c^2*d^m*m*log(c) + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
 a*b^2)*c^2*d^m*log(c))*x^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
+ a*b^2)*d^m*log(c))*x^m*log(x) + (4*b^3*d^m*x^m*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1))^2 - b^3*d^m*x^m*log(c^2*x^2)^2)*sqrt(c*x + 1)*sqrt(c*x - 1) -
 4*((a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + a^2*b*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1)) - (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
a*b^2)*log(c)^2)*d^m*m + (((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a
*b^2)*c^2*log(c)^2 - (a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + a^2
*b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*c^2)*d^m*m + ((b^3*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*c^2*log(c)^2 - (a*b^2*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1))^2 + a^2*b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*
c^2)*d^m)*x^2 + (a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + a^2*b*ar
ctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1)) + a*b^2)*log(c)^2)*d^m)*x^m - 4*(((b^3*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)) + a*b^2)*d^m*m - ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)
) + a*b^2)*c^2*d^m*m + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2
)*c^2*d^m)*x^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*d^m)
*x^m*log(x) + ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*d^m*m*
log(c) - ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*c^2*d^m*m*l
og(c) + (b^3*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + (b^3*arctan2(1,
sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*c^2*log(c))*d^m)*x^2 + (b^3*arctan2(1
, sqrt(c*x + 1)*sqrt(c*x - 1)) + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
)) + a*b^2)*log(c))*d^m)*x^m)*log(c^2*x^2))/((c^2*m + c^2)*x^2 - m - 1), x)
)/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arccsc}(cx) + a)^3 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)^3*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx = \int (d\,x)^m \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

```
[In] int((d*x)^m*(a + b*asin(1/(c*x)))^3,x)
[Out] int((d*x)^m*(a + b*asin(1/(c*x)))^3, x)
```

3.40 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \operatorname{Int}\left((dx)^m \left(a+b\csc^{-1}(cx)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arccsc(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx$$

```
[In] Int[(d*x)^m*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsc[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx$$

Mathematica [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.83 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b \operatorname{arccsc}(cx)\right)^2 dx$$

```
[In] int((d*x)^m*(a+b*arccsc(c*x))^2,x)
[Out] int((d*x)^m*(a+b*arccsc(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arccsc}(cx)+a)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
[Out] integral((b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 9.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b \operatorname{acsc}(cx)\right)^2 \, dx$$

```
[In] integrate((d*x)**m*(a+b*acsc(c*x))**2,x)
[Out] Integral((d*x)**m*(a + b*acsc(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 6.87 (sec) , antiderivative size = 551, normalized size of antiderivative = 34.44

$$\int (dx)^m \left(a + b\csc^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arccsc}(cx) + a)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/4*(4*b^2*d^m*x*x^m*arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))^2 - b^2*d^m*x*x^m*log(c^2*x^2)^2 + 4*(m + 1)*integrate((2
*sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*d^m*x^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1)) + (b^2*d^m*m + b^2*d^m - (b^2*c^2*d^m*m + b^2*c^2*d^m)*x^2)*x^m*log(x
)^2 + 2*(b^2*d^m*m*log(c) + b^2*d^m*log(c) - (b^2*c^2*d^m*m*log(c) + b^2*c^
2*d^m*log(c))*x^2)*x^m*log(x) + ((b^2*log(c)^2 - 2*a*b*arctan2(1, sqrt(c*x
+ 1)*sqrt(c*x - 1)))*d^m*m - ((b^2*c^2*log(c)^2 - 2*a*b*c^2*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1)))*d^m*m + (b^2*c^2*log(c)^2 - 2*a*b*c^2*arctan2(1,
sqrt(c*x + 1)*sqrt(c*x - 1)))*d^m)*x^2 + (b^2*log(c)^2 - 2*a*b*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1)))*d^m)*x^m - ((b^2*d^m*m + b^2*d^m - (b^2*c^2*d^
m*m + b^2*c^2*d^m)*x^2)*x^m*log(x) + (b^2*d^m*m*log(c) - (b^2*c^2*d^m*m*log
(c) + (b^2*c^2*log(c) + b^2*c^2)*d^m)*x^2 + (b^2*log(c) + b^2)*d^m)*x^m)*lo
g(c^2*x^2))/((c^2*m + c^2)*x^2 - m - 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b\csc^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arccsc}(cx) + a)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^2*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a + b\csc^{-1}(cx)\right)^2 \, dx = \int (d\,x)^m \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

```
[In] int((d*x)^m*(a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int((d*x)^m*(a + b*asin(1/(c*x)))^2, x)
```

3.41 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right) dx = \frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)} + \frac{b(dx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{c^2x^2}\right)}{cm(1+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arccsc(c*x))/d/(1+m)+b*(d*x)^m*hypergeom([1/2, -1/2*m],[1-
1/2*m],1/c^2/x^2)/c/m/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5329, 346, 371}

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right) dx = \frac{(dx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{d(m+1)} + \frac{b(dx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{c^2x^2}\right)}{cm(m+1)}$$

```
[In] Int[(d*x)^m*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(d*(1 + m)) + (b*(d*x)^m*Hypergeometric
2F1[1/2, -1/2*m, 1 - m/2, 1/(c^2*x^2)])/(c*m*(1 + m))
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
```

```
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)}+\frac{(bd)\int\frac{(dx)^{-1+m}}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{c(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)}-\frac{\left(b\left(\frac{1}{x}\right)^m(dx)^m\right)\text{Subst}\left(\int\frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c(1+m)}\\
&= \frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)}+\frac{b(dx)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{c^2x^2}\right)}{cm(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int(dx)^m\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{(dx)^m\left((1+m)x\left(a+b\csc^{-1}(cx)\right)+\frac{b\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{c\sqrt{1-\frac{1}{c^2x^2}}}\right)}{(1+m)^2}$$

```
[In] Integrate[(d*x)^m*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((d*x)^m*((1 + m)*x*(a + b*ArcCsc[c*x]) + (b*Sqrt[1 - c^2*x^2]*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(c*Sqrt[1 - 1/(c^2*x^2)])))/(1 +
 m)^2
```

Maple [F]

$$\int (dx)^m (a + b \operatorname{arccsc}(cx))\, dx$$

[In] int((d*x)^m*(a+b*arccsc(c*x)),x)

[Out] int((d*x)^m*(a+b*arccsc(c*x)),x)

Fricas [F]

$$\int (dx)^m \left(a + b \csc^{-1}(cx)\right)\, dx = \int (b \operatorname{arccsc}(cx) + a)(dx)^m\, dx$$

[In] integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="fricas")

[Out] integral((b*arccsc(c*x) + a)*(d*x)^m, x)

Sympy [F]

$$\int (dx)^m \left(a + b \csc^{-1}(cx)\right)\, dx = \int (dx)^m (a + b \operatorname{acsc}(cx))\, dx$$

[In] integrate((d*x)**m*(a+b*acsc(c*x)),x)

[Out] Integral((d*x)**m*(a + b*acsc(c*x)), x)

Maxima [F]

$$\int (dx)^m \left(a + b \csc^{-1}(cx)\right)\, dx = \int (b \operatorname{arccsc}(cx) + a)(dx)^m\, dx$$

[In] integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="maxima")

[Out] (d^m*x*x^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + (c^2*d^m*m + c^2*d^m)*
integrate(-sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(c^2*m - (c^4*m + c^4)*x^2 + c^2
), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))

Giac [F]

$$\int (dx)^m \left(a + b\csc^{-1}(cx)\right) \, dx = \int (b \operatorname{arccsc}(cx) + a)(dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a + b\csc^{-1}(cx)\right) \, dx = \int (d\,x)^m \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int((d*x)^m*(a + b*asin(1/(c*x))),x)
[Out] int((d*x)^m*(a + b*asin(1/(c*x))), x)
```

3.42 $\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \text{Int}\left(\frac{(dx)^m}{a+b\csc^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arccsc(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcCsc[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\ \mathrm{arccsc}\,(cx)}dx$$

```
[In] int((d*x)^m/(a+b*arccsc(c*x)),x)
[Out] int((d*x)^m/(a+b*arccsc(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx=\int \frac{(dx)^m}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arccsc(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\,\mathrm{acsc}\,(cx)}\,dx$$

```
[In] integrate((d*x)**m/(a+b*acsc(c*x)),x)
[Out] Integral((d*x)**m/(a + b*acsc(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arccsc(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arccsc(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \int \frac{(d\,x)^m}{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}\,dx$$

```
[In] int((d*x)^m/(a + b*asin(1/(c*x))),x)
[Out] int((d*x)^m/(a + b*asin(1/(c*x))), x)
```

3.43 $\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx=\operatorname{Int}\left(\frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arccsc(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsc[c*x])^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcCsc[c*x])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcCsc[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\ \mathrm{arccsc}\,(cx))^2}dx$$

```
[In] int((d*x)^m/(a+b*arccsc(c*x))^2,x)
[Out] int((d*x)^m/(a+b*arccsc(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(b\,\mathrm{arccsc}\,(cx)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(a+b\,\mathrm{acsc}\,(cx))^2}\,dx$$

```
[In] integrate((d*x)**m/(a+b*acsc(c*x))**2,x)
[Out] Integral((d*x)**m/(a + b*acsc(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 684, normalized size of antiderivative = 42.75

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(b\,\mathrm{arccsc}\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] (4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
 a)*d^m*x*x^m - (4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(
c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^
2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*l
og(x))*log(c^2*x^2))*integrate(4*((b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)
) + a)*d^m*m - ((b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*c^2*d^m*m +
 2*(b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*c^2*d^m)*x^2 + (b*arctan
2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*d^m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m
/(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 4*b^3*log(c)^2 + 8*a*b^
2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*c^2*log(c)^2 +
 (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*a*b^2*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1)) + a^2*b)*c^2)*x^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)
^2 - 4*(b^3*c^2*x^2 - b^3)*log(x)^2 + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) +
(b^3*c^2*x^2 - b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(
c))*log(x)), x))/(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log
(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*
log(x))*log(c^2*x^2))
```

Giac [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(b\,\mathrm{arccsc}\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arccsc(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2}\,dx = \int \frac{(d\,x)^m}{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

```
[In] int((d*x)^m/(a + b*asin(1/(c*x)))^2,x)
[Out] int((d*x)^m/(a + b*asin(1/(c*x)))^2, x)
```

3.44 $\int (d+ex)^3 (a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 167

$$\int (d+ex)^3 (a+b\csc^{-1}(cx))\,dx = \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} - \frac{bd^4\csc^{-1}(cx)}{4e} + \frac{(d+ex)^4(a+b\csc^{-1}(cx))}{4e} + \frac{bd(2c^2d^2+e^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{2c^3}$$

```
[Out] -1/4*b*d^4*arccsc(c*x)/e+1/4*(e*x+d)^4*(a+b*arccsc(c*x))/e+1/2*b*d*(2*c^2*d
^2+e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3+1/6*b*e*(9*c^2*d^2+e^2)*x*(1-1/c^2
/x^2)^(1/2)/c^3+1/2*b*d*e^2*x^2*(1-1/c^2/x^2)^(1/2)/c+1/12*b*e^3*x^3*(1-1/c
^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used

= {5335, 1582, 1489, 1821, 858, 222, 272, 65, 214}

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{(d+ex)^4\,(a+b\csc^{-1}(cx))}{4e} + \frac{bd\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)(2c^2d^2+e^2)}{2c^3} + \frac{bde^2x^2\sqrt{1-\frac{1}{c^2x^2}}}{2c} + \frac{be^3x^3\sqrt{1-\frac{1}{c^2x^2}}}{12c} + \frac{bex\sqrt{1-\frac{1}{c^2x^2}}(9c^2d^2+e^2)}{6c^3} - \frac{bd^4\csc^{-1}(cx)}{4e}$$

```
[In] Int[(d + e*x)^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(9*c^2*d^2 + e^2)*Sqrt[1 - 1/(c^2*x^2)]*x)/(6*c^3) + (b*d*e^2*Sqrt[1 -
 1/(c^2*x^2)]*x^2)/(2*c) + (b*e^3*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(12*c) - (b*d^
4*ArcCsc[c*x])/(4*e) + ((d + e*x)^4*(a + b*ArcCsc[c*x]))/(4*e) + (b*d*(2*c^
2*d^2 + e^2)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(2*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(d+ex)^4\,(a+b\csc^{-1}(cx))}{4e}+\frac{b\int\frac{(d+ex)^4}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{4ce}\\ &= \frac{(d+ex)^4\,(a+b\csc^{-1}(cx))}{4e}+\frac{b\int\frac{\left(e+\frac{d}{x}\right)^4x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{4ce}\\ &= \frac{(d+ex)^4\,(a+b\csc^{-1}(cx))}{4e}-\frac{b\text{Subst}\left(\int\frac{(e+dx)^4}{x^4\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{4ce}\end{aligned}$$

$$= \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4(a+b\csc^{-1}(cx))}{4e} + \frac{b\text{Subst}\left(\int \frac{-12de^3-2e^2\left(9d^2+\frac{e^2}{c^2}\right)x-12d^3ex^2-3d^4x^3}{x^3\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{12ce}$$

$$= \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4(a+b\csc^{-1}(cx))}{4e} - \frac{b\text{Subst}\left(\int \frac{4e^2\left(9d^2+\frac{e^2}{c^2}\right)+12de\left(2d^2+\frac{e^2}{c^2}\right)x+6d^4x^2}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{24ce}$$

$$= \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4(a+b\csc^{-1}(cx))}{4e} + \frac{b\text{Subst}\left(\int \frac{-12de\left(2d^2+\frac{e^2}{c^2}\right)-6d^4x}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{24ce}$$

$$= \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} + \frac{(d+ex)^4(a+b\csc^{-1}(cx))}{4e} - \frac{(bd^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce} - \frac{(bd(2c^2d^2+e^2))\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c^3}$$

$$= \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} - \frac{bd^4\csc^{-1}(cx)}{4e} + \frac{(d+ex)^4(a+b\csc^{-1}(cx))}{4e} - \frac{(bd(2c^2d^2+e^2))\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{4c^3}$$

$$= \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} - \frac{bd^4\csc^{-1}(cx)}{4e} + \frac{(d+ex)^4(a+b\csc^{-1}(cx))}{4e} + \frac{(bd(2c^2d^2+e^2))\,\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)}{2c}$$

$$= \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c} - \frac{bd^4\csc^{-1}(cx)}{4e} + \frac{(d+ex)^4(a+b\csc^{-1}(cx))}{4e} + \frac{bd(2c^2d^2+e^2)\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= \frac{3ac^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)+be\sqrt{1-\frac{1}{c^2x^2}}x(2e^2+c^2(18d^2+6dex+e^2x^2))+3bc^3x(4d^3+6d^2ex}{12c^3}$$

[In] Integrate[(d + e*x)^3*(a + b*ArcCsc[c*x]),x]

[Out] (3*a*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)) + 3*b*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcCsc[c*x] + 6*b*d*(2*c^2*d^2 + e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(12*c^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(147) = 294.

Time = 0.63 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.40

method	result
parts	$\frac{a(ex+d)^4}{4e}+\frac{b\,e^3\,\mathrm{arccsc}(cx)x^4}{4}+b\,e^2\,\mathrm{arccsc}\,(cx)\,x^3d+\frac{3be\,\mathrm{arccsc}(cx)x^2d^2}{2}+b\,\mathrm{arccsc}\,(cx)\,x\,d^3+\frac{b\,d^4}{}$
derivativedivides	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{bc\,\mathrm{arccsc}(cx)d^4}{4e}+b\,\mathrm{arccsc}(cx)d^3cx+\frac{3bce\,\mathrm{arccsc}(cx)d^2x^2}{2}+bc\,e^2\,\mathrm{arccsc}(cx)d\,x^3+\frac{bc\,e^3\,\mathrm{arccsc}(cx)x^4}{4}-\frac{b\sqrt{c^2x^2-1}\,d^4}{4e\sqrt{}}}{}$
default	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{bc\,\mathrm{arccsc}(cx)d^4}{4e}+b\,\mathrm{arccsc}(cx)d^3cx+\frac{3bce\,\mathrm{arccsc}(cx)d^2x^2}{2}+bc\,e^2\,\mathrm{arccsc}(cx)d\,x^3+\frac{bc\,e^3\,\mathrm{arccsc}(cx)x^4}{4}-\frac{b\sqrt{c^2x^2-1}\,d^4}{4e\sqrt{}}}{}$

[In] int((e*x+d)^3*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/4*a*(e*x+d)^4/e+1/4*b*e^3*arccsc(c*x)*x^4+b*e^2*arccsc(c*x)*x^3*d+3/2*b*e*arccsc(c*x)*x^2*d^2+b*arccsc(c*x)*x*d^3+1/4*b*d^4*arccsc(c*x)/e+1/12*b/c^3*e^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x-1/4*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^4*arctan(1/(c^2*x^2-1)^(1/2))+1/2*b/c^3*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d+b/c^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*ln(c*x+(c^2*x^2-1)^(1/2))+3/2*b/c^3*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2+1/2*b/c^4*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))+1/6*b/c^5*e^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.74

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{3\,ac^4e^3x^4+12\,ac^4de^2x^3+18\,ac^4d^2ex^2+12\,ac^4d^3x+3\left(bc^4e^3x^4+4\,bc^4de^2x^3+6\,bc^4d^2ex^2+4\,bc^4d^3x-4\,b\right.}{}$$

```
[In] integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*e^3*x^4 + 12*a*c^4*d*e^2*x^3 + 18*a*c^4*d^2*e*x^2 + 12*a*c^4*
d^3*x + 3*(b*c^4*e^3*x^4 + 4*b*c^4*d*e^2*x^3 + 6*b*c^4*d^2*e*x^2 + 4*b*c^4*
d^3*x - 4*b*c^4*d^3 - 6*b*c^4*d^2*e - 4*b*c^4*d*e^2 - b*c^4*e^3)*arccsc(c*x
) - 6*(4*b*c^4*d^3 + 6*b*c^4*d^2*e + 4*b*c^4*d*e^2 + b*c^4*e^3)*arctan(-c*x
 + sqrt(c^2*x^2 - 1)) - 6*(2*b*c^3*d^3 + b*c*d*e^2)*log(-c*x + sqrt(c^2*x^2
 - 1)) + (b*c^2*e^3*x^2 + 6*b*c^2*d*e^2*x + 18*b*c^2*d^2*e + 2*b*e^3)*sqrt(
c^2*x^2 - 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.17

$$\begin{aligned}
\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right)\,dx &= ad^3x+\frac{3ad^2ex^2}{2}+ade^2x^3+\frac{ae^3x^4}{4}+bd^3x\operatorname{acsc}(cx) \\
&+\frac{3bd^2ex^2\operatorname{acsc}(cx)}{2}+bde^2x^3\operatorname{acsc}(cx)+\frac{be^3x^4\operatorname{acsc}(cx)}{4} \\
&+\frac{bd^3\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}+\frac{3bd^2e\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c} \\
&+\frac{bde^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{c} \\
&+\frac{be^3\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}
\end{aligned}$$

```
[In] integrate((e*x+d)**3*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b*d**3*x*acs
c(c*x) + 3*b*d**2*e*x**2*acsc(c*x)/2 + b*d*e**2*x**3*acsc(c*x) + b*e**3*x**
4*acsc(c*x)/4 + b*d**3*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c + 3*b*d**2*e*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**
2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c) + b*d*e**2*Piecewise((x*sq
rt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x
**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)
/(2*c**2), True))/c + b*e**3*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*
sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2
+ 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.61

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= \frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + \frac{3}{2}\left(x^2\operatorname{arccsc}(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2e + \frac{1}{4}\left(4x^3\operatorname{arccsc}(cx) + \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde^2 + \frac{1}{12}\left(3x^4\operatorname{arccsc}(cx) + \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)be^3 + ad^3x + \frac{\left(2cx\operatorname{arccsc}(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^3}{2c}$$

```
[In] integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arccsc(c*x) + x*sq
rt(-1/(c^2*x^2) + 1)/c)*b*d^2*e + 1/4*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*
x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c
^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e^2 + 1/12*(3*x^4*arccsc(c
*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*
b*e^3 + a*d^3*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1)
- log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^3/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. 2(147) = 294.

Time = 2.48 (sec) , antiderivative size = 1130, normalized size of antiderivative = 6.77

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b*e^3*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a*e
^3*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 24*b*d*e^2*x^3*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^3*arcsin(1/(c*x))/c + 24*a*d*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1
)^3/c + 2*b*e^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 72*b*d^2*e*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + 72*a*d^2*e*x^2*(sqrt(-1/(c^
2*x^2) + 1) + 1)^2/c + 24*b*d*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 +
96*b*d^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c + 12*b*e^3*x^2*(s
qrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 96*a*d^3*x*(sqrt(-1/(c^2
*x^2) + 1) + 1)/c + 12*a*e^3*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 144*b
*d^2*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 72*b*d*e^2*x*(sqrt(-1/(c^2*x^2)
 + 1) + 1)*arcsin(1/(c*x))/c^3 + 72*a*d*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/
c^3 + 192*b*d^3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - 192*b*d^3*log(1/(abs(
c)*abs(x)))/c^2 + 18*b*e^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 144*b*d^2*e
*arcsin(1/(c*x))/c^3 + 144*a*d^2*e/c^3 + 96*b*d*e^2*log(sqrt(-1/(c^2*x^2) +
 1) + 1)/c^4 - 96*b*d*e^2*log(1/(abs(c)*abs(x)))/c^4 + 18*b*e^3*arcsin(1/(c
*x))/c^5 + 96*b*d^3*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) +
18*a*e^3/c^5 + 96*a*d^3/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 144*b*d^2*e/
(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 72*b*d*e^2*arcsin(1/(c*x))/(c^5*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)) + 72*a*d*e^2/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1
)) - 18*b*e^3/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 72*b*d^2*e*arcsin(1/(c
*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 72*a*d^2*e/(c^5*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2) - 24*b*d*e^2/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2) + 12*b*e^3*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 1
2*a*e^3/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 24*b*d*e^2*arcsin(1/(c*x
))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 24*a*d*e^2/(c^7*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3) - 2*b*e^3/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) +
 3*b*e^3*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 3*a*e^3
/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right) \, dx = \int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) (d+e\,x)^3 \, dx$$

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^3,x)
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^3, x)
```

3.45 $\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 123

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{b(6c^2d^2+e^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

```
[Out] -1/3*b*d^3*arccsc(c*x)/e+1/3*(e*x+d)^3*(a+b*arccsc(c*x))/e+1/6*b*(6*c^2*d^2
+e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3+b*d*e*x*(1-1/c^2/x^2)^(1/2)/c+1/6*b*
e^2*x^2*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5335, 1582, 1489, 1821, 858, 222, 272, 65, 214}

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)(6c^2d^2+e^2)}{6c^3} + \frac{bdex\sqrt{1-\frac{1}{c^2x^2}}}{c} + \frac{be^2x^2\sqrt{1-\frac{1}{c^2x^2}}}{6c} - \frac{bd^3\csc^{-1}(cx)}{3e}$$

```
[In] Int[(d + e*x)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*d*e*Sqrt[1 - 1/(c^2*x^2)]*x)/c + (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(6*c)
 - (b*d^3*ArcCsc[c*x])/(3*e) + ((d + e*x)^3*(a + b*ArcCsc[c*x]))/(3*e) + (b
*(6*c^2*d^2 + e^2)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(6*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
```

```
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^3\,(a+b\csc^{-1}(cx))}{3e} + \frac{b\int \frac{(d+ex)^3}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{3ce}$$

$$= \frac{(d+ex)^3\,(a+b\csc^{-1}(cx))}{3e} + \frac{b\int \frac{\left(e+\frac{d}{x}\right)^3 x}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{3ce}$$

$$= \frac{(d+ex)^3\,(a+b\csc^{-1}(cx))}{3e} - \frac{b\text{Subst}\left(\int \frac{(e+dx)^3}{x^3\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{3ce}$$

$$= \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c} + \frac{(d+ex)^3\,(a+b\csc^{-1}(cx))}{3e} + \frac{b\text{Subst}\left(\int \frac{-6de^2-e\left(6d^2+\frac{e^2}{c^2}\right)x-2d^3x^2}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6ce}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c} + \frac{(d+ex)^3\,(a+b\csc^{-1}(cx))}{3e} - \frac{b\text{Subst}\left(\int \frac{e\left(6d^2+\frac{e^2}{c^2}\right)+2d^3x}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6ce}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c} + \frac{(d+ex)^3\,(a+b\csc^{-1}(cx))}{3e} - \frac{(bd^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{3ce} - \frac{(b(6c^2d^2+e^2))\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6c^3}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e} - \frac{(b(6c^2d^2+e^2))\,\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{12c^3}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{(b(6c^2d^2+e^2))\,\text{Subst}\left(\int\frac{1}{c^2-c^2x^2}\,dx,x,\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{b(6c^2d^2+e^2)\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int(d+ex)^2\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{c^2x\left(be\sqrt{1-\frac{1}{c^2x^2}}(6d+ex)+2ac(3d^2+3dex+e^2x^2)\right)+2bc^3x(3d^2+3dex+e^2x^2)\csc^{-1}(cx)+b(6c^2d^2+}{6c^3}$$

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (c^2*x*(b*e*Sqrt[1 - 1/(c^2*x^2)]*(6*d + e*x) + 2*a*c*(3*d^2 + 3*d*e*x + e^
2*x^2)) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcCsc[c*x] + b*(6*c^2*d^2
+ e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(109) = 218.

Time = 0.57 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.47

method	result
parts	$\frac{a(ex+d)^3}{3e}+\frac{b\,e^2 \operatorname{arccsc}(cx)x^3}{3}+be\,\operatorname{arccsc}(cx)\,x^2d+b\,\operatorname{arccsc}(cx)\,x\,d^2+\frac{b\,d^3\operatorname{arccsc}(cx)}{3e}-\frac{b\sqrt{c^2x^2-}}{}$
derivativedivides	$\frac{\frac{a(cex+cd)^3}{3c^2e}+\frac{bc\,\operatorname{arccsc}(cx)d^3}{3e}+b\,\operatorname{arccsc}(cx)d^2cx+bce\,\operatorname{arccsc}(cx)d\,x^2+\frac{bc\,e^2\operatorname{arccsc}(cx)x^3}{3}-\frac{b\sqrt{c^2x^2-1}\,d^3\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}}{c}$
default	$\frac{\frac{a(cex+cd)^3}{3c^2e}+\frac{bc\,\operatorname{arccsc}(cx)d^3}{3e}+b\,\operatorname{arccsc}(cx)d^2cx+bce\,\operatorname{arccsc}(cx)d\,x^2+\frac{bc\,e^2\operatorname{arccsc}(cx)x^3}{3}-\frac{b\sqrt{c^2x^2-1}\,d^3\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}}{c}$

```
[In] int((e*x+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*(e*x+d)^3/e+1/3*b*e^2*arccsc(c*x)*x^3+b*e*arccsc(c*x)*x^2*d+b*arccsc(
c*x)*x*d^2+1/3*b*d^3*arccsc(c*x)/e-1/3*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*d^3*arctan(1/(c^2*x^2-1)^(1/2))+1/6*b/c^3*e^2*(c^2*x^2-1)
/((c^2*x^2-1)/c^2/x^2)^(1/2)+b/c^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))+b/c^3*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/
x^2)^(1/2)/x*d+1/6*b/c^4*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.70

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{2\,ac^3e^2x^3+6\,ac^3dex^2+6\,ac^3d^2x+2\left(bc^3e^2x^3+3\,bc^3dex^2+3\,bc^3d^2x-3\,bc^3d^2-3\,bc^3de-bc^3e^2\right)\operatorname{arccsc}(}{}$$

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x + 2*(b*c^3*e^2*x^3 +
 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^2 - 3*b*c^3*d*e - b*c^3*e^2)*a
rccsc(c*x) - 4*(3*b*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*arctan(-c*x + sqrt(c
^2*x^2 - 1)) - (6*b*c^2*d^2 + b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*e
^2*x + 6*b*c*d*e)*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.85

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\,\mathrm{acsc}\,(cx) + bdex^2\,\mathrm{acsc}\,(cx) + \frac{be^2x^3\,\mathrm{acsc}\,(cx)}{3} + \frac{bd^2\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c} + \frac{bde\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{c} + \frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate((e*x+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*acsc(c*x) + b*d*e*x**2*acs
c(c*x) + b*e**2*x**3*acsc(c*x)/3 + b*d**2*Piecewise((acosh(c*x), Abs(c**2*x
**2) > 1), (-I*asin(c*x), True))/c + b*d*e*Piecewise((sqrt(c**2*x**2 - 1)/c
, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/c + b*e**2*Piecewi
se((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1),
 (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*a
sin(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.61

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{3}\,ae^2x^3 + adex^2 + \left(x^2\,\mathrm{arccsc}\,(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bde + \frac{1}{12}\left(4\,x^3\,\mathrm{arccsc}\,(cx) + \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 + ad^2x + \frac{\left(2\,cx\,\mathrm{arccsc}\,(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2\,c}$$

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*
b*d*e + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^
2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2
) + 1) - 1)/c^2)/c)*b*e^2 + a*d^2*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/
(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^2/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(109) = 218$.

Time = 2.19 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.89

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right) dx$$
$$= \frac{1}{24}\left(\frac{be^2x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{ae^2x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c}-\frac{24\,bdex^2\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{c}\right.$$

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/24*(b*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + a*e^2*x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c - 24*b*d*e*x^2*(1/(c^2*x^2) - 1)*arcsin(
1/(c*x))/c + b*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 - 24*a*d*e*x^2*(1
/(c^2*x^2) - 1)/c + 12*b*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))
/c + 12*a*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c + 3*b*e^2*x*(sqrt(-1/(c^2*x^
2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 24*b*d*e*x*sqrt(-1/(c^2*x^2) + 1)/c^2 +
3*a*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 24*b*d^2*log(sqrt(-1/(c^2*x^2)
 + 1) + 1)/c^2 - 24*b*d^2*log(1/(abs(c)*abs(x)))/c^2 + 24*b*d*e*arcsin(1/(c
*x))/c^3 + 24*a*d*e/c^3 + 4*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 4*b
*e^2*log(1/(abs(c)*abs(x)))/c^4 + 12*b*d^2*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) + 12*a*d^2/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3*b*
e^2*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3*a*e^2/(c^5*x*(
sqrt(-1/(c^2*x^2) + 1) + 1)) - b*e^2/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^
2) + b*e^2*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + a*e^2
/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)^2\,dx$$

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^2,x)
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^2, x)
```

3.46 $\int (d+ex) \left(a+b\csc^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (d+ex) \left(a+b\csc^{-1}(cx)\right) \, dx = \frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} - \frac{bd^2\csc^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e} + \frac{bd\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] -1/2*b*d^2*arccsc(c*x)/e+1/2*(e*x+d)^2*(a+b*arccsc(c*x))/e+b*d*arctanh((1-1
/c^2/x^2)^(1/2))/c+1/2*b*e*x*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5335, 1582, 1410, 1821, 858, 222, 272, 65, 214}

$$\int (d+ex) \left(a+b\csc^{-1}(cx)\right) \, dx = \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e} + \frac{bd\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c} + \frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c} - \frac{bd^2\csc^{-1}(cx)}{2e}$$

```
[In] Int[(d + e*x)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c) - (b*d^2*ArcCsc[c*x])/(2*e) + ((d + e*x
)^2*(a + b*ArcCsc[c*x]))/(2*e) + (b*d*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1410

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol
] :> -Subst[Int[(d + e/x^n)^q*((a + c/x^(2*n))^p/x^2), x], x, 1/x] /; FreeQ
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{b\int\frac{(d+ex)^2}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{2ce}\\
&= \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{b\int\frac{\left(e+\frac{d}{x}\right)^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{2ce}\\
&= \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{b\text{Subst}\left(\int\frac{(e+dx)^2}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2ce}\\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{b\text{Subst}\left(\int\frac{-2de-d^2x}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2ce}\\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}\\
&\quad-\frac{(bd)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c}-\frac{\left(bd^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2ce}\\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}-\frac{bd^2\csc^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}\\
&\quad-\frac{(bd)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2c}\\
&= \frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}-\frac{bd^2\csc^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}\\
&\quad+(bcd)\text{Subst}\left(\int\frac{1}{c^2-c^2x^2}\,dx,x,\sqrt{1-\frac{1}{c^2x^2}}\right)
\end{aligned}
$$

$$=\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}-\frac{bd^2\csc^{-1}(cx)}{2e}+\frac{(d+ex)^2(a+b\csc^{-1}(cx))}{2e}+\frac{bd\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx = adx+\frac{1}{2}aex^2+\frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+bdx\csc^{-1}(cx)+\frac{1}{2}bex^2\csc^{-1}(cx)+\frac{bd\sqrt{1-\frac{1}{c^2x^2}}x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

```
[In] Integrate[(d + e*x)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x*
ArcCsc[c*x] + (b*e*x^2*ArcCsc[c*x])/2 + (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTan
h[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

method	result	size
parts	$a\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b\left(\frac{c\ \text{arccsc}(cx)x^2e}{2}+\text{arccsc}(cx)xcd+\frac{\sqrt{c^2x^2-1}\left(2dc\ln\left(cx+\sqrt{c^2x^2-1}\right)+e\sqrt{c^2x^2-1}\right)}{2c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}\right)}{c}$	110
derivativedivides	$\frac{\frac{a\left(dx\,c^2+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b\left(\text{arccsc}(cx)d\,c^2x+\frac{\text{arccsc}(cx)e\,c^2x^2}{2}+\frac{\sqrt{c^2x^2-1}\left(2dc\ln\left(cx+\sqrt{c^2x^2-1}\right)+e\sqrt{c^2x^2-1}\right)}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c}}{c}$	127
default	$\frac{\frac{a\left(dx\,c^2+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b\left(\text{arccsc}(cx)d\,c^2x+\frac{\text{arccsc}(cx)e\,c^2x^2}{2}+\frac{\sqrt{c^2x^2-1}\left(2dc\ln\left(cx+\sqrt{c^2x^2-1}\right)+e\sqrt{c^2x^2-1}\right)}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c}}{c}$	127

```
[In] int((e*x+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arccsc(c*x)*x^2*e+arccsc(c*x)*x*c*d+1/2/c^2/((
c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c^2*x^2-1)^(1/2)*(2*d*c*ln(c*x+(c^2*x^2-1)^(1/
2))+e*(c^2*x^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.55

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{ac^2ex^2+2\,ac^2dx-2\,bcd\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}be+\left(bc^2ex^2+2\,bc^2dx-2\,bc^2d-bc^2e\right)\mathrm{arcc}}{2\,c^2}$$

```
[In] integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*e*x^2 + 2*a*c^2*d*x - 2*b*c*d*log(-c*x + sqrt(c^2*x^2 - 1)) + sq
rt(c^2*x^2 - 1)*b*e + (b*c^2*e*x^2 + 2*b*c^2*d*x - 2*b*c^2*d - b*c^2*e)*arc
csc(c*x) - 2*(2*b*c^2*d + b*c^2*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/c^2
```

Sympy [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx = adx+\frac{aex^2}{2}+bdx\,\mathrm{acsc}\,(cx)+\frac{bex^2\,\mathrm{acsc}\,(cx)}{2} + \frac{bd\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c} + \frac{be\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

```
[In] integrate((e*x+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x + a*e*x**2/2 + b*d*x*acsc(c*x) + b*e*x**2*acsc(c*x)/2 + b*d*Piecewise
((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + b*e*Piecewise(
(sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, Tru
e))/(2*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{2}\,aex^2+\frac{1}{2}\left(x^2\operatorname{arccsc}\left(cx\right)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)be+adx$$
$$+\frac{\left(2\,cx\operatorname{arccsc}\left(cx\right)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2\,c}$$

```
[In] integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*b*e + a*d*
x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1
/(c^2*x^2) + 1) + 1))*b*d/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(73) = 146.

Time = 0.44 (sec) , antiderivative size = 341, normalized size of antiderivative = 4.11

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{8}\left(\frac{bex^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{aex^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c}+\frac{4\,bdx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c}\right.$$

```
[In] integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + a*e*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2/c + 4*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin
(1/(c*x))/c + 4*a*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c + 2*b*e*x*(sqrt(-1/(c^
2*x^2) + 1) + 1)/c^2 + 8*b*d*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - 8*b*d*lo
g(1/(abs(c)*abs(x)))/c^2 + 2*b*e*arcsin(1/(c*x))/c^3 + 2*a*e/c^3 + 4*b*d*ar
csin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 4*a*d/(c^3*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) - 2*b*e/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + b*e*arc
sin(1/(c*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a*e/(c^5*x^2*(sqrt(
-1/(c^2*x^2) + 1) + 1)^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)\,dx$$

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x),x)
[Out] int((a + b*asin(1/(c*x)))*(d + e*x), x)
```

3.47 $\int (a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int (a+b\csc^{-1}(cx))\,dx = ax + bx\csc^{-1}(cx) + \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] a*x+b*x*arccsc(c*x)+b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5323, 272, 65, 214}

$$\int (a+b\csc^{-1}(cx))\,dx = ax + \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c} + bx\csc^{-1}(cx)$$

```
[In] Int[a + b*ArcCsc[c*x],x]
```

```
[Out] a*x + b*x*ArcCsc[c*x] + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5323

```
Int[ArcCsc[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsc[c*x], x] + Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \csc^{-1}(cx)\,dx \\
&= ax + bx\csc^{-1}(cx) + \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x}\,dx}{c} \\
&= ax + bx\csc^{-1}(cx) - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2c} \\
&= ax + bx\csc^{-1}(cx) + (bc)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right) \\
&= ax + bx\csc^{-1}(cx) + \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx = ax + bx\csc^{-1}(cx) + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

```
[In] Integrate[a + b*ArcCsc[c*x],x]
```

```
[Out] a*x + b*x*ArcCsc[c*x] + (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

method	result	size
default	$ax + bx \operatorname{arccsc}(cx) + \frac{b \ln\left(cx + cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$	37
parts	$ax + bx \operatorname{arccsc}(cx) + \frac{b \ln\left(cx + cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$	37
derivativedivides	$\frac{acx + b\left(\operatorname{arccsc}(cx)cx + \ln\left(cx + cx\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{c}$	40

```
[In] int(a+b*arccsc(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b*x*arccsc(c*x)+b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(29) = 58.

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \left(a + b\csc^{-1}(cx)\right) dx = \frac{acx - 2\,bc\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + (bcx - bc)\operatorname{arccsc}(cx) - b\log\left(-cx + \sqrt{c^2x^2 - 1}\right)}{c}$$

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="fricas")
[Out] (a*c*x - 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arccsc(c*x)
 - b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \left(a + b\csc^{-1}(cx)\right) dx = ax + b\left(x\operatorname{acsc}(cx) + \frac{\begin{cases} \operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1 \\ -i\operatorname{asin}(cx) & \text{otherwise} \end{cases}}{c}\right)$$

```
[In] integrate(a+b*acsc(c*x),x)
[Out] a*x + b*(x*acsc(c*x) + Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \left(a + b\csc^{-1}(cx)\right)\,dx$$
$$= ax + \frac{\left(2\,cx\,\mathrm{arccsc}\,(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)b}{2\,c}$$

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(
-1/(c^2*x^2) + 1) + 1))*b/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.00

$$\int \left(a + b\csc^{-1}(cx)\right)\,dx$$
$$= \frac{1}{2}bc\left(\frac{2\,x\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}\right) + ax$$

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="giac")
```

```
[Out] 1/2*b*c*(2*x*arcsin(1/(c*x))/c + (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sq
rt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \left(a + b\csc^{-1}(cx)\right)\,dx = a\,x + b\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right) + \frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

```
[In] int(a + b*asin(1/(c*x)),x)
```

```
[Out] a*x + b*x*asin(1/(c*x)) + (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

3.48 $\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 257

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} - \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{i\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e}$$

```
[Out] -(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e+(a+b*arccsc(c*x))*
ln(1-I*(I/c/x+(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e+(a+b*arc
csc(c*x))*ln(1-I*(I/c/x+(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/c/d)/
e+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e-I*b*polylog(2,I*(I/c/x
+(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e-I*b*polylog(2,I*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/c/d)/e
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5333, 2598}

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx=\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(\sqrt{e^2-c^2d^2}+e\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{e}-\frac{ib\,\text{PolyLog}\left(2,\frac{i\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}-\frac{ib\,\text{PolyLog}\left(2,\frac{i\left(e+\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}+\frac{ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x),x]
```

```
[Out] ((a + b*ArcCsc[c*x])*Log[1 - (I*(e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcCsc[c*
x]))/(c*d)])/e + ((a + b*ArcCsc[c*x])*Log[1 - (I*(e + Sqrt[-(c^2*d^2) + e^2
])*E^(I*ArcCsc[c*x]))/(c*d)])/e - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*Arc
Csc[c*x])])/e - (I*b*PolyLog[2, (I*(e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcCsc
[c*x]))/(c*d)])/e - (I*b*PolyLog[2, (I*(e + Sqrt[-(c^2*d^2) + e^2])*E^(I*Ar
cCsc[c*x]))/(c*d)])/e + ((I/2)*b*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/e
```

Rule 2598

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rule 5333

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :> S
imp[(a + b*ArcCsc[c*x])*(Log[1 - I*(e - Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcCs
c[c*x])/(c*d))]/e), x] + (Dist[b/(c*e), Int[Log[1 - I*(e - Sqrt[(-c^2)*d^2
+ e^2])*(E^(I*ArcCsc[c*x])/(c*d))]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] + Di
st[b/(c*e), Int[Log[1 - I*(e + Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcCsc[c*x])/(
c*d))]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] - Dist[b/(c*e), Int[Log[1 - E^(2
```

```
*I*ArcCsc[c*x])]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] + Simp[(a + b*ArcCsc[c
*x])*(Log[1 - I*(e + Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcCsc[c*x])/(c*d))]/e),
 x] - Simp[(a + b*ArcCsc[c*x])*(Log[1 - E^(2*I*ArcCsc[c*x])]/e), x]) /; Fre
eQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\text{integral} = \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} - \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} + \frac{b\int\frac{\log\left(1-\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce} + \frac{b\int\frac{\log\left(1-\frac{i\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce} - \frac{b\int\frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}$$

$$= \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} - \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} - \frac{ib\,\text{PolyLog}\left(2,\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} - \frac{ib\,\text{PolyLog}\left(2,\frac{i\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} + \frac{ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.60

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \frac{a\log(d+ex)}{e} + \frac{b\left(i(\pi-2\csc^{-1}(cx))^2 + 32i\arcsin\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\arctan\left(\frac{(cd-e)\cot\left(\frac{1}{4}\left(\pi+2\csc^{-1}(cx)\right)\right)}{\sqrt{-c^2d^2+e^2}}\right) - 4\left(\pi-2\csc^{-1}(cx)+4\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(I*(Pi - 2*ArcCsc[c*x])^2 + (32*I)*ArcSin[Sqrt[1 +
e/(c*d)]/Sqrt[2]]*ArcTan[((c*d - e)*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[-(c^2
*d^2) + e^2]] - 4*(Pi - 2*ArcCsc[c*x] + 4*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]
)*Log[1 + (I*(e - Sqrt[-(c^2*d^2) + e^2]))/(c*d*E^(I*ArcCsc[c*x]))] - 4*(Pi
 - 2*ArcCsc[c*x] - 4*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]])*Log[1 + (I*(e + Sqr
t[-(c^2*d^2) + e^2]))/(c*d*E^(I*ArcCsc[c*x]))] - 8*ArcCsc[c*x]*Log[1 - E^((
2*I)*ArcCsc[c*x])] + 4*(Pi - 2*ArcCsc[c*x])*Log[e + d/x] + 8*ArcCsc[c*x]*Lo
g[e + d/x] + (8*I)*(PolyLog[2, (I*(-e + Sqrt[-(c^2*d^2) + e^2]))/(c*d*E^(I*
ArcCsc[c*x]))] + PolyLog[2, ((-I)*(e + Sqrt[-(c^2*d^2) + e^2]))/(c*d*E^(I*A
rcCsc[c*x]))]) + (4*I)*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]))
)/(8*e)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 867 vs. 2(316) = 632.

Time = 3.60 (sec) , antiderivative size = 868, normalized size of antiderivative = 3.38

method	result
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\left(-\frac{i\,\mathrm{dilog}\left(\frac{-cd\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-ie+\sqrt{c^2d^2-e^2}}{-ie+\sqrt{c^2d^2-e^2}}\right)c^3d^2}{e\left(c^2d^2-e^2\right)}-\frac{i\,\mathrm{dilog}\left(\frac{cd\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)+ie+\sqrt{c^2d^2-e^2}}{ie+\sqrt{c^2d^2-e^2}}\right)c^3d^2}{e\left(c^2d^2-e^2\right)}-\cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(-\frac{i\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{\mathrm{arccsc}(cx)\ln\left(\frac{cd\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)+ie-\sqrt{c^2d^2-e^2}}{ie-\sqrt{c^2d^2-e^2}}\right)c^2d^2}{e\left(c^2d^2-e^2\right)}+\frac{\mathrm{arccsc}(cx)\ln\left(\frac{cd\left(\frac{i}{cx}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(-\frac{i\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{\mathrm{arccsc}(cx)\ln\left(\frac{cd\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)+ie-\sqrt{c^2d^2-e^2}}{ie-\sqrt{c^2d^2-e^2}}\right)c^2d^2}{e\left(c^2d^2-e^2\right)}+\frac{\mathrm{arccsc}(cx)\ln\left(\frac{cd\left(\frac{i}{cx}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$

```
[In] int((a+b*arccsc(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(e*x+d)/e+b/c*(-I/e/(c^2*d^2-e^2)*dilog((-c*d*(I/c/x+(1-1/c^2/x^2)^(1/2
))-I*e+(c^2*d^2-e^2)^(1/2))/(-I*e+(c^2*d^2-e^2)^(1/2)))*c^3*d^2-I/e/(c^2*d^
2-e^2)*dilog((c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d^2-e^2)^(1/2))/(I*e
+(c^2*d^2-e^2)^(1/2)))*c^3*d^2-c/e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/
2))-I*c/e*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))+I*c/e*dilog(1+I/c/x+(1-1/c^2/x^2
)^(1/2))+I*c*e/(c^2*d^2-e^2)*dilog((-c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))-I*e+(c
^2*d^2-e^2)^(1/2))/(-I*e+(c^2*d^2-e^2)^(1/2)))+I*c*e/(c^2*d^2-e^2)*dilog((c
*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d^2-e^2)^(1/2))/(I*e+(c^2*d^2-e^2)^
(1/2)))-c*e*arccsc(c*x)/(c^2*d^2-e^2)*ln((-c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))-
I*e+(c^2*d^2-e^2)^(1/2))/(-I*e+(c^2*d^2-e^2)^(1/2)))-c*e*arccsc(c*x)/(c^2*d
^2-e^2)*ln((c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d^2-e^2)^(1/2))/(I*e+(
c^2*d^2-e^2)^(1/2)))+1/e*arccsc(c*x)/(c^2*d^2-e^2)*ln((-c*d*(I/c/x+(1-1/c^2
/x^2)^(1/2))-I*e+(c^2*d^2-e^2)^(1/2))/(-I*e+(c^2*d^2-e^2)^(1/2)))*c^3*d^2+1
/e*arccsc(c*x)/(c^2*d^2-e^2)*ln((c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d
^2-e^2)^(1/2))/(I*e+(c^2*d^2-e^2)^(1/2)))*c^3*d^2)
```

Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \int \frac{a+b\operatorname{acsc}(cx)}{d+ex}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/(e*x+d),x)
[Out] Integral((a + b*acsc(c*x))/(d + e*x), x)
```

Maxima [F]

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \int \frac{b\operatorname{arccsc}(cx)+a}{ex+d}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="maxima")
[Out] b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x + d), x) + a*log(e
*x + d)/e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \int \frac{a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)}{d+e\,x}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x),x)
[Out] int((a + b*asin(1/(c*x)))/(d + e*x), x)
```

3.49 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 102

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx=\frac{b\csc^{-1}(cx)}{de}-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}+\frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2-e^2}}$$

```
[Out] b*arccsc(c*x)/d/e+(-a-b*arccsc(c*x))/e/(e*x+d)+b*arctanh((c^2*d+e/x)/c/(c^2
*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d/(c^2*d^2-e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5335, 1582, 1489, 858, 222, 739, 212}

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx=-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}+\frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}}+\frac{b\csc^{-1}(cx)}{de}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^2,x]
```

```
[Out] (b*ArcCsc[c*x])/(d*e) - (a + b*ArcCsc[c*x])/(e*(d + e*x)) + (b*ArcTanh[(c^2
*d + e/x)/(c*Sqrt[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])])/(d*Sqrt[c^2*d^2 -
 e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a + b\csc^{-1}(cx)}{e(d+ex)} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)}\,dx}{ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{e(d+ex)} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)x^3}\,dx}{ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{e(d+ex)} + \frac{b\text{Subst}\left(\int \frac{x}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{e(d+ex)} - \frac{b\text{Subst}\left(\int \frac{1}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cde}$$

$$= \frac{b\csc^{-1}(cx)}{de} - \frac{a+b\csc^{-1}(cx)}{e(d+ex)} + \frac{b\text{Subst}\left(\int \frac{1}{d^2-\frac{e^2}{c^2}-x^2}\,dx, x, \frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{cd}$$

$$= \frac{b\csc^{-1}(cx)}{de} - \frac{a+b\csc^{-1}(cx)}{e(d+ex)} + \frac{b\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2-e^2}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.38

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a}{e(d+ex)} - \frac{b\csc^{-1}(cx)}{e(d+ex)} + \frac{b\arcsin\left(\frac{1}{cx}\right)}{de} + \frac{b\log(d+ex)}{d\sqrt{c^2d^2-e^2}} - \frac{b\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d\sqrt{c^2d^2-e^2}}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^2,x]
```

```
[Out] -(a/(e*(d + e*x))) - (b*ArcCsc[c*x])/(e*(d + e*x)) + (b*ArcSin[1/(c*x)])/(d
*e) + (b*Log[d + e*x])/(d*Sqrt[c^2*d^2 - e^2]) - (b*Log[e + c*(c*d - Sqrt[c
^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])*x])/(d*Sqrt[c^2*d^2 - e^2])
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.88

method	result
parts	$-\frac{a}{(ex+d)e}+\frac{b\left(-\frac{c^2\,\mathrm{arccsc}(cx)}{(cex+cd)e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)}{c}$
derivativedivides	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\mathrm{arccsc}(cx)}{(cex+cd)e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)}{c}$
default	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\mathrm{arccsc}(cx)}{(cex+cd)e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)}{c}$

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/(e*x+d)/e+b/c*(-c^2/(c*e*x+c*d)/e*arccsc(c*x)+1/e*(c^2*x^2-1)^(1/2)*(arc
tan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)-ln(2*(((c^2*d^2-e^2)/e^2
)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d)))/((c^2*x^2-1)/c^2/x^2)^
(1/2)/x/d/((c^2*d^2-e^2)/e^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(96) = 192$.

Time = 0.33 (sec) , antiderivative size = 475, normalized size of antiderivative = 4.66

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx$$
$$= \left[-\frac{ac^2d^3-ade^2-\sqrt{c^2d^2-e^2}(be^2x+bde)\log\left(\frac{c^3d^2x+cde+\sqrt{c^2d^2-e^2}(c^2dx+e)+\left(c^2d^2+\sqrt{c^2d^2-e^2}cd-e^2\right)\sqrt{c^2x^2-1}}{ex+d}\right)+}{c^2d^4e-d^2e^3+(c^2d^3e}\right.$$
$$-\frac{ac^2d^3-ade^2+2\sqrt{-c^2d^2+e^2}(be^2x+bde)\arctan\left(-\frac{\sqrt{-c^2d^2+e^2}\sqrt{c^2x^2-1}e-\sqrt{-c^2d^2+e^2}(cex+cd)}{c^2d^2-e^2}\right)+(bc^2d^3-b}{c^2d^4e-d^2e^3+(c^2d^3e^2-d}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-(a*c^2*d^3 - a*d*e^2 - sqrt(c^2*d^2 - e^2)*(b*e^2*x + b*d*e)*log((c^3*d^2
*x + c*d*e + sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 + sqrt(c^2*d^2 -
```

```
e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) + (b*c^2*d^3 - b*d*e^2)*arccs
c(c*x) + 2*(b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x + sq
rt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x), -(a*c^2*
d^3 - a*d*e^2 + 2*sqrt(-c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*arctan(-(sqrt(-c^2
*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))/(c^2*
d^2 - e^2)) + (b*c^2*d^3 - b*d*e^2)*arccsc(c*x) + 2*(b*c^2*d^3 - b*d*e^2 +
(b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2
*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

Sympy [F]

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex)^2}\,dx = \int \frac{a + b\operatorname{acsc}(cx)}{(d + ex)^2}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**2,x)
[Out] Integral((a + b*acsc(c*x))/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex)^2}\,dx = \int \frac{b\operatorname{arccsc}(cx) + a}{(ex + d)^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="maxima")
[Out] -((c^2*e^2*x + c^2*d*e)*integrate(x*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))
/(c^2*e^2*x^3 + c^2*d*e*x^2 - e^2*x - d*e + (c^2*e^2*x^3 + c^2*d*e*x^2 - e^
2*x - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) + arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1)))*b/(e^2*x + d*e) - a/(e^2*x + d*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{\left(d+e\,x\right)^2}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^2,x)
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^2, x)
```

3.50 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 172

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3} dx = -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d(c^2d^2-e^2)\left(e+\frac{d}{x}\right)}+\frac{b\csc^{-1}(cx)}{2d^2e}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}+\frac{b(2c^2d^2-e^2)\operatorname{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2(c^2d^2-e^2)^{3/2}}$$

```
[Out] 1/2*b*arccsc(c*x)/d^2/e+1/2*(-a-b*arccsc(c*x))/e/(e*x+d)^2+1/2*b*(2*c^2*d^2
-e^2)*arctanh((c^2*d+e/x)/c/(c^2*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^2/(c
^2*d^2-e^2)^(3/2)-1/2*b*c*e*(1-1/c^2/x^2)^(1/2)/d/(c^2*d^2-e^2)/(e+d/x)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5335, 1582, 1489, 1665, 858, 222, 739, 212}

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3} dx = -\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}+\frac{b(2c^2d^2-e^2)\operatorname{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{2d^2(c^2d^2-e^2)^{3/2}}-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d(c^2d^2-e^2)\left(\frac{d}{x}+e\right)}+\frac{b\csc^{-1}(cx)}{2d^2e}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^3,x]
```

```
[Out] -1/2*(b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(d*(c^2*d^2 - e^2)*(e + d/x)) + (b*ArcCs
c[c*x])/(2*d^2*e) - (a + b*ArcCsc[c*x])/(2*e*(d + e*x)^2) + (b*(2*c^2*d^2 -
 e^2)*ArcTanh[(c^2*d + e/x)/(c*Sqrt[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])])
/(2*d^2*(c^2*d^2 - e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
```

```
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}-\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^2}\,dx}{2ce}\\
&= -\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}-\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)^2x^4}\,dx}{2ce}\\
&= -\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}+\frac{b\text{Subst}\left(\int\frac{x^2}{(e+dx)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2ce}\\
&= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}-\frac{(bc)\text{Subst}\left(\int\frac{e-\left(d-\frac{e^2}{c^2d}\right)x}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2e\left(c^2d^2-e^2\right)}\\
&= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}+\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2cd^2e}\\
&\quad-\frac{\left(bc\left(2-\frac{e^2}{c^2d^2}\right)\right)\text{Subst}\left(\int\frac{1}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2\left(c^2d^2-e^2\right)}\\
&= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)}+\frac{b\csc^{-1}(cx)}{2d^2e}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}\\
&\quad+\frac{\left(bc\left(2-\frac{e^2}{c^2d^2}\right)\right)\text{Subst}\left(\int\frac{1}{d^2-\frac{e^2}{c^2}-x^2}\,dx,x,\frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2\left(c^2d^2-e^2\right)}
\end{aligned}
$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\csc^{-1}(cx)}{2d^2e} - \frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} + \frac{b(2c^2d^2-e^2)\operatorname{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\left(c^2d^2-e^2\right)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.45

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx = \frac{1}{2}\left(-\frac{a}{e(d+ex)^2} - \frac{bce\sqrt{1-\frac{1}{c^2x^2}}\,x}{d\left(c^2d^2-e^2\right)(d+ex)} - \frac{b\csc^{-1}(cx)}{e(d+ex)^2} + \frac{b\arcsin\left(\frac{1}{cx}\right)}{d^2e} + \frac{b(2c^2d^2-e^2)\log(d+ex)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}} - \frac{b(2c^2d^2-e^2)\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}}\right)$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d^2 - e^2)*
(d + e*x)) - (b*ArcCsc[c*x])/(e*(d + e*x)^2) + (b*ArcSin[1/(c*x)])/(d^2*e)
+ (b*(2*c^2*d^2 - e^2)*Log[d + e*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2
- e^2]) - (b*(2*c^2*d^2 - e^2)*Log[e + c*(c*d - Sqrt[c^2*d^2 - e^2]*Sqrt[1
- 1/(c^2*x^2)])*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2 - e^2]))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(159) = 318$.

Time = 3.24 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.33

method	result
parts	$-\frac{a}{2(ex+d)^2 e}+\frac{b\left(-\frac{c^3 \operatorname{arccsc}(cx)}{2(cex+cd)^2 e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\, c^3d^3+\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\, c^3d^2ex-2\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{a\,c^3}{2(cex+cd)^2 e}+b\,c^3\left(-\frac{\operatorname{arccsc}(cx)}{2(cex+cd)^2 e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\, c^3d^3+\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\, c^3d^2ex-2\right.}{}\right.}{}$
default	$\frac{-\frac{a\,c^3}{2(cex+cd)^2 e}+b\,c^3\left(-\frac{\operatorname{arccsc}(cx)}{2(cex+cd)^2 e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\, c^3d^3+\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\, c^3d^2ex-2\right.}{}\right.}{}$

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/(e*x+d)^2/e+b/c*(-1/2*c^3/(c*e*x+c*d)^2/e*arccsc(c*x)+1/2/e*(c^2*x^2
-1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*c^3*d^3+ar
ctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*c^3*d^2*e*x-2*ln(2*(((c
^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*c^3*d^3-
2*ln(2*(((c^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d
))*c^3*d^2*e*x-arctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*c*d*e^
2-arctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*e^3*c*x-(c^2*x^2-1)
^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*c*d*e^2+ln(2*(((c^2*d^2-e^2)/e^2)^(1/2)*(c
^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*c*d*e^2+ln(2*(((c^2*d^2-e^2)/e^2)
^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*e^3*c*x)/((c^2*x^2-1)/c^
2/x^2)^(1/2)/x/d^2/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(156) = 312$.

Time = 0.56 (sec) , antiderivative size = 1111, normalized size of antiderivative = 6.46

$$\int \frac{a + b\csc^{-1}(cx)}{(d+ex)^3}\,dx$$
$$= \left[-\frac{ac^4d^6 + bc^3d^5e - 2\,ac^2d^4e^2 - bcd^3e^3 + ad^2e^4 + (bc^3d^3e^3 - bcde^5)x^2 - (2\,bc^2d^4e - bd^2e^3 + (2\,bc^2d^2e^3 -}{}\right.$$

$$-\frac{ac^4d^6 + bc^3d^5e - 2\,ac^2d^4e^2 - bcd^3e^3 + ad^2e^4 + (bc^3d^3e^3 - bcde^5)x^2 + 2\,(2\,bc^2d^4e - bd^2e^3 + (2\,bc^2d^2e^3 -}{}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*c^4*d^6 + b*c^3*d^5*e - 2*a*c^2*d^4*e^2 - b*c*d^3*e^3 + a*d^2*e^4
+ (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d
^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(c^2*d^2 - e^2)*
log((c^3*d^2*x + c*d*e + sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 + sqr
t(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) + 2*(b*c^3*d^4*e^
2 - b*c*d^2*e^4)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arccsc(c*x)
+ 2*(b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2
*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*arctan(-
c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3*e^3 - b*
d*e^5)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^
6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3
*e^6)*x), -1/2*(a*c^4*d^6 + b*c^3*d^5*e - 2*a*c^2*d^4*e^2 - b*c*d^3*e^3 + a
*d^2*e^4 + (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 + 2*(2*b*c^2*d^4*e - b*d^2*e^3 +
 (2*b*c^2*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(-c^2
*d^2 + e^2)*arctan(-(sqrt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d
^2 + e^2)*(c*e*x + c*d))/(c^2*d^2 - e^2)) + 2*(b*c^3*d^4*e^2 - b*c*d^2*e^4)
*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arccsc(c*x) + 2*(b*c^4*d^6 -
 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2*e^4 + b*e^6)*x^
2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*arctan(-c*x + sqrt(c^2*x
^2 - 1)) + (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3*e^3 - b*d*e^5)*x)*sqrt(c
^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 - 2*c^2*d^
4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3*e^6)*x)]
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx=\int \frac{a+b\operatorname{acsc}(cx)}{(d+ex)^3}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx=\int \frac{b\operatorname{arccsc}(cx)+a}{(ex+d)^3}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(c^2*e^3*x^2 + 2*c^2*d*e^2*x + c^2*d^2*e)*integrate(1/2*x*e^(1/2*lo
g(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2*x -
 d^2*e + (c^2*d^2*e - e^3)*x^2 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2*x
 - d^2*e + (c^2*d^2*e - e^3)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) + ar
ctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/
2*a/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{\left(d+e\,x\right)^3}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^3,x)
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^3, x)
```

3.51 $\int x^2\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 496

$$\begin{aligned}
&\int x^2\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx \\
&= \frac{4bd\sqrt{d+ex}(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{4b(d+ex)^{3/2}\,(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{2d^2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} \\
&\quad -\frac{4d(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3}+\frac{2(d+ex)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^3} \\
&\quad +\frac{4b(5c^2d^2-9e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad -\frac{4bd(9c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad -\frac{32bd^4\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

```
[Out] 2/3*d^2*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3-4/5*d*(e*x+d)^(5/2)*(a+b*arccsc
(c*x))/e^3+2/7*(e*x+d)^(7/2)*(a+b*arccsc(c*x))/e^3-4/35*b*(e*x+d)^(3/2)*(-c
^2*x^2+1)/c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/105*b*d*(-c^2*x^2+1)*(e*x+d)^(1/2)/
c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/105*b*(5*c^2*d^2-9*e^2)*EllipticE(1/2*(-c*x+1
)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)
/c^4/e^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/105*b*d*(9*c^2*d
^2-e^2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*
(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*
x+d)^(1/2)-32/105*b*d^4*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/
```

```
(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1/c
^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.40, number of steps used = 31, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {45, 5355, 12, 6853, 6874, 757, 858, 733, 435, 430, 972, 946, 174, 552, 551, 847}

$$\begin{aligned}
&\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx \\
&=\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} \\
&+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3} \\
&-\frac{32bd^4\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} \\
&-\frac{32bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} \\
&+\frac{32bd^2\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&-\frac{4b\sqrt{1-c^2x^2}(c^2d^2+3e^2)\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^4e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&-\frac{4bd\sqrt{1-c^2x^2}(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} \\
&+\frac{4bd(1-c^2x^2)\sqrt{d+ex}}{105c^3ex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{4b(1-c^2x^2)(d+ex)^{3/2}}{35c^3ex\sqrt{1-\frac{1}{c^2x^2}}}
\end{aligned}$$

```
[In] Int[x^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (4*b*d*Sqrt[d + e*x]*(1 - c^2*x^2))/(105*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) - (
4*b*(d + e*x)^(3/2)*(1 - c^2*x^2))/(35*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*
d^2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (4*d*(d + e*x)^(5/2)*(a
+ b*ArcCsc[c*x]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^3)
 + (32*b*d^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(
```

d + e*x))/(c*d + e)]) - (4*b*(c^2*d^2 + 3*e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x ^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*c^4*e^2* Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (32*b*d^3*Sqrt[(c* (d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt [2]], (2*e)/(c*d + e)])/(105*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^ 2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^4*e^2* Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d^4*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/ (c*d + e)])/(105*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} \\
&\quad+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}+\frac{b\int\frac{2(d+ex)^{3/2}\left(8d^2-12dex+15e^2x^2\right)}{105e^3\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{c} \\
&= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} \\
&\quad+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}+\frac{(2b)\int\frac{(d+ex)^{3/2}\left(8d^2-12dex+15e^2x^2\right)}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{105ce^3}
\end{aligned}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{(d+ex)^{3/2}(8d^2-12dex+15e^2x^2)}{x\sqrt{1-c^2x^2}}\,dx}{105ce^3\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \left(-\frac{12de(d+ex)^{3/2}}{\sqrt{1-c^2x^2}} + \frac{8d^2(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}} + \frac{15e^2x(d+ex)^{3/2}}{\sqrt{1-c^2x^2}}\right)dx}{105ce^3\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3} + \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}\,dx}{105ce^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(8bd\sqrt{1-c^2x^2}\right)\int \frac{(d+ex)^{3/2}}{\sqrt{1-c^2x^2}}\,dx}{35ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{x(d+ex)^{3/2}}{\sqrt{1-c^2x^2}}\,dx}{7ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{16bd\sqrt{d+ex}(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{4b(d+ex)^{3/2}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3} + \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int \left(\frac{2de}{\sqrt{d+ex}\sqrt{1-c^2x^2}} + \frac{d^2}{x\sqrt{d+ex}\sqrt{1-c^2x^2}} + \frac{e^2x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{105ce^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(16bd\sqrt{1-c^2x^2}\right)\int \frac{\frac{1}{2}(-3c^2d^2-e^2)-2c^2dex}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{105c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(4b\sqrt{1-c^2x^2}\right)\int \frac{\left(-\frac{3e}{2}-\frac{3}{2}c^2dx\right)\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{4bd\sqrt{d+ex}(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{4b(d+ex)^{3/2}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$+ \frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} - \frac{4d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3}$$

$$+ \frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^3} + \frac{\left(16bd^4\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{105ce^3\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$- \frac{\left(32bd^2\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{105ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(32bd^3\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{105ce^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$+ \frac{\left(8b\sqrt{1-c^2x^2}\right)\int \frac{3c^2de+\frac{3}{4}c^2(c^2d^2+3e^2)x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{105c^5e\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int \frac{x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{105ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$+ \frac{\left(8bd(cd-e)(cd+e)\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{105c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$
\begin{aligned}
&= \frac{4bd\sqrt{d+ex}\left(1-c^2x^2\right)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{4b(d+ex)^{3/2}\left(1-c^2x^2\right)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} \\
&- \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3} \\
&+ \frac{\left(16bd^4\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{105ce^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{105ce^2\sqrt{1-\frac{1}{c^2x^2}}x} \\
&- \frac{\left(16bd^3\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{105ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\left(c^2d^2+3e^2\right)\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{35c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(8b\left(3c^2de^2-\frac{3}{4}c^2d\left(c^2d^2+3e^2\right)\right)\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{105c^5e^2\sqrt{1-\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(64bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{105c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}} \\
&- \frac{\left(64bd^3\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{105c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{\left(16bd(cd-e)(cd+e)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{105c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$= \frac{4bd\sqrt{d+ex}(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{4b(d+ex)^{3/2}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}$$
$$- \frac{4d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^3}$$
$$+ \frac{64bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{64bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{16bd(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{\left(32bd^4\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}dx,x,\sqrt{1-cx}\right)}{105ce^3\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(32bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{105c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$
$$- \frac{\left(4b(c^2d^2+3e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{35c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$
$$+ \frac{\left(32bd^3\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{105c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{\left(16b\left(3c^2de^2-\frac{3}{4}c^2d(c^2d^2+3e^2)\right)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right.}{105c^6e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{4bd\sqrt{d+ex}(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{4b(d+ex)^{3/2}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}$$
$$- \frac{4d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^3}$$
$$+ \frac{32bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{4b(c^2d^2+3e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{32bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{16bd(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{4bd(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{\left(32bd^4\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{105ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$
\begin{aligned}
&= \frac{4bd\sqrt{d+ex}(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{4b(d+ex)^{3/2}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} \\
&\quad - \frac{4d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^3} \\
&\quad + \frac{32bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad - \frac{4b(c^2d^2+3e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad - \frac{32bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{16bd(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{4bd(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{32bd^4\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 44.49 (sec) , antiderivative size = 870, normalized size of antiderivative = 1.75

$$
\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx = -\frac{ad^3\sqrt{d+ex}B_{-\frac{ex}{d}}\left(3,\frac{3}{2}\right)}{e^3\sqrt{1+\frac{ex}{d}}}
$$

$$
+\frac{b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(-\frac{4\left(-5c^2d^2+9e^2\right)\sqrt{1-\frac{1}{c^2x^2}}}{105e^2}-\frac{16c^3d^3\csc^{-1}(cx)}{105e^3}-\frac{2}{7}c^3x^3\csc^{-1}(cx)-\frac{2c^2x^2\left(2e\sqrt{1-\frac{1}{c^2x^2}}+cd\csc^{-1}(cx)\right)}{35e}-\frac{8cx\left(cde\sqrt{1-\frac{1}{c^2x^2}}-c^2d^2\right.}{105e^2}}{\sqrt{d+ex}}\right.}{}
$$

```
[In] Integrate[x^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] -((a*d^3*Sqrt[d + e*x]*Beta[-((e*x)/d), 3, 3/2])/(e^3*Sqrt[1 + (e*x)/d])) +
 (b*(-((c*(e + d/x)*x*((-4*(-5*c^2*d^2 + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(105
*e^2) - (16*c^3*d^3*ArcCsc[c*x])/(105*e^3) - (2*c^3*x^3*ArcCsc[c*x])/7 - (2
*c^2*x^2*(2*e*Sqrt[1 - 1/(c^2*x^2)] + c*d*ArcCsc[c*x]))/(35*e) - (8*c*x*(c*
d*e*Sqrt[1 - 1/(c^2*x^2)] - c^2*d^2*ArcCsc[c*x]))/(105*e^2)))/Sqrt[d + e*x]
) - (2*Sqrt[e + d/x]*Sqrt[c*x]*((2*(9*c^3*d^3*e - c*d*e^3)*Sqrt[(c*d + c*e*
x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*c
^4*d^4 + 5*c^2*d^2*e^2 - 9*e^4)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*
x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1
 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-5*c^3*d^3*e + 9*c*d*e^3)*
Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*
x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e
*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]],
(c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (
c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqrt[(c*d +
c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]
*(-2 + c^2*x^2))))/(105*e^3*Sqrt[d + e*x])))/c^4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. 2(445) = 890.

Time = 12.33 (sec) , antiderivative size = 1204, normalized size of antiderivative = 2.43

method	result	size
derivativedivides	Expression too large to display	1204
default	Expression too large to display	1204
parts	Expression too large to display	1225

```
[In] int(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/7*(e*x+d)^(7/2)-2/5*d*(e*x+d)^(5/2)+1/3*d^2*(e*x+d)^(3/2))+b*(1
/7*arccsc(c*x)*(e*x+d)^(7/2)-2/5*arccsc(c*x)*d*(e*x+d)^(5/2)+1/3*arccsc(c*x
)*d^2*(e*x+d)^(3/2)+2/105/c^4*(3*(c/(c*d-e))^(1/2)*c^3*(e*x+d)^(7/2)-7*(c/(
c*d-e))^(1/2)*c^3*d*(e*x+d)^(5/2)+4*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c
*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((
c*d-e)/(c*d+e))^(1/2))*c^3*d^3+5*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e
*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d
-e)/(c*d+e))^(1/2))*c^3*d^3-8*d^3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(
e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c
*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^3+5*(c/(c*d-e))^(1/2)*c^3
```

```
*d^2*(e*x+d)^(3/2)-5*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)
/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))
^(1/2))*c^2*d^2*e+5*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/
(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^
(1/2))*c^2*d^2*e-(c/(c*d-e))^(1/2)*c^3*d^3*(e*x+d)^(1/2)+8*((-c*(e*x+d)+c*d
-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/
2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2-9*((-c*(e*x+d)+c*d-e)
/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2-3*(c/(c*d-e))^(1/2)*c*e^
2*(e*x+d)^(3/2)+9*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c
*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1
/2))*e^3-9*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^
(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^
3+(c/(c*d-e))^(1/2)*c*d*e^2*(e*x+d)^(1/2))/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d
)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

Fricas [F]

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex+d}\left(b\operatorname{arccsc}\left(cx\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x + d), x)
```

Sympy [F]

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2(a+b\operatorname{acsc}\left(cx\right))\sqrt{d+ex}\,dx$$

```
[In] integrate(x**2*(a+b*acsc(c*x))*(e*x+d)**(1/2),x)
[Out] Integral(x**2*(a + b*acsc(c*x))*sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

```
[In] int(x^2*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
[Out] int(x^2*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.52 $\int x\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 404

$$\begin{aligned}
&\int x\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx\\
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2}\\
&\quad + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} - \frac{8bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}\\
&\quad + \frac{4b(3c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\\
&\quad + \frac{8bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

```
[Out] -2/3*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^2+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*
x))/e^2-4/15*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/x/(1-1/c^2/x^2)^(1/2)-8/15*b*
d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(
1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/
2)+4/15*b*(3*c^2*d^2-e^2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(
c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e/x/(1-1/c^
2/x^2)^(1/2)/(e*x+d)^(1/2)+8/15*b*d^3*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2)
,2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/
c/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.24, number of steps used = 24, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {45, 5355, 12, 6853, 6874, 757, 858, 733, 435, 430, 972, 946, 174, 552, 551}

$$\begin{aligned}
&\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx \\
&= -\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} \\
&+\frac{8bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} \\
&+\frac{8bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^2ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} \\
&-\frac{8bd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&+\frac{4b\sqrt{1-c^2x^2}(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} \\
&-\frac{4b(1-c^2x^2)\sqrt{d+ex}}{15c^3x\sqrt{1-\frac{1}{c^2x^2}}}
\end{aligned}$$

```
[In] Int[x*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*Sqrt[1 - 1/(c^2*x^2)]*x) - (2*d*
(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) + (2*(d + e*x)^(5/2)*(a + b*Ar
cCsc[c*x]))/(5*e^2) - (8*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcS
in[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2*e*Sqrt[1 - 1/(c^2*x^2)
]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]
*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)
])/(15*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*(c*d - e)*(c*d +
 e)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1
 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) + (8*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipt
icPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c*e^2*Sqrt[1 -
 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
```

```
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
```

```
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2} \\
&\quad +\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{b\int \frac{2(d+ex)^{3/2}(-2d+3ex)}{15e^2\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{c} \\
&= -\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2} \\
&\quad +\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{(2b)\int \frac{(d+ex)^{3/2}(-2d+3ex)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{15ce^2} \\
&= -\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} \\
&\quad +\frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{(d+ex)^{3/2}(-2d+3ex)}{x\sqrt{1-c^2x^2}}\,dx}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= -\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} \\
&\quad +\frac{\left(2b\sqrt{1-c^2x^2}\right)\int \left(\frac{3e(d+ex)^{3/2}}{\sqrt{1-c^2x^2}}-\frac{2d(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}\right)dx}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= -\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} \\
&\quad -\frac{\left(4bd\sqrt{1-c^2x^2}\right)\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}\,dx}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{(d+ex)^{3/2}}{\sqrt{1-c^2x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$
$$+ \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}$$
$$- \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int\left(\frac{2de}{\sqrt{d+ex}\sqrt{1-c^2x^2}} + \frac{d^2}{x\sqrt{d+ex}\sqrt{1-c^2x^2}} + \frac{e^2x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(4b\sqrt{1-c^2x^2}\right)\int\frac{\frac{1}{2}(-3c^2d^2-e^2)-2c^2dex}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$
$$+ \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} - \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int\frac{x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15c\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(4bd^3\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(8bd\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{15ce\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(8bd^2\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15ce\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(2b(cd-e)(cd+e)\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2}$$

$$+ \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} - \frac{\left(4bd^3\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$- \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{15ce\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4bd^2\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$- \frac{\left(16bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$+ \frac{\left(16bd^2\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+ \frac{\left(4b(cd-e)(cd+e)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^4e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$
\begin{aligned}
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2} \\
&+ \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} \\
&- \frac{16bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&+ \frac{16bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{4b(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{\left(8bd^3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(8bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}} \\
&- \frac{\left(8bd^2\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$
$$+ \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} - \frac{8bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{8bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{4b(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{\left(8bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}dx,x,\sqrt{1-cx}\right)}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$
$$+ \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} - \frac{8bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{8bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{4b(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{8bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.42 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.91

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{15}\left(\frac{4b\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}{c}+\frac{2a\sqrt{d+ex}(-2d^2+dex+3e^2x^2)}{e^2}+\frac{2b\sqrt{d+ex}(-2d^2+dex+3e^2x^2)\csc^{-1}(cx)}{e^2}-\frac{4ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-2cd(cd-e)E\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(-c^2d^2-2cde+e^2)\,\text{Ellipti}\right.}{c^3e^2\sqrt{-\frac{c}{cd+}}}\right.$$

```
[In] Integrate[x*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((4*b*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (2*a*Sqrt[d + e*x]*(-2*d^2
 + d*e*x + 3*e^2*x^2))/e^2 + (2*b*Sqrt[d + e*x]*(-2*d^2 + d*e*x + 3*e^2*x^2
)*ArcCsc[c*x])/e^2 - ((4*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*
e*x)/(c*d + e)]*(-2*c*d*(c*d - e)*EllipticE[I*ArcSinh[Sqrt[-(c/(c*d + e))]*
Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (-(c^2*d^2) - 2*c*d*e + e^2)*Ellipti
cF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + 2*
c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x
]], (c*d + e)/(c*d - e)]))/(c^3*e^2*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^
2)]*x))/15
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(363) = 726.

Time = 9.99 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.04

method	result
derivativedivides	$\frac{-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+\frac{(ex+d)^{\frac{3}{2}}d}{3}\right)-2b\left(-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}-2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^{\frac{3}{2}}\right.}{}\right.}{}$
default	$\frac{-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+\frac{(ex+d)^{\frac{3}{2}}d}{3}\right)-2b\left(-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}-2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^{\frac{3}{2}}\right.}{}\right.}{}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{(ex+d)^{\frac{3}{2}}d}{3}\right)}{e^2}+\frac{2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}-\frac{4\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^{\frac{3}{2}}}{15}-\right.}{}$

```
[In] int(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(-1/5*(e*x+d)^(5/2)+1/3*(e*x+d)^(3/2)*d)-b*(-1/5*arccsc(c*x)*(e*x
+d)^(5/2)+1/3*arccsc(c*x)*(e*x+d)^(3/2)*d-2/15/c^3*((c/(c*d-e))^(1/2)*c^2*(
e*x+d)^(5/2)-2*(c/(c*d-e))^(1/2)*c^2*d*(e*x+d)^(3/2)-d^2*((-c*(e*x+d)+c*d-e
)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)
*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2-2*((-c*(e*x+d)+c*d-e)/(c*d-
e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*
d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2+2*d^2*((-c*(e*x+d)+c*d-e)/(c*d
-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(
c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2+(c/(c*
d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c
*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((
c*d-e)/(c*d+e))^(1/2))*c*d*e-2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x
+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e
)/(c*d+e))^(1/2))*c*d*e+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d
+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*e^2*(e*x+d)^(1/2))/(c/(c*d-e))^(1/2)/x/((c
^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

Fricas [F]

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex+d}(b\operatorname{arccsc}(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsc(c*x) + a*x)*sqrt(e*x + d), x)
```

Sympy [F]

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x(a+b\operatorname{acsc}(cx))\sqrt{d+ex}\,dx$$

```
[In] integrate(x*(a+b*acsc(c*x))*(e*x+d)**(1/2),x)
[Out] Integral(x*(a + b*acsc(c*x))*sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex+d}(b\operatorname{arccsc}(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x\left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)\sqrt{d+e\,x}\,dx$$

```
[In] int(x*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
[Out] int(x*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.53 $\int \sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 315

$$\int \sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx = \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^
(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(1-
1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*d*EllipticF(1/2*(-c*x+1)^(
1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2
+1)^(1/2)/c^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/3*b*d^2*EllipticPi(1/2*
(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/
2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5335, 1588, 972, 733, 430, 947, 174, 552, 551, 858, 435}

$$\begin{aligned}
&\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx\\
&=\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}\\
&\quad-\frac{4bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}\\
&\quad-\frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}\\
&\quad-\frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}
\end{aligned}$$

```
[In] Int[Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e) - (4*b*Sqrt[d + e*x]*Sqrt[1 -
 c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2
*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*d*Sqrt[(c*(d
 + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2
]], (2*e)/(c*d + e)])/(3*c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*
d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{(2b)\int\frac{(d+ex)^{3/2}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{3ce}$$

$$=\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{(d+ex)^{3/2}}{x\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$=\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\left(\frac{2de}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}+\frac{d^2}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}+\frac{e^2x}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\right)dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$=\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{\left(4bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{\left(2bd^2\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{\left(2be\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{x}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$- \frac{\left(2bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{\left(2bd^2\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$- \frac{\left(8bd\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$- \frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$- \frac{\left(4bd^2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{3ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$- \frac{\left(4b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$

$$+ \frac{\left(4bd\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}}$$

$$- \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$- \frac{\left(4bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \sqrt{1-cx}\right)}{3ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$
$$- \frac{4bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 657 vs. 2(315) = 630.

Time = 32.09 (sec) , antiderivative size = 657, normalized size of antiderivative = 2.09

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{2a(d+ex)^{3/2}}{3e}$$
$$- \frac{b(cd+cex)\left(-\frac{2\left(2e\sqrt{1-\frac{1}{c^2x^2}}+cd\csc^{-1}(cx)+cex\csc^{-1}(cx)\right)}{e} + \frac{4d\sqrt{-c^2\left(1-\frac{1}{c^2x^2}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{(cd+e)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{cd+cex}{cd+e}}} - \frac{4(-cd+\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (2*a*(d + e*x)^(3/2))/(3*e) - (b*(c*d + c*e*x)*((-2*(2*e*Sqrt[1 - 1/(c^2*x^
2)] + c*d*ArcCsc[c*x] + c*e*x*ArcCsc[c*x]))/e + (4*d*Sqrt[-(c^2*(1 - 1/(c^2
*x^2))*x^2)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/((c
*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*d + c*e*x)/(c*d + e)]) - (4*(-(c*d)
 + e)*Sqrt[-(c^2*(1 - 1/(c^2*x^2))*x^2)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*d + c*e*x
)/(c*d + e)]) + ((c^2*(1 - 1/(c^2*x^2))*x^2*(c*d + c*e*x) + c^2*d*x*Sqrt[-(
c^2*(1 - 1/(c^2*x^2))*x^2)]*Sqrt[(c*d + c*e*x)/(c*d + e)]*EllipticF[ArcSin[
Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/
(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(
c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c
*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d)
+ e)] + c*e*x*Sqrt[-(c^2*(1 - 1/(c^2*x^2))*x^2)]*Sqrt[(c*d + c*e*x)/(c*d +
e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])*Sec[2*Ar
cCsc[c*x]]*Sin[4*ArcCsc[c*x]])/(c^2*(1 - 1/(c^2*x^2))*(e + d/x)*x^2)))/(3*c
^2*Sqrt[d + e*x])
```

Maple [A] (verified)

Time = 8.52 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\frac{2\left(2d\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\operatorname{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\operatorname{Ellipti}\right.}{}\right.}{}$
default	$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\frac{2\left(2d\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\operatorname{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\operatorname{Ellipti}\right.}{}\right.}{}$
parts	$\frac{2a(ex+d)^{\frac{3}{2}}}{3e}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\frac{2\left(2d\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\operatorname{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\operatorname{Elli}\right.}{}\right.}{}$

```
[In] int((a+b*arccsc(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arccsc(c*x)+2/3/c^2*(2*d*Elli
pticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c-EllipticE(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-d*EllipticPi((
e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^
(1/2))*c+EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))
*e-EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e)*((
-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)/(c/(c*d
-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2
)))
```

Fricas [F]

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}\left(b\operatorname{arccsc}\left(cx\right)+a\right)dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a), x)
```

Sympy [F]

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(a+b\operatorname{acsc}\left(cx\right)\right)\sqrt{d+ex}\,dx$$

```
[In] integrate((a+b*acsc(c*x))*(e*x+d)**(1/2),x)
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx = \int \sqrt{ex+d}(b\operatorname{arccsc}\left(cx\right)+a)\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.54 $\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \operatorname{Int}\left(\frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx$$

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 38.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x,x]
[Out] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\sqrt{ex+d}}{x}dx$$

```
[In] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
[Out] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))*(e*x+d)**(1/2)/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.48

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex+d}(b\operatorname{arccsc}(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*sqrt(d)*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)) + b*integrate(sqrt
(e*x + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) + 2*sqrt(e*x + d)*a
```

Giac [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex+d}(b\operatorname{arccsc}(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}}{x}\,dx$$

```
[In] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x,x)
```

```
[Out] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x, x)
```

3.55 $\int \frac{\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \mathrm{Int}\left(\frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx$$

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\ \mathrm{arccsc}\,(cx))\,\sqrt{ex+d}}{x^2}dx$$

```
[In] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
[Out] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 22.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,\sqrt{d+ex}}{x^2}\,dx$$

```
[In] integrate((a+b*acsc(c*x))*(e*x+d)**(1/2)/x**2,x)
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.19

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*e*x*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)) + 2*b*sqrt(d)*x*i
ntegrate(sqrt(e*x + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x) - 2*
sqrt(e*x + d)*a*sqrt(d))/(sqrt(d)*x)
```

Giac [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}}{x^2}\,dx$$

```
[In] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x^2, x)
```

3.56 $\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 372

$$
\begin{aligned}
\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = & -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} \\
& + \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} - \frac{28bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
& - \frac{4b(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
& - \frac{4bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

```
[Out] 2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*x))/e-4/15*b*e*(-c^2*x^2+1)*(e*x+d)^(1/2)/c
^3/x/(1-1/c^2/x^2)^(1/2)-28/15*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(
1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(1-1/c^2/x^2
)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*(-c*
x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c
^2*x^2+1)^(1/2)/c^4/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*d^3*EllipticP
i(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e
))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5335, 1588, 972, 733, 430, 947, 174, 552, 551, 858, 435, 945, 1598}

$$\int (d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} - \frac{4bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{28bd\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b\sqrt{1-c^2x^2}(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4be(1-c^2x^2)\sqrt{d+ex}}{15c^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

```
[In] Int[(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (-4*b*e*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*
(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) - (28*b*d*Sqrt[d + e*x]*Sqrt[1 -
 c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^
2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*(2*c^2*d^2
+ e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqr
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[d + e*x]) - (4*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellip
ticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*c*e*Sqrt[1 - 1
/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
```

```
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} + \frac{(2b)\int \frac{(d+ex)^{5/2}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{5ce}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{5/2}}{x\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{d^3}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{3de^2x}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{e^3x^2}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\right)dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} + \frac{\left(6bd^2\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{\left(2bd^3\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{\left(6bde\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{x}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{\left(2be^2\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{x^2}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} + \frac{\left(6bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\left(6bd^2\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\left(2be\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{-\frac{ex}{c^2}+2dx^2}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{\left(2bd^3\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\left(12bd^2\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$
$$- \frac{12bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{\left(2be\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{-\frac{e}{c^2}+2dx}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(4bd^3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(12bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$
$$+ \frac{\left(12bd^2\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$
$$- \frac{12bd\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{\left(4bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(2b\left(-2d^2-\frac{e^2}{c^2}\right)\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15c\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(4bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e}$$
$$- \frac{12bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{4bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{\left(8bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$
$$+ \frac{\left(4b\left(-2d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e}$$
$$- \frac{28bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{4b(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{4bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.33 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.90

$$\int (d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{15}\left(\frac{4be\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}{c}\right.$$

$$+\frac{6a(d+ex)^{5/2}}{e}+\frac{6b(d+ex)^{5/2}\csc^{-1}(cx)}{e}$$

$$-\frac{4ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-7cd(cd-e)E\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(9c^2d^2-7cde+e^2)\,\text{EllipticF}\right.}{c^3e\sqrt{-\frac{c}{cd+e}}\sqrt{}}$$

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((4*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (6*a*(d + e*x)^(5/2))/e
+ (6*b*(d + e*x)^(5/2)*ArcCsc[c*x])/e - ((4*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d)
 + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(-7*c*d*(c*d - e)*EllipticE[I*ArcSinh[Sq
rt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (9*c^2*d^2 - 7*c*
d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d +
e)/(c*d - e)] - 3*c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d +
 e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^3*e*Sqrt[-(c/(c*d + e))]*Sq
rt[1 - 1/(c^2*x^2)]*x))/15
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. 2(335) = 670.

Time = 9.27 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.15

method	result
derivativedivides	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}+\frac{6d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}}\right.}{5}\right.}{}$
default	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}+\frac{6d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}}\right.}{5}\right.}{}$
parts	$\frac{2a(ex+d)^{\frac{5}{2}}}{5e}+\frac{2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}-\frac{4\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^{\frac{3}{2}}}{15}+\frac{6d^2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-}{cd+e}}}{5}\right.}{}$

[In] int((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)

[Out] 2/e*(1/5*a*(e*x+d)^(5/2)+b*(1/5*arccsc(c*x)*(e*x+d)^(5/2)+2/15/c^3*((c/(c*d-e))^(1/2)*c^2*(e*x+d)^(5/2)+9*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-3*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2-2*(c/(c*d-e))^(1/2)*c^2*d*(e*x+d)^(3/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*e^2*(e*x+d)^(1/2))/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))

Fricas [F(-1)]

Timed out.

$$\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \text{Timed out}$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int (a+b\operatorname{acsc}(cx))\,(d+ex)^{\frac{3}{2}} \, dx$$

```
[In] integrate((e*x+d)**(3/2)*(a+b*acsc(c*x)),x)
[Out] Integral((a + b*acsc(c*x))*(d + e*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int (ex+d)^{\frac{3}{2}} (b\operatorname{arccsc}(cx)+a) \, dx$$

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^(3/2)*(b*arccsc(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) (d+e\,x)^{3/2} \, dx$$

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^(3/2),x)
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^(3/2), x)
```

3.57 $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}} dx$

Optimal result

Integrand size = 21, antiderivative size = 714

$$\begin{aligned}
&\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx \\
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}}+\frac{4bd\sqrt{d+ex}(1-c^2x^2)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} \\
&+\frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{e^4}-\frac{6d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^4} \\
&+\frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^4}-\frac{24bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&+\frac{4b(2c^2d^2-9e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&+\frac{64bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&-\frac{32bd(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+\frac{64bd^4\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35ce^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

```
[Out] 2*d^2*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^4-6/5*d*(e*x+d)^(5/2)*(a+b*arccsc(c
*x))/e^4+2/7*(e*x+d)^(7/2)*(a+b*arccsc(c*x))/e^4-2*d^3*(a+b*arccsc(c*x))*(e
```

```
*x+d)^(1/2)/e^4-4/35*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e/(1-1/c^2/x^2)^(1/2)
+4/21*b*d*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1-1/c^2/x^2)^(1/2)-24/35*b*
d^2*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)
^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))
^(1/2)+4/105*b*(2*c^2*d^2-9*e^2)*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/
2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^3/x/(1-1/c^2/x
^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+64/35*b*d^3*EllipticF(1/2*(-c*x+1)^(1/2
)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)
^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-32/105*b*d*(c*d-e)*(c*d+
e)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+
d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^
(1/2)+64/35*b*d^4*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e
))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^4/x/(1-1/c^2/x^2
)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used

$= \{45, 5355, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551, 847, 858, 956, 1668\}$

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$$
$$= -\frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}+\frac{2d^2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4}$$
$$-\frac{6d(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^4}+\frac{2(d+ex)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^4}$$
$$+\frac{64bd^4\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35ce^4x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$
$$+\frac{64bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$
$$-\frac{24bd^2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+\frac{4b\sqrt{1-c^2x^2}(2c^2d^2-9e^2)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^4e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$-\frac{32bd\sqrt{1-c^2x^2}(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$
$$+\frac{4bd(1-c^2x^2)\sqrt{d+ex}}{21c^3e^2x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{4b(1-c^2x^2)\sqrt{d+ex}}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}}$$

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(35*c^3*e*Sqrt[1 - 1/(c^2*x^2)]) + (4*b*
d*Sqrt[d + e*x]*(1 - c^2*x^2))/(21*c^3*e^2*Sqrt[1 - 1/(c^2*x^2)]*x) - (2*d^
3*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(a + b*Ar
cCsc[c*x]))/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^4) + (2*(d
 + e*x)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^4) - (24*b*d^2*Sqrt[d + e*x]*Sqrt[1
 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*
c^2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*(2*c^
2*d^2 - 9*e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^4*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c
*(d + e*x))/(c*d + e)]) + (64*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 -
c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*c^2
*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d*(c*d - e)*(c*d + e)*S
qrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^4*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
```

```
+ e*x]) + (64*b*d^4*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipti
cPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*c*e^4*Sqrt[1 -
1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 956

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)/(Sqr
t[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
+ 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d
*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
```

```
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}+\frac{b\int\frac{2\sqrt{d+ex}(-16d^3+8d^2ex-6de^2x^2+5e^3x^3)}{35e^4\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{c}$$

$$= -\frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}+\frac{(2b)\int\frac{\sqrt{d+ex}(-16d^3+8d^2ex-6de^2x^2+5e^3x^3)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{35ce^4}$$

$$= -\frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}+\frac{\left(2b\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}(-16d^3+8d^2ex-6de^2x^2+5e^3x^3)}{x\sqrt{1-c^2x^2}}\,dx}{35ce^4\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}+\frac{\left(2b\sqrt{1-c^2x^2}\right)\int\left(\frac{8d^2e\sqrt{d+ex}}{\sqrt{1-c^2x^2}}-\frac{16d^3\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}-\frac{6de^2x\sqrt{d+ex}}{\sqrt{1-c^2x^2}}+\frac{5e^3x^2\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\right)dx}{35ce^4\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{\left(32bd^3\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}\,dx}{35ce^4\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{35ce^3\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{\left(12bd\sqrt{1-c^2x^2}\right)\int\frac{x\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{35ce^2\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{\left(2b\sqrt{1-c^2x^2}\right)\int\frac{x^2\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{7ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}} + \frac{8bd\sqrt{d+ex}(1-c^2x^2)}{35c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{e^4} - \frac{6d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^4} + \frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^4} - \frac{(32bd^4\sqrt{1-c^2x^2})\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{35ce^4\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{(32bd^3\sqrt{1-c^2x^2})\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{35ce^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{(8bd\sqrt{1-c^2x^2})\int\frac{-\frac{e}{2}-\frac{1}{2}c^2dx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{35c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{(2b\sqrt{1-c^2x^2})\int\frac{2d+3ex+c^2dx^2}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{(32bd^2\sqrt{d+ex}\sqrt{1-c^2x^2})\,\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}} + \frac{4bd\sqrt{d+ex}(1-c^2x^2)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{e^4} - \frac{6d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^4} + \frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^4} - \frac{32bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{(32bd^4\sqrt{1-c^2x^2})\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{35ce^4\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{(4b\sqrt{1-c^2x^2})\int\frac{-\frac{7}{2}c^2de^2+\frac{1}{2}c^2e(2c^2d^2-9e^2)x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{105c^5e^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{(4bd^2\sqrt{1-c^2x^2})\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{35ce^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{(4bd(cd-e)(cd+e)\sqrt{1-c^2x^2})\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{35c^3e^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(64bd^3\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}} + \frac{4bd\sqrt{d+ex}(1-c^2x^2)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$- \frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{e^4}$$

$$- \frac{6d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^4} + \frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^4}$$

$$- \frac{32bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$

$$+ \frac{64bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+ \frac{\left(64bd^4\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{35ce^4\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$+ \frac{\left(4bd(cd-e)(cd+e)\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{105c^3e^3\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$- \frac{\left(2b(2c^2d^2-9e^2)\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{105c^3e^3\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$+ \frac{\left(8bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$- \frac{\left(8bd(cd-e)(cd+e)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{35c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$
\begin{aligned}
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}} + \frac{4bd\sqrt{d+ex}(1-c^2x^2)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} \\
&\quad - \frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2d^2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} \\
&\quad - \frac{6d(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^4} + \frac{2(d+ex)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^4} \\
&\quad - \frac{24bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad + \frac{64bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{8bd(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{\left(64bd^4\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{35ce^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{\left(4b(2c^2d^2-9e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{105c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}} \\
&\quad - \frac{\left(8bd(cd-e)(cd+e)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{105c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}} + \frac{4bd\sqrt{d+ex}(1-c^2x^2)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} \\
&\quad - \frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2d^2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} \\
&\quad - \frac{6d(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^4} + \frac{2(d+ex)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^4} \\
&\quad - \frac{24bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad + \frac{4b(2c^2d^2-9e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad + \frac{64bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{32bd(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{64bd^4\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35ce^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 33.90 (sec) , antiderivative size = 873, normalized size of antiderivative = 1.22

$$
\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \frac{ad^4\sqrt{1+\frac{ex}{d}}B_{-\frac{ex}{d}}\left(4,\frac{1}{2}\right)}{e^4\sqrt{d+ex}}
$$

$$
+ \frac{b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(-\frac{4\left(16c^2d^2+9e^2\right)\sqrt{1-\frac{1}{c^2x^2}}}{105e^3}+\frac{32c^3d^3\csc^{-1}(cx)}{35e^4}-\frac{2c^3x^3\csc^{-1}(cx)}{7e}-\frac{4c^2x^2\left(e\sqrt{1-\frac{1}{c^2x^2}}-3cd\csc^{-1}(cx)\right)}{35e^2}+\frac{4cx\left(5cde\sqrt{1-\frac{1}{c^2x^2}}-12c^2 \cdots\right.}{105e^3}\cdots\right.}{\sqrt{d+ex}}\cdots\right.}{\cdots}
$$

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (a*d^4*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 4, 1/2])/(e^4*Sqrt[d + e*x]) + (b
*(-((c*(e + d/x)*x*((-4*(16*c^2*d^2 + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(105*e^
3) + (32*c^3*d^3*ArcCsc[c*x])/(35*e^4) - (2*c^3*x^3*ArcCsc[c*x])/(7*e) - (4
*c^2*x^2*(e*Sqrt[1 - 1/(c^2*x^2)] - 3*c*d*ArcCsc[c*x]))/(35*e^2) + (4*c*x*(
5*c*d*e*Sqrt[1 - 1/(c^2*x^2)] - 12*c^2*d^2*ArcCsc[c*x]))/(105*e^3)))/Sqrt[d
 + e*x]) + (2*Sqrt[e + d/x]*Sqrt[c*x]*((2*(40*c^3*d^3*e + 8*c*d*e^3)*Sqrt[(
c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sq
rt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2))
 + (2*(48*c^4*d^4 + 16*c^2*d^2*e^2 + 9*e^4)*Sqrt[(c*d + c*e*x)/(c*d + e)]*S
qrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-16*c^3*d^3*e
- 9*c*d*e^3)*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqr
t[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sq
rt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/
(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(
c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x
*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[
1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d
/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(105*e^4*Sqrt[d + e*x])))/c^4
```

Maple [A] (verified)

Time = 10.84 (sec) , antiderivative size = 1233, normalized size of antiderivative = 1.73

method	result	size
derivativedivides	Expression too large to display	1233
default	Expression too large to display	1233
parts	Expression too large to display	1251

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^4*(-a*(-1/7*(e*x+d)^(7/2)+3/5*d*(e*x+d)^(5/2)-d^2*(e*x+d)^(3/2)+d^3*(e*
x+d)^(1/2))-b*(-1/7*arccsc(c*x)*(e*x+d)^(7/2)+3/5*arccsc(c*x)*d*(e*x+d)^(5/
2)-arccsc(c*x)*d^2*(e*x+d)^(3/2)+arccsc(c*x)*d^3*(e*x+d)^(1/2)+2/105/c^4*(-
3*(c/(c*d-e))^(1/2)*c^3*(e*x+d)^(7/2)+14*(c/(c*d-e))^(1/2)*c^3*d*(e*x+d)^(5
/2)-19*(c/(c*d-e))^(1/2)*c^3*d^2*(e*x+d)^(3/2)+24*((-c*(e*x+d)+c*d-e)/(c*d-
e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*
d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3+16*((-c*(e*x+d)+c*d-e)/(c*d-e)
)^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-
e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3-48*d^3*((-c*(e*x+d)+c*d-e)/(c*d-
e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c
*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^3+8*(c/(c
*d-e))^(1/2)*c^3*d^3*(e*x+d)^(1/2)-16*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((
-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),
```

```
((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e+16*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((
-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),
((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e+3*(c/(c*d-e))^(1/2)*c*e^2*(e*x+d)^(3/2)-
((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ellip
ticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2+9*((-
c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elliptic
E((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2-8*(c/(c*
d-e))^(1/2)*c*d*e^2*(e*x+d)^(1/2)-9*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c
*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((
c*d-e)/(c*d+e))^(1/2))*e^3+9*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d
)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/
(c*d+e))^(1/2))*e^3)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^
2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^3}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^3*arccsc(c*x) + a*x^3)/sqrt(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{x^3(a+b\operatorname{acsc}(cx))}{\sqrt{d+ex}}\,dx$$

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
[Out] Integral(x**3*(a + b*acsc(c*x))/sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{d + ex}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^3}{\sqrt{ex + d}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^3/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{d + ex}}\,dx = \int \frac{x^3\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d + e\,x}}\,dx$$

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.58 $\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 530

$$\begin{aligned}\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx &= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}-\frac{4d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}\\ &+\frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3}+\frac{4bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}\\ &-\frac{32bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\\ &+\frac{4b(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\\ &-\frac{32bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\end{aligned}$$

```
[Out] -4/3*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*
x))/e^3+2*d^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/15*b*(-c^2*x^2+1)*(e*x+
d)^(1/2)/c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/5*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2
^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/
x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-32/15*b*d^2*EllipticF(1/2*(
-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*
(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/15*b*(c*d-
```

```
e)*(c*d+e)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*
(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^2/x/(1-1/c^2/x^2)^(1/2)/
(e*x+d)^(1/2)-32/15*b*d^3*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(
e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1
/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {45, 5355, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551, 847, 858}

$$\begin{aligned}
&\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx \\
&= \frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} \\
&\quad + \frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3} \\
&\quad - \frac{32bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} \\
&\quad - \frac{32bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} \\
&\quad + \frac{4bd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad + \frac{4b\sqrt{1-c^2x^2}(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} \\
&\quad - \frac{4b(1-c^2x^2)\sqrt{d+ex}}{15c^3ex\sqrt{1-\frac{1}{c^2x^2}}}
\end{aligned}$$

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*
d^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^3 - (4*d*(d + e*x)^(3/2)*(a + b*Ar
cCsc[c*x]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3) + (4*
b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]]
, (2*e)/(c*d + e)])/(5*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(
c*d + e)]) - (32*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elli
pticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2*e^2*Sqrt[1 -
```

```
1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x)
)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*
e)/(c*d + e)])/(15*c^4*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d
^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqr
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sq
rt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{b\int \frac{2\sqrt{d+ex}(8d^2-4dex+3e^2x^2)}{15e^3\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{c}$$

$$= \frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(2b)\int \frac{\sqrt{d+ex}(8d^2-4dex+3e^2x^2)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{15ce^3}$$

$$= \frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex}(8d^2-4dex+3e^2x^2)}{x\sqrt{1-c^2x^2}}\,dx}{15ce^3\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \left(-\frac{4de\sqrt{d+ex}}{\sqrt{1-c^2x^2}} + \frac{8d^2\sqrt{d+ex}}{x\sqrt{1-c^2x^2}} + \frac{3e^2x\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\right)dx}{15ce^3\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}\,dx}{15ce^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(8bd\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{x\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{\left(16bd^3\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15ce^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(4b\sqrt{1-c^2x^2}\right)\int \frac{-\frac{e}{2}-\frac{1}{2}c^2dx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(16bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{16bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{\left(16bd^3\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{15ce^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2bd\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2b(cd-e)(cd+e)\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(32bd^2\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$
\begin{aligned}
&= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} \\
&\quad - \frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{16bd\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad - \frac{32bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^2e^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\
&\quad - \frac{\left(32bd^3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{15ce^3\sqrt{1-\frac{1}{c^2x^2}}\,x} \\
&\quad - \frac{\left(4bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2e^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}} \\
&\quad + \frac{\left(4b(cd-e)(cd+e)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^4e^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}
\end{aligned}
$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}$$
$$- \frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$
$$+ \frac{4bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{32bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{4b(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{\left(32bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \sqrt{1-cx}\right)}{15ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}$$
$$- \frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$
$$+ \frac{4bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{32bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{4b(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{32bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.01 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.48

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = -\frac{ad^3\sqrt{1+\frac{ex}{d}}B_{-\frac{ex}{d}}\left(3,\frac{1}{2}\right)}{e^3\sqrt{d+ex}}$$

$$+\frac{b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(\frac{4cd\sqrt{1-\frac{1}{c^2x^2}}}{5e^2}-\frac{16c^2d^2\csc^{-1}(cx)}{15e^3}-\frac{2c^2x^2\csc^{-1}(cx)}{5e}-\frac{4cx\left(e\sqrt{1-\frac{1}{c^2x^2}}-2cd\csc^{-1}(cx)\right)}{15e^2}\right)}{\sqrt{d+ex}}-\frac{2\sqrt{e+\frac{d}{x}}\sqrt{cx}\left(\frac{2\left(7c^2d^2e+e^3\right)\sqrt{\frac{cd+}{cd\cdot}}}{}\right.}{}\right.}{}$$

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] -((a*d^3*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 3, 1/2])/(e^3*Sqrt[d + e*x])) +
 (b*(-((c*(e + d/x)*x*((4*c*d*Sqrt[1 - 1/(c^2*x^2)])/(5*e^2) - (16*c^2*d^2*
ArcCsc[c*x])/(15*e^3) - (2*c^2*x^2*ArcCsc[c*x])/(5*e) - (4*c*x*(e*Sqrt[1 -
1/(c^2*x^2)] - 2*c*d*ArcCsc[c*x]))/(15*e^2)))/Sqrt[d + e*x]) - (2*Sqrt[e +
d/x]*Sqrt[c*x]*((2*(7*c^2*d^2*e + e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1
 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqr
t[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*c^3*d^3 + 3*c*d*e^2)*
Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c
*x)^(3/2)) - (6*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^
2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d
 + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d
+ c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d +
 c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)
] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, Arc
Sin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[
e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(15*e^3*Sqrt[d + e*x])))/c^3
```

Maple [A] (verified)

Time = 9.16 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{2(ex+d)^{\frac{3}{2}}d}{3}+d^2\sqrt{ex+d}\right)+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{2\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\operatorname{arccsc}(cx)d^2\sqrt{ex+d}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2}{15}\right.}{}$
default	$\frac{2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{2(ex+d)^{\frac{3}{2}}d}{3}+d^2\sqrt{ex+d}\right)+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{2\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\operatorname{arccsc}(cx)d^2\sqrt{ex+d}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2}{15}\right.}{}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{2(ex+d)^{\frac{3}{2}}d}{3}+d^2\sqrt{ex+d}\right)}{e^3}+\frac{2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{2\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\operatorname{arccsc}(cx)d^2\sqrt{ex+d}+\frac{2\sqrt{\frac{c}{cd-e}}}{}\right.}{}$

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/5*(e*x+d)^(5/2)-2/3*(e*x+d)^(3/2)*d+d^2*(e*x+d)^(1/2))+b*(1/5*a
rccsc(c*x)*(e*x+d)^(5/2)-2/3*arccsc(c*x)*(e*x+d)^(3/2)*d+arccsc(c*x)*d^2*(e
*x+d)^(1/2)+2/15/c^3*((c/(c*d-e))^(1/2)*c^2*(e*x+d)^(5/2)+4*d^2*((-c*(e*x+d
)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d
)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2+3*((-c*(e*x+d)+c*d-e
)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)
*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-8*d^2*((-c*(e*x+d)+c*d-
e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/
2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2
-2*(c/(c*d-e))^(1/2)*c^2*d*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/
2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(
1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((
-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),
((c*d-e)/(c*d+e))^(1/2))*c*d*e+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+((-c
*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF
((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(
1/2)*e^2*(e*x+d)^(1/2))/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)
+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \text{Timed out}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{x^2(a+b\,\mathrm{acsc}\,(cx))}{\sqrt{d+ex}}\,dx$$

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
[Out] Integral(x**2*(a + b*acsc(c*x))/sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^2/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex}}\,dx = \int \frac{x^2\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d + e\,x}}\,dx$$

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.59 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 344

$$\begin{aligned}\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = & -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}\\ & -\frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}\\ & +\frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\\ & +\frac{8bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\end{aligned}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^2-2*d*(a+b*arccsc(c*x))*(e*x+d)^(1/2)
/e^2-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*
(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*
d+e))^(1/2)+8/3*b*d*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e)
)^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)
^(1/2)/(e*x+d)^(1/2)+8/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1
/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^2/x
/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {45, 5355, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551}

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2} + \frac{8bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{8bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}$$

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-2*d*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*Ar
cCsc[c*x]))/(3*e^2) - (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin
[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x
*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt
[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3
*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (8*b*d^2*Sqrt[(c*(d + e*x))
/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(3*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} \\
&\quad+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}+\frac{b\int\frac{2(-2d+ex)\sqrt{d+ex}}{3e^2\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{c} \\
&= -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}+\frac{(2b)\int\frac{(-2d+ex)\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{3ce^2} \\
&= -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2} \\
&\quad+\frac{\left(2b\sqrt{1-c^2x^2}\right)\int\frac{(-2d+ex)\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}\,dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$= -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int\left(\frac{e\sqrt{d+ex}}{\sqrt{1-c^2x^2}} - \frac{2d\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}\right)dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} - \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} - \frac{\left(4bd^2\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(4b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$= -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{\left(4bd^2\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(8bd\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{\left(8bd^2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}dx,x,\sqrt{1-cx}\right)}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{\left(8bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}dx,x,\sqrt{1-cx}\right)}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{8bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.11

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(a(-2d+ex)(d+ex)+b(-2d+ex)(d+ex)\csc^{-1}(cx)-\frac{2bde\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{1-c^2x^2}}\right.}{}$$

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
[Out] (2*(a*(-2*d + e*x)*(d + e*x) + b*(-2*d + e*x)*(d + e*x)*ArcCsc[c*x] - (2*b*
d*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[
Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[1 - c^2*x^2] + (2*b*e*Sqrt[1
 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d - e)]*Sqrt[(e - c*e*x)/(c*d + e)]
*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*(d + e*x))/(c*d - e)]], (c*d - e)/(c*d
 + e)] - e*EllipticF[ArcSin[Sqrt[(c*(d + e*x))/(c*d - e)]], (c*d - e)/(c*d
+ e)]))/(c*(-1 + c*x)*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]) - (4*b*c*d^2*Sqrt[1
 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1
 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[1 - c^2*x^2]))/(3*e^2*Sqrt[d + e*x
])
```

Maple [A] (verified)

Time = 8.05 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\operatorname{arccsc}(cx)d\sqrt{ex+d}+\frac{2\left(d\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c+\text{El}\right.}{}\right.}{}$
default	$\frac{-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\operatorname{arccsc}(cx)d\sqrt{ex+d}+\frac{2\left(d\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c+\text{El}\right.}{}\right.}{}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-d\sqrt{ex+d}\right)}{e^2}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-\operatorname{arccsc}(cx)d\sqrt{ex+d}-\frac{2\left(d\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c+\text{Ellipt}\right.}{}\right.}{}$

```
[In] int(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(-1/3*(e*x+d)^(3/2)+d*(e*x+d)^(1/2))-b*(-1/3*(e*x+d)^(3/2)*arccsc
(c*x)+arccsc(c*x)*d*(e*x+d)^(1/2)+2/3/c^2*(d*EllipticF((e*x+d)^(1/2)*(c/(c*
d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^
(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-2*d*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))
^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c-EllipticF((e*x+
d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e+EllipticE((e*x+d)^(1/
2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e)*((-c*(e*x+d)+c*d+e)/(c*d+e
))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+
d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsc(c*x) + a*x)/sqrt(e*x + d), x)
```

Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{x(a+b\operatorname{acsc}(cx))}{\sqrt{d+ex}}\,dx$$

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{\sqrt{ex+d}}\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.60 $\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 212

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] 2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e-4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2
),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c
^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*
2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)
^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

= {5335, 1588, 958, 733, 430, 947, 174, 552, 551}

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e} - \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)
]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e
)])/(c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d*Sqrt[(c*(d + e*x))
/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && !GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !In
tegerQ[p] && !IntegerQ[q] && PosQ[n]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e} + \frac{(2b)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{ce}$$

$$= \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{x\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{c\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$+ \frac{\left(2bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e} + \frac{\left(2bd\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(4b\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e}$$
$$- \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{\left(4bd\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{\left(4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \sqrt{1-cx}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$
= \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
$$

Mathematica [A] (warning: unable to verify)

Time = 5.23 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.15

$$
\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\left(\frac{a(d+ex)}{e} + \frac{b\left((d+ex)\csc^{-1}(cx) + \frac{2cd\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{1-c^2x^2}}\right)}{e} + \frac{2bcx^2\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\dots\right.\right.}{\dots}\right)}{\sqrt{d+ex}}
$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*((a*(d + e*x))/e + (b*((d + e*x)*ArcCsc[c*x] + (2*c*d*Sqrt[1 - 1/(c^2*x^
2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)])/Sqrt[1 - c^2*x^2]))/e + (2*b*c*x^2*Sqrt[1 + c*x]*Sq
rt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)]*(Cos[ArcCsc[c*x]/2] - Sin[ArcCsc[c*x]/2])^3*(Cos[ArcCsc[c*x]/2]
+ Sin[ArcCsc[c*x]/2]))/(Sqrt[1 - c*x]*(-1 + c^2*x^2))))/Sqrt[d + e*x]
```

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{2a\sqrt{ex+d}+2b\left(\sqrt{ex+d}\,\operatorname{arccsc}(cx)+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticPi}\left(\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}}{e}$
default	$\frac{2a\sqrt{ex+d}+2b\left(\sqrt{ex+d}\,\operatorname{arccsc}(cx)+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticPi}\left(\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}}{e}$
parts	$\frac{2a\sqrt{ex+d}}{e}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arccsc}(cx)+\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{Elliptic}\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}}{e}$

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(a*(e*x+d)^(1/2)+b*((e*x+d)^(1/2)*arccsc(c*x)+2/c*((-c*(e*x+d)+c*d-e)/(
c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*(EllipticF((e*x+d)^(1/2)*(
c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))-EllipticPi((e*x+d)^(1/2)*(c/(c*d-
e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)))/((c^2*(e*x+d)
^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/(c/(c*d-e))^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx=\int \frac{b\operatorname{arccsc}(cx)+a}{\sqrt{ex+d}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/sqrt(e*x + d), x)
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx=\int \frac{a+b\operatorname{acsc}(cx)}{\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/sqrt(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{\sqrt{d + ex}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{a + b\csc^{-1}(cx)}{\sqrt{d + ex}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{\sqrt{ex + d}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{\sqrt{d + ex}}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{\sqrt{d + e\,x}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(1/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(1/2), x)
```

3.61 $\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx=\operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.69 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a + b \csc^{-1}(cx)}{x\sqrt{d + ex}}\, dx = \int \frac{a + b \csc^{-1}(cx)}{x\sqrt{d + ex}}\, dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{x\sqrt{ex + d}}dx$$

```
[In] int((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)
[Out] int((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{a + b \csc^{-1}(cx)}{x\sqrt{d + ex}}\, dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{\sqrt{ex + d}x}\, dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e*x^2 + d*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \csc^{-1}(cx)}{x\sqrt{d + ex}}\, dx = \text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/x/(e*x+d)**(1/2),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.19

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex+d}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(d)*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(e*x + d)
*x), x) + a*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)))/sqrt(d)
```

Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex+d}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x + d)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{d+e\,x}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(1/2)), x)
```

3.62 $\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arccsc}\,(cx)}{x^2\sqrt{ex+d}}dx$$

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
[Out] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex+d}x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e*x^3 + d*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 19.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{x^2\sqrt{d+ex}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(1/2),x)
[Out] Integral((a + b*acsc(c*x))/(x**2*sqrt(d + e*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.24

$$\int \frac{a + b \csc^{-1}(cx)}{x^2\sqrt{d + ex}}\,dx = \int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex + d}x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*d^(3/2)*x*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(
e*x + d)*x^2), x) - a*e*x*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)) -
2*sqrt(e*x + d)*a*sqrt(d))/(d^(3/2)*x)
```

Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a + b \csc^{-1}(cx)}{x^2\sqrt{d + ex}}\,dx = \int \frac{b \operatorname{arccsc}(cx) + a}{\sqrt{ex + d}x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x + d)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a + b \csc^{-1}(cx)}{x^2\sqrt{d + ex}}\,dx = \int \frac{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{d + e\,x}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(1/2)), x)
```

3.63 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 551

$$
\begin{aligned}
\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx &= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} \\
&+\frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4} \\
&+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}+\frac{32bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&-\frac{8bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&-\frac{4b(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&-\frac{64bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

```
[Out] -2*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^4+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*x)
)/e^4+2*d^3*(a+b*arccsc(c*x))/e^4/(e*x+d)^(1/2)+6*d^2*(a+b*arccsc(c*x))*(e*
x+d)^(1/2)/e^4-4/15*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1-1/c^2/x^2)^(1
/2)+32/15*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2
))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d
)/(c*d+e))^(1/2)-8*b*d^2*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c
*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c
```

```
^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*(-c*x+1)^(
1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2
+1)^(1/2)/c^4/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-64/5*b*d^3*EllipticPi
(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e)
)^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^4/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {45, 5355, 12, 6853, 6874, 733, 430, 946, 174, 552, 551, 858, 435, 945, 1598}

$$\begin{aligned}\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx &= \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}\\ &-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}\\ &-\frac{64bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}\\ &-\frac{8bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}\\ &+\frac{32bd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\\ &-\frac{4b\sqrt{1-c^2x^2}(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}\\ &-\frac{4b(1-c^2x^2)\sqrt{d+ex}}{15c^3e^2x\sqrt{1-\frac{1}{c^2x^2}}}\end{aligned}$$

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*e^2*Sqrt[1 - 1/(c^2*x^2)]*x) + (
2*d^3*(a + b*ArcCsc[c*x]))/(e^4*Sqrt[d + e*x]) + (6*d^2*Sqrt[d + e*x]*(a +
b*ArcCsc[c*x]))/e^4 - (2*d*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/e^4 + (2*(d
 + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^4) + (32*b*d*Sqrt[d + e*x]*Sqrt[1 -
 c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^
2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (8*b*d^2*Sqr
t[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(c^2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]
) - (4*b*(2*c^2*d^2 + e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*
```

```
EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*e^3*Sqrt
[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (64*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)
]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d
 + e)])/(5*c*e^4*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} \\
&\quad - \frac{2d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} \\
&\quad + \frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^4} + \frac{b\int \frac{2(16d^3+8d^2ex-2de^2x^2+e^3x^3)}{5e^4\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{c} \\
&= \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} \\
&\quad - \frac{2d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} \\
&\quad + \frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^4} + \frac{(2b)\int \frac{16d^3+8d^2ex-2de^2x^2+e^3x^3}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{5ce^4} \\
&= \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} \\
&\quad + \frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^4} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{16d^3+8d^2ex-2de^2x^2+e^3x^3}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{5ce^4\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$= \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^4} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int\left(\frac{8d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}} + \frac{16d^3}{x\sqrt{d+ex}\sqrt{1-c^2x^2}} - \frac{2de^2x}{\sqrt{d+ex}\sqrt{1-c^2x^2}} + \frac{e^3x^2}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{5ce^4\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^4} + \frac{\left(32bd^3\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{5ce^4\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{5ce^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int\frac{x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{5ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int\frac{x^2}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^4} + \frac{\left(32bd^3\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{5ce^4\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{5ce^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4bd^2\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{5ce^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int\frac{ex-2c^2dx^2}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(32bd^2\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$
\begin{aligned}
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} \\
&\quad - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4} \\
&\quad - \frac{32bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{\left(64bd^3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{5ce^4\sqrt{1-\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int\frac{e-2c^2dx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} \\
&\quad + \frac{\left(8bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}} \\
&\quad - \frac{\left(8bd^2\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$
\begin{aligned}
= &-\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} \\
&-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4} \\
&+\frac{8bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&-\frac{8bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&-\frac{\left(4bd\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{15ce^3\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{\left(2b(2c^2d^2+e^2)\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{15c^3e^3\sqrt{1-\frac{1}{c^2x^2}}x} \\
&-\frac{\left(64bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{5ce^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^4} + \frac{8bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{8bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{64bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{(8bd\sqrt{d+ex}\sqrt{1-c^2x^2})\,\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}} - \frac{\left(4b(2c^2d^2+e^2)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^4} + \frac{32bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{8bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4b(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{64bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.38 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.48

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \frac{ad^4\left(1+\frac{ex}{d}\right)^{3/2}B_{-\frac{ex}{d}}\left(4,-\frac{1}{2}\right)}{e^4(d+ex)^{3/2}}$$

$$+ \frac{b\left(-\frac{c^2\left(e+\frac{d}{x}\right)^2x^2\left(\frac{32cd\sqrt{1-\frac{1}{c^2x^2}}}{15e^3}-\frac{32c^2d^2\csc^{-1}(cx)}{5e^4}+\frac{2c^2d^2\csc^{-1}(cx)}{e^3\left(e+\frac{d}{x}\right)}-\frac{2c^2x^2\csc^{-1}(cx)}{5e^2}-\frac{2cx\left(2e\sqrt{1-\frac{1}{c^2x^2}}-9cd\csc^{-1}(cx)\right)}{15e^3}\right)}{(d+ex)^{3/2}} - \frac{2\left(e+\frac{d}{x}\right)^{3/2}(cx)^3 \cdots}{\cdots}\right)}{\cdots}$$

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 4, -1/2])/(e^4*(d + e*x)^(3/2))
 + (b*(-((c^2*(e + d/x)^2*x^2*((32*c*d*Sqrt[1 - 1/(c^2*x^2)])/(15*e^3) - (3
2*c^2*d^2*ArcCsc[c*x])/(5*e^4) + (2*c^2*d^2*ArcCsc[c*x])/(e^3*(e + d/x)) -
(2*c^2*x^2*ArcCsc[c*x])/(5*e^2) - (2*c*x*(2*e*Sqrt[1 - 1/(c^2*x^2)] - 9*c*d
*ArcCsc[c*x]))/(15*e^3)))/(d + e*x)^(3/2)) - (2*(e + d/x)^(3/2)*(c*x)^(3/2)
*((2*(32*c^2*d^2*e + e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*E
llipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*
x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(48*c^3*d^3 + 8*c*d*e^2)*Sqrt[(c*d +
c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) -
 (16*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[
(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/S
qrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt
[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c
*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*
d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*S
qrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
- c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sq
rt[c*x]*(-2 + c^2*x^2))))/(15*e^4*(d + e*x)^(3/2))))/c^4
```

Maple [A] (verified)

Time = 10.08 (sec) , antiderivative size = 880, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+(ex+d)^{\frac{3}{2}}d-3d^2\sqrt{ex+d}-\frac{d^3}{\sqrt{ex+d}}\right)-2b\left(-\frac{\text{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\text{arccsc}(cx)(ex+d)^{\frac{3}{2}}d-3\,\text{arccsc}(cx)d^2\sqrt{}\right.}{}$
default	$\frac{-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+(ex+d)^{\frac{3}{2}}d-3d^2\sqrt{ex+d}-\frac{d^3}{\sqrt{ex+d}}\right)-2b\left(-\frac{\text{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\text{arccsc}(cx)(ex+d)^{\frac{3}{2}}d-3\,\text{arccsc}(cx)d^2\sqrt{}\right.}{}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-(ex+d)^{\frac{3}{2}}d+3d^2\sqrt{ex+d}+\frac{d^3}{\sqrt{ex+d}}\right)}{e^4}+\frac{2b\left(\frac{\text{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\text{arccsc}(cx)(ex+d)^{\frac{3}{2}}d+3\,\text{arccsc}(cx)d^2\sqrt{ex}\right.}{}$

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^4*(-a*(-1/5*(e*x+d)^(5/2)+(e*x+d)^(3/2)*d-3*d^2*(e*x+d)^(1/2)-d^3/(e*x+
d)^(1/2))-b*(-1/5*arccsc(c*x)*(e*x+d)^(5/2)+arccsc(c*x)*(e*x+d)^(3/2)*d-3*a
rccsc(c*x)*d^2*(e*x+d)^(1/2)-arccsc(c*x)*d^3/(e*x+d)^(1/2)-2/15/c^3*((c/(c*
d-e))^(1/2)*c^2*(e*x+d)^(5/2)+24*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-
c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),(
(c*d-e)/(c*d+e))^(1/2))*c^2+8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+
d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)
/(c*d+e))^(1/2))*c^2*d^2-48*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*
x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(
c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2-2*(c/(c*d-e))^(1/2)*c^2*d
*(e*x+d)^(3/2)-8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*
d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/
2))*c*d*e+8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))
^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c
*d*e+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(
1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))
^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*e^2*(e*x+d)^(1/2))/(c
/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)
^(1/2)))
```

Fricas [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^3}{(ex + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^3*arccsc(c*x) + a*x^3)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^2
), x)
```

Sympy [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex)^{3/2}}\,dx = \int \frac{x^3(a + b\,\mathrm{acsc}\,(cx))}{(d + ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
[Out] Integral(x**3*(a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^3}{(ex + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(d+e\,x\right)^{3/2}}\,dx$$

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.64 $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 369

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{20bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{32bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3-2*d^2*(a+b*arccsc(c*x))/e^3/(e*x+d)
^(1/2)-4*d*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/3*b*EllipticE(1/2*(-c*x+1)
^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/
c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+20/3*b*d*EllipticF(
1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(
1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+32/3*b*
d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(
e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d
)^(1/2)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {45, 5355, 12, 6853, 6874, 733, 430, 946, 174, 552, 551, 858, 435}

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{32bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{20bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}$$

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (-2*d^2*(a + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a +
b*ArcCsc[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (4*
b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(3*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*
d + e)]) + (20*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elliptic
F[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e^2*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[d + e*x]) + (32*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1
- c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(
3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} \\
&\quad + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{b\int \frac{2(-8d^2-4dex+e^2x^2)}{3e^3\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{c} \\
&= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} \\
&\quad + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{(2b)\int \frac{-8d^2-4dex+e^2x^2}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{3ce^3}
\end{aligned}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{-8d^2-4dex+e^2x^2}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \left(-\frac{4de}{\sqrt{d+ex}\sqrt{1-c^2x^2}} - \frac{8d^2}{x\sqrt{d+ex}\sqrt{1-c^2x^2}} + \frac{e^2x}{\sqrt{d+ex}\sqrt{1-c^2}}\right.}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(8bd\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2bd\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(16bd\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}$$
$$+ \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$
$$+ \frac{16bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{\left(32bd^2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x}$$
$$- \frac{\left(4b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$
$$+ \frac{\left(4bd\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}$$
$$+ \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{20bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{\left(32bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{20bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{32bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.08 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.03

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = -\frac{ad^3\left(1+\frac{ex}{d}\right)^{3/2}B_{-\frac{ex}{d}}\left(3,-\frac{1}{2}\right)}{e^3(d+ex)^{3/2}} + \frac{b\left(-\frac{c^2\left(e+\frac{d}{x}\right)^2x^2\left(-\frac{4\sqrt{1-\frac{1}{c^2x^2}}}{3e^2}+\frac{16cd\csc^{-1}(cx)}{3e^3}-\frac{2cd\csc^{-1}(cx)}{e^2\left(e+\frac{d}{x}\right)}-\frac{2cx\csc^{-1}(cx)}{3e^2}\right)}{(d+ex)^{3/2}} + \frac{2\left(e+\frac{d}{x}\right)^{3/2}(cx)^{3/2}\left(\frac{10cde\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\ldots\right.}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}(cx)^{3/\ldots}}\ldots\right)}{\ldots}\right)}{\ldots}$$

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] -((a*d^3*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 3, -1/2])/(e^3*(d + e*x)^(3/2
))) + (b*(-((c^2*(e + d/x)^2*x^2*((-4*Sqrt[1 - 1/(c^2*x^2)])/(3*e^2) + (16*
c*d*ArcCsc[c*x])/(3*e^3) - (2*c*d*ArcCsc[c*x])/(e^2*(e + d/x)) - (2*c*x*Arc
Csc[c*x])/(3*e^2)))/(d + e*x)^(3/2)) + (2*(e + d/x)^(3/2)*(c*x)^(3/2)*((10*
c*d*e*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*
(c*x)^(3/2)) + (2*(8*c^2*d^2 + e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 -
c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sq
rt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*e*Cos[2*ArcCsc[c*x]]*((
c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1
- c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x
```

```
*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d
+ e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]
- e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))
/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqr
t[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)
]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(3*e^3
*(d + e*x)^(3/2))))/c^3
```

Maple [A] (verified)

Time = 9.30 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.19

method	result
derivativedivides	$2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-2\operatorname{arccsc}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsc}(cx)d^2}{\sqrt{ex+d}}-\frac{2\left(4d\operatorname{EllipticF}\left(\sqrt{ex+d}\right.\right.}{}\right.$ …
default	$2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-2\operatorname{arccsc}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsc}(cx)d^2}{\sqrt{ex+d}}-\frac{2\left(4d\operatorname{EllipticF}\left(\sqrt{ex+d}\right.\right.}{}\right.$ …
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)}{e^3}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-2\operatorname{arccsc}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsc}(cx)d^2}{\sqrt{ex+d}}-\frac{2\left(4d\operatorname{EllipticF}\left(\sqrt{ex}\right.\right.}{}\right.}{}$ …

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/3*(e*x+d)^(3/2)-2*d*(e*x+d)^(1/2)-d^2/(e*x+d)^(1/2))+b*(1/3*(e*
x+d)^(3/2)*arccsc(c*x)-2*arccsc(c*x)*d*(e*x+d)^(1/2)-arccsc(c*x)*d^2/(e*x+d
)^(1/2)-2/3/c^2*(4*d*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*
d+e))^(1/2))*c+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^
(1/2))*c*d-8*d*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/
(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c-EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/
2),((c*d-e)/(c*d+e))^(1/2))*e+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c
*d-e)/(c*d+e))^(1/2))*e)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*
d-e)/(c*d-e))^(1/2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2
*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \int \frac{x^2(a+b\,\mathrm{acsc}\,(cx))}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.65 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 238

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] 2*d*(a+b*arccsc(c*x))/e^2/(e*x+d)^(1/2)+2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e
^2-4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(
e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d
)^(1/2)-8*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(
1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^2/x/(1-1/c^2/x^2)^(1
/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules

used = {45, 5355, 12, 6853, 6874, 733, 430, 946, 174, 552, 551}

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} - \frac{8bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*d*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*Arc
Csc[c*x]))/e^2 - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellip
ticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c^2*e*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[d + e*x]) - (8*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^
2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*e^
2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{b\int \frac{2(2d+ex)}{e^2\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{c}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{(2b)\int \frac{2d+ex}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{ce^2}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{2d+ex}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{ce^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \left(\frac{e}{\sqrt{d+ex}\sqrt{1-c^2x^2}} + \frac{2d}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{ce^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} + \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{ce^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(4b\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{\left(8bd\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}dx,x,\sqrt{1-cx}\right)}{ce^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{\left(8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}dx,x,\sqrt{1-cx}\right)}{ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.57 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.95

$$\int\frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}dx = \frac{2\left(\frac{a(2d+ex)}{\sqrt{d+ex}}+\frac{b(2d+ex)\csc^{-1}(cx)}{\sqrt{d+ex}}-\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right),\frac{cd+}{cd-}\right.\right.}{c\sqrt{-\frac{c}{cd+e}}\sqrt{1}}\right.}{e^2}$$

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*(2*d + e*x))/Sqrt[d + e*x] + (b*(2*d + e*x)*ArcCsc[c*x])/Sqrt[d + e*
x] - ((2*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*
```

```
(EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d -
e)] - 2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x
]], (c*d + e)/(c*d - e)]))/(c*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^2)]*x)
))/e^2
```

Maple [A] (verified)

Time = 6.61 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.18

method	result
parts	$\frac{2a\left(\sqrt{ex+d}+\frac{d}{\sqrt{ex+d}}\right)}{e^2}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arccsc}(cx)+\frac{\operatorname{arccsc}(cx)d}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d},\cdots\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(\cdots}{c^2e^2\cdots}}}\cdots\right.}{e^2}$
derivativedivides	$\frac{-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\operatorname{arccsc}(cx)-\frac{\operatorname{arccsc}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+\cdots}\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c\cdots}{c\cdots}}}\cdots\right.}{e^2}$
default	$\frac{-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\operatorname{arccsc}(cx)-\frac{\operatorname{arccsc}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+\cdots}\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c\cdots}{c\cdots}}}\cdots\right.}{e^2}$

```
[In] int(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/e^2*((e*x+d)^(1/2)+d/(e*x+d)^(1/2))+2*b/e^2*((e*x+d)^(1/2)*arccsc(c*x)+
arccsc(c*x)*d/(e*x+d)^(1/2)+2/c*(-(c*(e*x+d)-c*d+e)/(c*d-e))^(1/2)*(-(c*(e*
x+d)-c*d-e)/(c*d+e))^(1/2)*(EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d
-e)/(c*d+e))^(1/2))-2*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e
)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)))/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c
^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/(c/(c*d-e))^(1/2))
```

Fricas [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx=\int \frac{(b\operatorname{arccsc}(cx)+a)x}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsc(c*x) + a*x)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^2), x
)
```

Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \int \frac{x(a+b\operatorname{acsc}(cx))}{(d+ex)^{\frac{3}{2}}}\,dx$$

[In] integrate(x*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)

[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x)**(3/2), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)

Giac [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x}{(ex+d)^{\frac{3}{2}}}\,dx$$

[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")

[Out] integrate((b*arccsc(c*x) + a)*x/(e*x + d)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \int \frac{x\left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{(d+e\,x)^{3/2}}\,dx$$

[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)

[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)

3.66 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] -2*(a+b*arccsc(c*x))/e/(e*x+d)^(1/2)+4*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1
/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/
2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5335, 1588, 947, 174, 552, 551}

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCsc[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(c*(d + e*x))/(c*d +
 e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(
c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} - \frac{(2b)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}\,dx}{ce}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} + \frac{\left(4b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} + \frac{\left(4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \sqrt{1-cx}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2, \arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \frac{-2(-1+c^2x^2)\left(a+b\csc^{-1}(cx)\right)+4bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\right.}{e\sqrt{d+ex}\,(-1+c^2x^2)}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(-1 + c^2*x^2)*(a + b*ArcCsc[c*x]) + 4*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c
*x]/Sqrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x]*(-1 + c^2*x^2))
```

Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.81

method	result	siz
derivativedivides	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	21
default	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	21
parts	$-\frac{2a}{\sqrt{ex+d}\,e}+\frac{2b\left(-\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\operatorname{EllipticPi}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	21

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arccsc(c*x)+2/c/((c^2*(e*x+d)^2-2
*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/d/(c/(c*d-e))^(1/2)*((-c*(
e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))
^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{b\operatorname{arccsc}(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{a+b\operatorname{acsc}(cx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{b\operatorname{arccsc}(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \int \frac{a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{3/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(3/2), x)
```

3.67 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx=\operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.78 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\ \operatorname{arccsc}\left(cx\right)}{x\left(ex+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
[Out] int((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx=\int \frac{b\operatorname{arccsc}\left(cx\right)+a}{\left(ex+d\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{x(d + ex)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/x/(e*x+d)**(3/2),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.62

$$\int \frac{a + b\csc^{-1}(cx)}{x(d + ex)^{3/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{(ex + d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] ((b*d^(3/2)*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e*x^2 + d*x
)*sqrt(e*x + d)), x) + a*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)))*sq
rt(e*x + d) + 2*a*sqrt(d))/(sqrt(e*x + d)*d^(3/2))
```

Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a + b\csc^{-1}(cx)}{x(d + ex)^{3/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{(ex + d)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(d+e\,x\right)^{3/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(3/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(3/2)), x)
```

3.68 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \mathrm{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a + b\csc^{-1}(cx)}{x^2(d + ex)^{3/2}}\, dx = \int \frac{a + b\csc^{-1}(cx)}{x^2(d + ex)^{3/2}}\, dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.83 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a + b\ \mathrm{arccsc}\,(cx)}{x^2\,(ex + d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
[Out] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{a + b\csc^{-1}(cx)}{x^2(d + ex)^{3/2}}\, dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{(ex + d)^{\frac{3}{2}}x^2}\, dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2),
 x)
```

Sympy [N/A]

Not integrable

Time = 84.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{x^2\,(d+ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(3/2),x)
[Out] Integral((a + b*acsc(c*x))/(x**2*(d + e*x)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 145, normalized size of antiderivative = 6.90

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="maxima")
[Out] 1/2*(2*(b*d^2*e*x^2 + b*d^3*x)*sqrt(d)*integrate(arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1))/((e*x^3 + d*x^2)*sqrt(e*x + d)), x) - 2*(3*a*e*x + a*d)*sqrt(
e*x + d)*sqrt(d) - 3*(a*e^2*x^2 + a*d*e*x)*log(e*x/(e*x + 2*sqrt(e*x + d)*s
qrt(d) + 2*d)))/((d^2*e*x^2 + d^3*x)*sqrt(d))
```

Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(3/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(3/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(3/2)), x)
```

3.69 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 602

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = -\frac{4bd^2(1-c^2x^2)}{3ce^2(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$
$$+\frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}}-\frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}$$
$$+\frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^4}+\frac{8bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e^3(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$-\frac{4b(2c^2d^2-e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^3(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+\frac{32bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^3\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$
$$+\frac{64bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^4\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] 2/3*d^3*(a+b*arccsc(c*x))/e^4/(e*x+d)^(3/2)+2/3*(e*x+d)^(3/2)*(a+b*arccsc(c
*x))/e^4-6*d^2*(a+b*arccsc(c*x))/e^4/(e*x+d)^(1/2)-4/3*b*d^2*(-c^2*x^2+1)/c
/e^2/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-6*d*(a+b*arccsc(c*x)
)*(e*x+d)^(1/2)/e^4+8/3*b*d^2*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*
(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/e^3/(c^2*d^2-e^2)/x/(1-
1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*(2*c^2*d^2-e^2)*EllipticE(
1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x
```

```
^2+1)^(1/2)/c^2/e^3/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))
^(1/2)+32/3*b*d*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1
/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(
1/2)/(e*x+d)^(1/2)+64/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/
2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^4/x/
(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {45, 5355, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551, 849, 858, 430, 1665}

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx=\frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}}-\frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}+\frac{64bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}+\frac{8bd^2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e^3x\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{4b\sqrt{1-c^2x^2}(2c^2d^2-e^2)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{32bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{4bd^2(1-c^2x^2)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}$$

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (-4*b*d^2*(1 - c^2*x^2))/(3*c*e^2*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*S
qrt[d + e*x]) + (2*d^3*(a + b*ArcCsc[c*x]))/(3*e^4*(d + e*x)^(3/2)) - (6*d^
2*(a + b*ArcCsc[c*x]))/(e^4*Sqrt[d + e*x]) - (6*d*Sqrt[d + e*x]*(a + b*ArcC
sc[c*x]))/e^4 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^4) + (8*b*d^2*
Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)])/(3*e^3*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d +
 e*x))/(c*d + e)]) - (4*b*(2*c^2*d^2 - e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]
```

```
*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e^3*(c^2
*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (32*b*
d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
 c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
 + e*x]) + (64*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipt
icPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*e^4*Sqrt[1 -
1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} \\
&\quad + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4} + \frac{b\int \frac{2(-16d^3-24d^2ex-6de^2x^2+e^3x^3)}{3e^4\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{c} \\
&= \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} \\
&\quad + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4} + \frac{(2b)\int \frac{-16d^3-24d^2ex-6de^2x^2+e^3x^3}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{3ce^4} \\
&= \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} \\
&\quad + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{-16d^3-24d^2ex-6de^2x^2+e^3x^3}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3ce^4\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} \\
&\quad - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4} \\
&\quad + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \left(-\frac{24d^2e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}} - \frac{16d^3}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}} - \frac{6de^2x}{(d+ex)^{3/2}\sqrt{1-c^2x^2}} + \frac{e^3x^2}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\right)dx}{3ce^4\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} \\
&\quad + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4} - \frac{\left(32bd^3\sqrt{1-c^2x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3ce^4\sqrt{1-\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{ce^3\sqrt{1-\frac{1}{c^2x^2}}x} \\
&\quad - \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int \frac{x}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{x^2}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{68bd^2(1-c^2x^2)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} \\
&- \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4} \\
&- \frac{\left(32bd^3\sqrt{1-c^2x^2}\right)\int\left(-\frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}}+\frac{1}{dx\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{3ce^4\sqrt{1-\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(32bcd^2\sqrt{1-c^2x^2}\right)\int\frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x} \\
&- \frac{\left(8bd\sqrt{1-c^2x^2}\right)\int\frac{-\frac{e}{2}-\frac{1}{2}c^2dx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{1-c^2x^2}\right)\int\frac{\frac{d}{2}+\frac{1}{2}\left(\frac{2c^2d^2}{e}-e\right)x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{68bd^2(1-c^2x^2)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} \\
&- \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} \\
&+ \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4} - \frac{\left(32bd^2\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3ce^4\sqrt{1-\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(32bd^2\sqrt{1-c^2x^2}\right)\int\frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(4bcd^2\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(16bcd^2\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(4bd\left(1-\frac{c^2d^2}{e^2}\right)\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(2b\left(-1+\frac{2c^2d^2}{e^2}\right)\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x} \\
&- \frac{\left(4bd(cd-e)(cd+e)\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{ce^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$= -\frac{4bd^2(1-c^2x^2)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}}$$

$$- \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}$$

$$+ \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4} - \frac{\left(32bd^2\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{3ce^4\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$- \frac{\left(64bcd^2\sqrt{1-c^2x^2}\right)\int\frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$- \frac{\left(8bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$+ \frac{\left(32bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$- \frac{\left(4b\left(-1+\frac{2c^2d^2}{e^2}\right)\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2e\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$- \frac{\left(8bd\left(1-\frac{c^2d^2}{e^2}\right)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2e\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+ \frac{\left(8bd(cd-e)(cd+e)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{c^2e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$
\begin{aligned}
&= -\frac{4bd^2(1-c^2x^2)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} \\
&- \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4} \\
&+ \frac{24bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&- \frac{4b(2c^2d^2-e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&+ \frac{32bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{\left(64bd^2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{3ce^4\sqrt{1-\frac{1}{c^2x^2}}x} \\
&+ \frac{\left(32bcd^2\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{3e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4bd^2(1-c^2x^2)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} \\
&- \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4} \\
&+ \frac{24bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&- \frac{4b(2c^2d^2-e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&+ \frac{32bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+ \frac{\left(64bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{3ce^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{\left(64bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4bd^2(1-c^2x^2)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} \\
&\quad - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4} \\
&\quad + \frac{8bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad - \frac{4b(2c^2d^2-e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad + \frac{32bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{64bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.21 (sec) , antiderivative size = 887, normalized size of antiderivative = 1.47

$$
\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \frac{ad^4\left(1+\frac{ex}{d}\right)^{5/2}B_{-\frac{ex}{d}}\left(4,-\frac{3}{2}\right)}{e^4(d+ex)^{5/2}}
$$

$$
+ \frac{b\left(-\frac{c^3\left(e+\frac{d}{x}\right)^3x^3\left(-\frac{4\sqrt{1-\frac{1}{c^2x^2}}}{3e\left(-c^2d^2+e^2\right)}+\frac{32cd\csc^{-1}(cx)}{3e^4}-\frac{2cd\csc^{-1}(cx)}{3e^2\left(e+\frac{d}{x}\right)^2}-\frac{2cx\csc^{-1}(cx)}{3e^3}-\frac{2\left(-2c^2d^2e\sqrt{1-\frac{1}{c^2x^2}}-7c^3d^3\csc^{-1}(cx)+7cde^2\csc^{-1}(cx)\right)}{3e^3\left(-c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)}\right)}{(d+ex)^{5/2}} + \cdots\right.}{\cdots}
$$

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 4, -3/2])/(e^4*(d + e*x)^(5/2))
 + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 - 1/(c^2*x^2)])/(3*e*(-(c^2*d^2)
+ e^2)) + (32*c*d*ArcCsc[c*x])/(3*e^4) - (2*c*d*ArcCsc[c*x])/(3*e^2*(e + d/
```

```
x)^2) - (2*c*x*ArcCsc[c*x])/(3*e^3) - (2*(-2*c^2*d^2*e*Sqrt[1 - 1/(c^2*x^2)
] - 7*c^3*d^3*ArcCsc[c*x] + 7*c*d*e^2*ArcCsc[c*x]))/(3*e^3*(-(c^2*d^2) + e^
2)*(e + d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*((2*(8*c^
3*d^3*e - 8*c*d*e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipt
icF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]
*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(16*c^4*d^4 - 16*c^2*d^2*e^2 - e^4)*Sqrt[(
c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x
]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3
/2)) + (2*e^3*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sq
rt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x
]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*S
qrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)
/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/
(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*
x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]
*Sqrt[c*x]*(-2 + c^2*x^2))))/(3*(c*d - e)*e^4*(c*d + e)*(d + e*x)^(5/2))))/
c^4
```

Maple [A] (verified)

Time = 9.56 (sec) , antiderivative size = 1067, normalized size of antiderivative = 1.77

method	result	size
derivativedivides	Expression too large to display	1067
default	Expression too large to display	1067
parts	Expression too large to display	1080

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^4*(-a*(-1/3*(e*x+d)^(3/2)+3*d*(e*x+d)^(1/2)+3*d^2/(e*x+d)^(1/2)-1/3*d^3
/(e*x+d)^(3/2))-b*(-1/3*(e*x+d)^(3/2)*arccsc(c*x)+3*arccsc(c*x)*d*(e*x+d)^(
1/2)+3*arccsc(c*x)*d^2/(e*x+d)^(1/2)-1/3*arccsc(c*x)*d^3/(e*x+d)^(3/2)+2/3/
c^2*(8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2
)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^
3*(e*x+d)^(1/2)-16*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(
c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(
c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^
3*d^2*(e*x+d)^2+2*(c/(c*d-e))^(1/2)*c^3*d^3*(e*x+d)-7*((-c*(e*x+d)+c*d-e)/(
c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c
/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2*(e*x+d)^(1/2)-((-c*(e*x+d)
+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)
^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2*(e*x+d)^(1/2)+16*
((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ellip
ticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c
```

```
*d-e))^(1/2))*c*d*e^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^3*d^4+((-c*(e*x+d)+
c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^
(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^3*(e*x+d)^(1/2)-((-c*(e*
x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^3*(e*x+d)^(1/2)+(c/
(c*d-e))^(1/2)*c*d^2*e^2)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/x
/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^3}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsc(c*x) + a*x^3)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 +
 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^3}{(ex + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex)^{5/2}}\,dx = \int \frac{x^3\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(d + e\,x)^{5/2}}\,dx$$

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.70 $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 440

$$\begin{aligned}\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx &= \frac{4bd(1-c^2x^2)}{3ce\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\ &- \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} \\ &- \frac{4bd\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e^2\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} \\ &- \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\ &- \frac{32bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}\end{aligned}$$

```
[Out] -2/3*d^2*(a+b*arccsc(c*x))/e^3/(e*x+d)^(3/2)+4*d*(a+b*arccsc(c*x))/e^3/(e*x
+d)^(1/2)+4/3*b*d*(-c^2*x^2+1)/c/e/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x
+d)^(1/2)+2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/3*b*d*EllipticE(1/2*(-c*x
+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/
2)/e^2/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4*b*El
lipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c
*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
-32/3*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2)
)*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1/c^2/x^2)^(1/2)/
(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.810, Rules used = {45, 5355, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551, 849, 858, 430}

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = -\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}$$
$$+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}-\frac{4bd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e^2x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$-\frac{32bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$
$$-\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$
$$+\frac{4bd(1-c^2x^2)}{3cex\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{d+ex}}$$

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*d*(1 - c^2*x^2))/(3*c*e*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
 + e*x]) - (2*d^2*(a + b*ArcCsc[c*x]))/(3*e^3*(d + e*x)^(3/2)) + (4*d*(a +
b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))
/e^3 - (4*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x
]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e^2*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*
x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[
1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c^
2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d*Sqrt[(c*(d + e*x))/(
c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)])/(3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
```

```
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} \\
&\quad+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}+\frac{b\int\frac{2(8d^2+12dex+3e^2x^2)}{3e^3\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{c} \\
&= -\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} \\
&\quad+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}+\frac{(2b)\int\frac{8d^2+12dex+3e^2x^2}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{3ce^3} \\
&= -\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} \\
&\quad+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}+\frac{\left(2b\sqrt{1-c^2x^2}\right)\int\frac{8d^2+12dex+3e^2x^2}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= -\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} \\
&\quad+\frac{\left(2b\sqrt{1-c^2x^2}\right)\int\left(\frac{12de}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}+\frac{8d^2}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}+\frac{3e^2x}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\right)dx}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(8bd\sqrt{1-c^2x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{x}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{12bd(1-c^2x^2)}{ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{\left(16bd^2\sqrt{1-c^2x^2}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}} + \frac{1}{dx\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(16bcd\sqrt{1-c^2x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{1-c^2x^2}\right)\int \frac{-\frac{e}{2}-\frac{1}{2}c^2dx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{12bd(1-c^2x^2)}{ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{\left(16bd\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(16bd\sqrt{1-c^2x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2bcd\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(8bcd\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b(cd-e)(cd+e)\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{4bd(1-c^2x^2)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{\left(16bd\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(32bcd\sqrt{1-c^2x^2}\right)\int\frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}} - \frac{\left(16bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}} - \frac{\left(4b(cd-e)(cd+e)\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{c^2e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{4bd(1-c^2x^2)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{12bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{\left(32bd\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(16bcd\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{3e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{4bd(1-c^2x^2)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}$$
$$+ \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{12bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{\left(32bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}dx,x,\sqrt{1-cx}\right)}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{\left(32bd\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$= \frac{4bd(1-c^2x^2)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}$$
$$+ \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} - \frac{4bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{32bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.07 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.95

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = -\frac{ad^3\left(1+\frac{ex}{d}\right)^{5/2}B_{-\frac{ex}{d}}\left(3,-\frac{3}{2}\right)}{e^3(d+ex)^{5/2}} + \frac{b\left(-\frac{c^3\left(e+\frac{d}{x}\right)^3x^3\left(\frac{4cd\sqrt{1-\frac{1}{c^2x^2}}}{3e^2\left(-c^2d^2+e^2\right)}-\frac{16\csc^{-1}(cx)}{3e^3}+\frac{2\csc^{-1}(cx)}{3e\left(e+\frac{d}{x}\right)^2}+\frac{4\left(-cde\sqrt{1-\frac{1}{c^2x^2}}-2c^2d^2\csc^{-1}(cx)+2e^2\csc^{-1}(cx)\right)}{3e^2\left(-c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)}\right)}{(d+ex)^{5/2}} - \frac{2\left(e+\frac{d}{x}\right)^{5/2}(cx)^{5/2}\left(\frac{2(3 \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] -((a*d^3*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 3, -3/2])/(e^3*(d + e*x)^(5/2
))) + (b*(-((c^3*(e + d/x)^3*x^3*((4*c*d*Sqrt[1 - 1/(c^2*x^2)])/(3*e^2*(-(c
^2*d^2) + e^2)) - (16*ArcCsc[c*x])/(3*e^3) + (2*ArcCsc[c*x])/(3*e*(e + d/x)
^2) + (4*(-(c*d*e*Sqrt[1 - 1/(c^2*x^2)]) - 2*c^2*d^2*ArcCsc[c*x] + 2*e^2*Ar
cCsc[c*x]))/(3*e^2*(-(c^2*d^2) + e^2)*(e + d/x))))/(d + e*x)^(5/2)) - (2*(e
 + d/x)^(5/2)*(c*x)^(5/2)*((2*(3*c^2*d^2*e - 3*e^3)*Sqrt[(c*d + c*e*x)/(c*d
 + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*
d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*c^3*d^3
- 9*c*d*e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2,
ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqr
t[e + d/x]*(c*x)^(3/2)) + (2*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 +
c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elliptic
F[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e
- c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcS
in[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSi
n[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))
/(-(c*d) + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elli
pticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^
2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(3*(c*d - e)*e^3*(c*d + e
)*(d + e*x)^(5/2))))/c^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. 2(401) = 802.

Time = 9.03 (sec) , antiderivative size = 1026, normalized size of antiderivative = 2.33

method	result	size
derivativedivides	Expression too large to display	1026
default	Expression too large to display	1026
parts	Expression too large to display	1041

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*((e*x+d)^(1/2)-1/3*d^2/(e*x+d)^(3/2)+2*d/(e*x+d)^(1/2))+b*((e*x+d)
^(1/2)*arccsc(c*x)-1/3*arccsc(c*x)*d^2/(e*x+d)^(3/2)+2*arccsc(c*x)*d/(e*x+d
)^(1/2)+2/3/c*(4*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*
d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/
2))*c^2*d^2*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c
*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*
d+e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-
c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),
1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-(c
/(c*d-e))^(1/2)*c^2*d*(e*x+d)^2+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*
x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-
e)/(c*d+e))^(1/2))*c*d*e*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*(
(-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2)
,((c*d-e)/(c*d+e))^(1/2))*c*d*e*(e*x+d)^(1/2)+2*(c/(c*d-e))^(1/2)*c^2*d^2*(
e*x+d)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1
/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2*
(e*x+d)^(1/2)+8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d
+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d
+e))^(1/2)/(c/(c*d-e))^(1/2))*e^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2*d^3+(
c/(c*d-e))^(1/2)*d*e^2)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/x/(
(c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx=\int \frac{(b\operatorname{arccsc}(cx)+a)x^2}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 +
 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{(d + ex)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{(d + ex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{(d + ex)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^2}{(ex + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{(d + ex)^{5/2}}\,dx = \int \frac{x^2\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{(d + e\,x)}^{5/2}}\,dx$$

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.71 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 314

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = -\frac{4b(1-c^2x^2)}{3c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}+\frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}}$$

$$-\frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}}$$

$$+\frac{8b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] 2/3*d*(a+b*arccsc(c*x))/e^2/(e*x+d)^(3/2)-2*(a+b*arccsc(c*x))/e^2/(e*x+d)^(
1/2)-4/3*b*(-c^2*x^2+1)/c/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
+4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x
+d)^(1/2)*(-c^2*x^2+1)^(1/2)/e/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+
d)/(c*d+e))^(1/2)+8/3*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/
(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^2/x/(1-1/c
^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {45, 5355, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551}

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = -\frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} + \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3ex\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{8b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} - \frac{4b(1-c^2x^2)}{3cx\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{d+ex}}$$

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (-4*b*(1 - c^2*x^2))/(3*c*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d +
e*x]) + (2*d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x)^(3/2)) - (2*(a + b*ArcCs
c[c*x]))/(e^2*Sqrt[d + e*x]) + (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*Ellipti
cE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e*(c^2*d^2 - e^2)*Sq
rt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*Sqrt[(c*(d + e*
x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]
], (2*e)/(c*d + e)])/(3*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
```

```
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{b\int \frac{2(-2d-3ex)}{3e^2\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{c} \\
&= \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{(2b)\int \frac{-2d-3ex}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{3ce^2}
\end{aligned}
$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int\frac{-2d-3ex}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int\left(-\frac{3e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}} - \frac{2d}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\right)dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} - \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int\frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{1-c^2x^2}\right)\int\frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b(1-c^2x^2)}{c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} - \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int\left(-\frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}} + \frac{1}{dx\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4bc\sqrt{1-c^2x^2}\right)\int\frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{e\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b(1-c^2x^2)}{c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} - \frac{\left(4b\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{1-c^2x^2}\right)\int\frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2bc\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{e\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b(1-c^2x^2)}{3c(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} - \frac{\left(4b\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(8bc\sqrt{1-c^2x^2}\right)\int\frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx}{3e(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{e(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$= -\frac{4b(1-c^2x^2)}{3c(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{\left(8b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4bc\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}\,dx}{3e(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b(1-c^2x^2)}{3c(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{\left(8b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx, x, \sqrt{1-cx}\right)}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{\left(8b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2cex^2}{-c^2d-ce}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3e(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{-\frac{c^2(d+ex)}{-c^2d-ce}}}$$

$$= -\frac{4b(1-c^2x^2)}{3c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}}$$
$$- \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{8b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.46 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.10

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \frac{4bc\sqrt{1-\frac{1}{c^2x^2}}x}{3\left(c^2d^2-e^2\right)\sqrt{d+ex}} - \frac{2a(2d+3ex)}{3e^2(d+ex)^{3/2}} - \frac{2b(2d+3ex)\csc^{-1}(cx)}{3e^2(d+ex)^{3/2}}$$
$$+ \frac{4ib\sqrt{-\frac{c}{cd+e}}\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(cdE\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right) - cd\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{\right.\right.\right.}{3c^2de^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(3*(c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*a*(2
*d + 3*e*x))/(3*e^2*(d + e*x)^(3/2)) - (2*b*(2*d + 3*e*x)*ArcCsc[c*x])/(3*e
^2*(d + e*x)^(3/2)) + (((4*I)/3)*b*Sqrt[-(c/(c*d + e))]*Sqrt[(e*(1 + c*x))/
(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(c*d*EllipticE[I*ArcSinh[Sqrt[-(c
/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - c*d*EllipticF[I*ArcSinh
[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + 2*(c*d + e)*El
lipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d +
 e)/(c*d - e)]))/(c^2*d*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(284) = 568$.

Time = 9.51 (sec) , antiderivative size = 900, normalized size of antiderivative = 2.87

method	result
derivativedivides	$\frac{-2a\left(\frac{1}{\sqrt{ex+d}}-\frac{d}{3(ex+d)^{\frac{3}{2}}}\right)-2b\left(\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}-\frac{\operatorname{arccsc}(cx)d}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^{2}d(ex+d)^{2}-\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\ \mathrm{Ellipt}\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-2a\left(\frac{1}{\sqrt{ex+d}}-\frac{d}{3(ex+d)^{\frac{3}{2}}}\right)-2b\left(\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}-\frac{\operatorname{arccsc}(cx)d}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^{2}d(ex+d)^{2}-\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\ \mathrm{Ellipt}\cdots\right.}{\cdots}\right.}{\cdots}$
parts	$\frac{2a\left(-\frac{1}{\sqrt{ex+d}}+\frac{d}{3(ex+d)^{\frac{3}{2}}}\right)}{e^{2}}+\frac{2b\left(-\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}+\frac{\operatorname{arccsc}(cx)d}{3(ex+d)^{\frac{3}{2}}}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^{2}d(ex+d)^{2}}{3}-\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(1/(e*x+d)^(1/2)-1/3*d/(e*x+d)^(3/2))-b*(1/(e*x+d)^(1/2)*arccsc(c
*x)-1/3*arccsc(c*x)*d/(e*x+d)^(3/2)-2/3/c*((c/(c*d-e))^(1/2)*c^2*d*(e*x+d)^
2-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ell
ipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*(e*
x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^
(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^
2*d^2*(e*x+d)^(1/2)+2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e
)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(
c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-2*(c/(c*d-e))^(1/
2)*c^2*d^2*(e*x+d)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(
c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(
1/2))*c*d*e*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c
*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*
d+e))^(1/2))*c*d*e*(e*x+d)^(1/2)+(c/(c*d-e))^(1/2)*c^2*d^3-2*((-c*(e*x+d)+c
*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^
(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*
e^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*d*e^2)/d/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x
+d)^(1/2)/(c*d+e)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^
2)^(1/2)))
```

Fricas [F]

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex)^{5/2}} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)x}{(ex + d)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
[Out] integral((b*x*arccsc(c*x) + a*x)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d
^2*e*x + d^3), x)
```

Sympy [F]

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex)^{5/2}} \, dx = \int \frac{x(a + b \operatorname{acsc}(cx))}{(d + ex)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex)^{5/2}} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)x}{(ex + d)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(d+e\,x\right)^{5/2}}\,dx$$

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.72 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 298

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \frac{4be(1-c^2x^2)}{3cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3cde\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] -2/3*(a+b*arccsc(c*x))/e/(e*x+d)^(3/2)+4/3*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e^
2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2
^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d
^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/3*b*EllipticPi(1/
2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(
1/2)*(-c^2*x^2+1)^(1/2)/c/d/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5335, 1588, 972, 759, 21, 733, 435, 947, 174, 552, 551}

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3dx\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3cdex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{4be(1-c^2x^2)}{3cdx\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{d+ex}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*e*(1 - c^2*x^2))/(3*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
 + e*x]) - (2*(a + b*ArcCsc[c*x]))/(3*e*(d + e*x)^(3/2)) - (4*b*Sqrt[d + e*
x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
e)])/(3*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d +
 e)]) + (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2,
ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*d*e*Sqrt[1 - 1/(c^2*x
^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
```

```
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{(2b)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}\,dx}{3ce}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{1}{dx\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\right)dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3cd\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3cde\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{4be(1-c^2x^2)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{\left(4b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3cd\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{3cde\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{4be(1-c^2x^2)}{3cd\,(c^2d^2-e^2)\,\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3cd\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx,x,\sqrt{1-cx}\right)}{3cde\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{4be(1-c^2x^2)}{3cd\,(c^2d^2-e^2)\,\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{\left(4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{3cde\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{\left(4b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{3c^2d\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$

$$= \frac{4be(1-c^2x^2)}{3cd\,(c^2d^2-e^2)\,\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\,(c^2d^2-e^2)\,\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3cde\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 608 vs. 2(298) = 596.

Time = 21.29 (sec) , antiderivative size = 608, normalized size of antiderivative = 2.04

$$\int\frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \frac{2\left(-\frac{a}{e}+\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(d+ex)^2}{c^2d^3-de^2}-\frac{bex^2\csc^{-1}(cx)}{d^2}-\frac{b(d+ex)^2\csc^{-1}(cx)}{d^2e}+\frac{2bx(d+ex)\left(-cde\sqrt{1-\frac{1}{c^2x^2}}+(\right.}{c^2d^4-d^2e}\right.}{}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (2*(-(a/e) + (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*(d + e*x)^2)/(c^2*d^3 - d*e^2) -
(b*e*x^2*ArcCsc[c*x])/d^2 - (b*(d + e*x)^2*ArcCsc[c*x])/(d^2*e) + (2*b*x*(d
 + e*x)*(-(c*d*e*Sqrt[1 - 1/(c^2*x^2)]) + (c^2*d^2 - e^2)*ArcCsc[c*x]))/(c^
2*d^4 - d^2*e^2) + (2*b*d*((c*(d + e*x))/(c*d + e))^(3/2)*Sqrt[1 - c^2*x^2]
*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/((c*d - e)*
e*Sqrt[1 - 1/(c^2*x^2)]*x) - (2*b*c*(d + e*x)*Cos[2*ArcCsc[c*x]]*((d + e*x)
*(-1 + c^2*x^2) + c*d*x*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ell
ipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (x*(1 + c*x)*Sqrt[
(c*(d + e*x))/(c*d - e)]*Sqrt[(e - c*e*x)/(c*d + e)]*((c*d + e)*EllipticE[A
rcSin[Sqrt[(c*(d + e*x))/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[Ar
cSin[Sqrt[(c*(d + e*x))/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*
x))/(-(c*d) + e)] + e*x*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ell
ipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(d*(c*d - e)*(
c*d + e)*Sqrt[1 - 1/(c^2*x^2)]*(-2 + c^2*x^2))))/(3*(d + e*x)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(270) = 540$.

Time = 7.16 (sec) , antiderivative size = 875, normalized size of antiderivative = 2.94

method	result
derivativedivides	$\frac{-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arccsc}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2\sqrt{ex+d}-2\sqrt{\frac{-c(}{}}\cdots}{3}\cdots\right.}{\cdots}$
default	$\frac{-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arccsc}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2\sqrt{ex+d}-2\sqrt{\frac{-c(}{}}\cdots}{3}\cdots\right.}{\cdots}$
parts	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}e}+\frac{2b\left(-\frac{\operatorname{arccsc}(cx)}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{c}{cd-e}}c^2d(ex+d)^2-\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arccsc(c*x)+2/3/c*(((-c*(e*
x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)
-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elli
pticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*(e*x
```

```
+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(
1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(
1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2*d*(e*x+
d)^2+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*
EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e*(e
*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))
^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c
*d*e*(e*x+d)^(1/2)+2*(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)-((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*e^2*(e
*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2*d^3+(c/(c*d-e))^(1/2)*d*e^2)/(c*d-e)/(c/(
c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/d^2/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c
^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e
*x + d^3), x)
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{(d+ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(5/2),x)
[Out] Integral((a + b*acsc(c*x))/(d + e*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{a + b \csc^{-1}(cx)}{(d + ex)^{5/2}} \, dx = \int \frac{b \operatorname{arccsc}(cx) + a}{(ex + d)^{\frac{5}{2}}} \, dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \csc^{-1}(cx)}{(d + ex)^{5/2}} \, dx = \int \frac{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^{5/2}} \, dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(5/2),x)
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(5/2), x)
```

3.73 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \mathrm{Int}\left(\frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 30.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x\left(ex+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
[Out] int((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.43

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\left(ex+d\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e
*x^2 + d^3*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/x/(e*x+d)**(5/2),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 177, normalized size of antiderivative = 8.43

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="maxima")
[Out] 1/3*(3*(b*d^2*e^2*x^2 + 2*b*d^3*e*x + b*d^4)*sqrt(d)*integrate(arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^3 + 2*d*e*x^2 + d^2*x)*sqrt(e*x + d)),
x) + 2*(3*a*e*x + 4*a*d)*sqrt(e*x + d)*sqrt(d) + 3*(a*e^2*x^2 + 2*a*d*e*x +
 a*d^2)*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)))/((d^2*e^2*x^2 + 2*d
^3*e*x + d^4)*sqrt(d))
```

Giac [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(5/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a + b\csc^{-1}(cx)}{x(d + ex)^{5/2}}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,{\left(d + e\,x\right)}^{5/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(5/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(5/2)), x)
```

3.74 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 27.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.87 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x^{2}\left(ex+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
[Out] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.52

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\left(ex+d\right)^{\frac{5}{2}}x^{2}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e
*x^3 + d^3*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 179, normalized size of antiderivative = 8.52

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*(b*d^3*e*x^2 + b*d^4*x)*sqrt(d)*integrate(arctan2(1, sqrt(c*x + 1
)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^3 + d^2*x^2)*sqrt(e*x + d)), x) - 5*(a
*e^2*x^2 + a*d*e*x)*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)))*sqrt(e*
x + d) - 2*(15*a*e^2*x^2 + 20*a*d*e*x + 3*a*d^2)*sqrt(d))/((d^3*e*x^2 + d^4
*x)*sqrt(e*x + d)*sqrt(d))
```

Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(5/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(5/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(5/2)), x)
```

3.75 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 540

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \frac{4be(1-c^2x^2)}{15cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} + \frac{16bce(1-c^2x^2)}{15\,(c^2d^2-e^2)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{4be(1-c^2x^2)}{5cd^2\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{4b(7c^2d^2-3e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15\,(c^2d^3-de^2)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] -2/5*(a+b*arccsc(c*x))/e/(e*x+d)^(5/2)+4/15*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e
^2)/x/(e*x+d)^(3/2)/(1-1/c^2/x^2)^(1/2)+16/15*b*c*e*(-c^2*x^2+1)/(c^2*d^2-e
^2)^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*e*(-c^2*x^2+1)/c/d^2/(c^2*d
^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*(7*c^2*d^2-3*e^2)*Ellipt
icE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c
^2*x^2+1)^(1/2)/(c^2*d^3-d*e^2)^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))
^(1/2)+4/15*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2
))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/x/(1-1/c^2/
x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1
```

```
/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d^2/e
/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.18, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5335, 1588, 972, 759, 849, 858, 733, 435, 430, 21, 947, 174, 552, 551}

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}} + \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15dx\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{d+ex}} - \frac{16bc^2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)^2\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cd^2ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}} + \frac{16bce(1-c^2x^2)}{15x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)^2\sqrt{d+ex}} + \frac{4be(1-c^2x^2)}{5cd^2x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{d+ex}} + \frac{4be(1-c^2x^2)}{15cdx\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)(d+ex)^{3/2}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (4*b*e*(1 - c^2*x^2))/(15*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
e*x)^(3/2)) + (16*b*c*e*(1 - c^2*x^2))/(15*(c^2*d^2 - e^2)^2*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[d + e*x]) + (4*b*e*(1 - c^2*x^2))/(5*c*d^2*(c^2*d^2 - e^2)*S
qrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*(a + b*ArcCsc[c*x]))/(5*e*(d + e
*x)^(5/2)) - (16*b*c^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqr
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*(c^2*d^2 - e^2)^2*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*
x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*(c^2
*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*S
qrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*
```

```
x*Sqrt[d + e*x]) + (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ell
ipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*c*d^2*e*Sqrt
[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
```

```
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}}-\frac{(2b)\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{5/2}}\,dx}{5ce}\\
&= -\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}}-\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{x(d+ex)^{5/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}x}\\
&= -\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}}\\
&\quad-\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\left(-\frac{e}{d(d+ex)^{5/2}\sqrt{-\frac{1}{c^2}+x^2}}-\frac{e}{d^2(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}+\frac{1}{d^2x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\right)dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}x}\\
&= -\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}}+\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd^2\sqrt{1-\frac{1}{c^2x^2}}x}\\
&\quad+\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{(d+ex)^{5/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$= \frac{4be(1-c^2x^2)}{15cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}} + \frac{4be(1-c^2x^2)}{5cd^2\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{\left(4b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{-\frac{d}{2}-\frac{ex}{2}}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd^2\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(4b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{-\frac{3d}{2}+\frac{ex}{2}}{(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15cd\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(2b\sqrt{1-c^2x^2}\right)\int\frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{4be(1-c^2x^2)}{15cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}} + \frac{16bce(1-c^2x^2)}{15\,(c^2d^2-e^2)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{4be(1-c^2x^2)}{5cd^2\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}} + \frac{\left(8b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{\frac{1}{4}\left(3d^2+\frac{e^2}{c^2}\right)+dex}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15cd\left(d^2-\frac{e^2}{c^2}\right)^2\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd^2\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{\left(4b\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= \frac{4be(1-c^2x^2)}{15cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}}+\frac{16bce(1-c^2x^2)}{15\,(c^2d^2-e^2)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$+\frac{4be(1-c^2x^2)}{5cd^2\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$-\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}}+\frac{\left(8b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15c\left(d^2-\frac{e^2}{c^2}\right)^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$+\frac{\left(2b\left(-d^2+\frac{e^2}{c^2}\right)\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{15cd\left(d^2-\frac{e^2}{c^2}\right)^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$+\frac{\left(4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\,dx,x,\sqrt{1-cx}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$-\frac{\left(4b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{5c^2d^2\left(d^2-\frac{e^2}{c^2}\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$

$$= \frac{4be(1-c^2x^2)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}} + \frac{16bce(1-c^2x^2)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{4be(1-c^2x^2)}{5cd^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}}$$
$$- \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{\left(16b\sqrt{d+ex}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2\left(d^2-\frac{e^2}{c^2}\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{c}}}}$$
$$- \frac{\left(4b\left(-d^2+\frac{e^2}{c^2}\right)\sqrt{\frac{d+ex}{d+\frac{e}{c}}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{2ex^2}{c\left(d+\frac{e}{c}\right)}}}dx,x,\frac{\sqrt{1-cx}}{\sqrt{2}}\right)}{15c^2d\left(d^2-\frac{e^2}{c^2}\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

$$= \frac{4be(1-c^2x^2)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}} + \frac{16bce(1-c^2x^2)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{4be(1-c^2x^2)}{5cd^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}}$$
$$- \frac{16bc^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$- \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}$$
$$+ \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Mathematica [A] (warning: unable to verify)

Time = 34.05 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.86

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2a}{5e(d+ex)^{5/2}} + \frac{b\left(-\frac{c^4\left(e+\frac{d}{x}\right)^4 x^4\left(\frac{4\left(-7c^2d^2+3e^2\right)\sqrt{1-\frac{1}{c^2x^2}}}{15c^2d^2\left(-c^2d^2+e^2\right)^2}+\frac{2\csc^{-1}(cx)}{5c^3d^3e}-\frac{2e^2\csc^{-1}(cx)}{5c^3d^3\left(e+\frac{d}{x}\right)^3}-\frac{2\left(2cde^2\sqrt{1-\frac{1}{c^2x^2}}-9c^2d^2e\csc^{-1}(cx)+9e^3\csc^{-1}(cx)\right)}{15c^3d^3\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)^2}-\frac{2\left(-16c^3d^3e\sqrt{1-\cdots}\right.}{\cdots}\right.}{(d+ex)^{7/2}}\cdots}{\cdots}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (-2*a)/(5*e*(d + e*x)^(5/2)) + (b*(-((c^4*(e + d/x)^4*x^4*((4*(-7*c^2*d^2 +
 3*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(15*c^2*d^2*(-(c^2*d^2) + e^2)^2) + (2*ArcCs
c[c*x])/(5*c^3*d^3*e) - (2*e^2*ArcCsc[c*x])/(5*c^3*d^3*(e + d/x)^3) - (2*(2
*c*d*e^2*Sqrt[1 - 1/(c^2*x^2)] - 9*c^2*d^2*e*ArcCsc[c*x] + 9*e^3*ArcCsc[c*x
]))/(15*c^3*d^3*(c^2*d^2 - e^2)*(e + d/x)^2) - (2*(-16*c^3*d^3*e*Sqrt[1 - 1
/(c^2*x^2)] + 8*c*d*e^3*Sqrt[1 - 1/(c^2*x^2)] + 9*c^4*d^4*ArcCsc[c*x] - 18*
c^2*d^2*e^2*ArcCsc[c*x] + 9*e^4*ArcCsc[c*x]))/(15*c^3*d^3*(c^2*d^2 - e^2)^2
*(e + d/x))))/(d + e*x)^(7/2)) + (2*(e + d/x)^(7/2)*(c*x)^(7/2)*((2*(c^2*d^
2*e - e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin
[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e +
d/x]*(c*x)^(3/2)) + (2*(3*c^3*d^3 + c*d*e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*
Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
 e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-7*c^2*d^2*e
+ 3*e^3)*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c
*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqr
t[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(
c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d
 - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d
- e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqr
t[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 -
c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*
Sqrt[c*x]*(-2 + c^2*x^2))))/(15*c*d*(c*d - e)^2*e*(c*d + e)^2*(d + e*x)^(7/
2))))/c
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. 2(491) = 982.

Time = 9.34 (sec) , antiderivative size = 1618, normalized size of antiderivative = 3.00

method	result	size
derivativedivides	Expression too large to display	1618
default	Expression too large to display	1618
parts	Expression too large to display	1642

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arccsc(c*x)-2/15/c*(7*(c/(c
*d-e))^(1/2)*c^4*d^3*(e*x+d)^3-6*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e
*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d
-e)/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1
/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^
(1/2),((c*d-e)/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^4*d^
4*(e*x+d)^(3/2)-13*(c/(c*d-e))^(1/2)*c^4*d^4*(e*x+d)^2-7*((-c*(e*x+d)+c*d-e
)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)
*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3*e*(e*x+d)^(3/2)+7*((-c*
(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3*e*(e*x+d)^
(3/2)+5*(c/(c*d-e))^(1/2)*c^4*d^5*(e*x+d)-3*(c/(c*d-e))^(1/2)*c^2*d*e^2*(e*
x+d)^3+2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1
/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*
d^2*e^2*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d
+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)+6*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*(
(-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2
),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3
/2)+(c/(c*d-e))^(1/2)*c^4*d^6+5*(c/(c*d-e))^(1/2)*c^2*d^2*e^2*(e*x+d)^2+3*(
(-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ellipt
icF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^3*(e*x+d
)^(3/2)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(
1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d
*e^3*(e*x+d)^(3/2)-8*(c/(c*d-e))^(1/2)*c^2*d^3*e^2*(e*x+d)-3*((-c*(e*x+d)+c
*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^
(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*
e^4*(e*x+d)^(3/2)-2*(c/(c*d-e))^(1/2)*c^2*d^4*e^2+3*(c/(c*d-e))^(1/2)*d*e^4
*(e*x+d)+(c/(c*d-e))^(1/2)*d^2*e^4)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(3/2)
/(c*d+e)/(c^2*d^2-e^2)/d^3/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c
^2/e^2/x^2)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for more d
etails)
```

Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{7/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(7/2),x)
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(7/2), x)
```

3.76 $\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 206

$$\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}+\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{b(42c^2d+25e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{560c^6\sqrt{c^2x^2}}$$

```
[Out] 1/5*d*x^5*(a+b*arccsc(c*x))+1/7*e*x^7*(a+b*arccsc(c*x))+1/560*b*(42*c^2*d+2
5*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)+1/560*b*(42*c^2*d
+25*e)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)+1/840*b*(42*c^2*d+25*e)*x^
4*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/42*b*e*x^6*(c^2*x^2-1)^(1/2)/c/(c
^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used

$= \{14, 5347, 12, 470, 327, 223, 212\}$

$$\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)dx = \frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right) + \frac{bx\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(42c^2d+25e)}{560c^6\sqrt{c^2x^2}} + \frac{bex^6\sqrt{c^2x^2-1}}{42c\sqrt{c^2x^2}} + \frac{bx^2\sqrt{c^2x^2-1}(42c^2d+25e)}{560c^5\sqrt{c^2x^2}} + \frac{bx^4\sqrt{c^2x^2-1}(42c^2d+25e)}{840c^3\sqrt{c^2x^2}}$$

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(42*c^2*d + 25*e)*x^2*Sqrt[-1 + c^2*x^2])/(560*c^5*Sqrt[c^2*x^2]) + (b*(
42*c^2*d + 25*e)*x^4*Sqrt[-1 + c^2*x^2])/(840*c^3*Sqrt[c^2*x^2]) + (b*e*x^6
*Sqrt[-1 + c^2*x^2])/(42*c*Sqrt[c^2*x^2]) + (d*x^5*(a + b*ArcCsc[c*x]))/5 +
 (e*x^7*(a + b*ArcCsc[c*x]))/7 + (b*(42*c^2*d + 25*e)*x*ArcTanh[(c*x)/Sqrt[
-1 + c^2*x^2]])/(560*c^6*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^4(7d+5ex^2)}{35\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^4(7d+5ex^2)}{\sqrt{-1+c^2x^2}}\,dx}{35\sqrt{c^2x^2}}\\
&= \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)-\frac{\left(bc\left(-42d-\frac{25e}{c^2}\right)x\right)\int\frac{x^4}{\sqrt{-1+c^2x^2}}\,dx}{210\sqrt{c^2x^2}}\\
&= \frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)-\frac{\left(b\left(-42d-\frac{25e}{c^2}\right)x\right)\int\frac{x^2}{\sqrt{-1+c^2x^2}}\,dx}{280c\sqrt{c^2x^2}}\\
&= \frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}+\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}\\
&\quad+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)-\frac{\left(b\left(-42d-\frac{25e}{c^2}\right)x\right)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{560c^3\sqrt{c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} + \frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} \\
&\quad + \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right) + \frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right) \\
&\quad - \frac{\left(b\left(-42d-\frac{25e}{c^2}\right)x\right)\operatorname{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{560c^3\sqrt{c^2x^2}} \\
&= \frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}} + \frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} \\
&\quad + \frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right) + \frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right) + \frac{b(42c^2d+25e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{560c^6\sqrt{c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.68

$$
\int x^4\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx
$$
$$
= \frac{48ac^7x^5(7d+5ex^2) + bc^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2(75e+2c^2(63d+25ex^2)+c^4(84dx^2+40ex^4)) + 48bc^7x^5(7d+5ex^2)}{1680c^7}
$$

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (48*a*c^7*x^5*(7*d + 5*e*x^2) + b*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*(75*e + 2*c
^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5*e
*x^2)*ArcCsc[c*x] + 3*b*(42*c^2*d + 25*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x
])/(1680*c^7)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.59

method	result
parts	$a\left(\frac{1}{7}e\,x^7+\frac{1}{5}d\,x^5\right)+\frac{b\,\operatorname{arccsc}(cx)e\,x^7}{7}+\frac{b\,\operatorname{arccsc}(cx)x^5d}{5}+\frac{b(c^2x^2-1)x^4e}{42c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b(c^2x^2-1)x^2d}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{5b(c^2x^2-1)x^2}{168c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\,\operatorname{arccsc}(cx)d\,c^5x^5}{5}+\frac{b\,c^5\operatorname{arccsc}(cx)e\,x^7}{7}+\frac{b\left(c^2x^2-1\right)c^2x^2d}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\,c^2\left(c^2x^2-1\right)x^4e}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\left(c^2x^2-1\right)d}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{5b\left(c^2x^2-\right.}{168\sqrt{\frac{c^2x}{c}}}}{c^5}$
default	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\,\operatorname{arccsc}(cx)d\,c^5x^5}{5}+\frac{b\,c^5\operatorname{arccsc}(cx)e\,x^7}{7}+\frac{b\left(c^2x^2-1\right)c^2x^2d}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\,c^2\left(c^2x^2-1\right)x^4e}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\left(c^2x^2-1\right)d}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{5b\left(c^2x^2-\right.}{168\sqrt{\frac{c^2x}{c}}}}{c^5}$

```
[In] int(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*e*x^7+1/5*d*x^5)+1/7*b*arccsc(c*x)*e*x^7+1/5*b*arccsc(c*x)*x^5*d+1/4
2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^4*e+1/20*b/c^3*(c^2*x^2-1
)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*d+5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^
2/x^2)^(1/2)*x^2*e+3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d+5/1
12*b/c^7*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e+3/40*b/c^6*(c^2*x^2-1)^(
1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))+5/112*b/c^8*
(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.93

$$\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{240\,ac^7ex^7+336\,ac^7dx^5+48\,(5\,bc^7ex^7+7\,bc^7dx^5-7\,bc^7d-5\,bc^7e)\,\mathrm{arccsc}\,(cx)-96\,(7\,bc^7d+5\,bc^7e)\,\mathrm{arct}}{}$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 + 48*(5*b*c^7*e*x^7 + 7*b*c^7*d*x
^5 - 7*b*c^7*d - 5*b*c^7*e)*arccsc(c*x) - 96*(7*b*c^7*d + 5*b*c^7*e)*arctan
(-c*x + sqrt(c^2*x^2 - 1)) - 3*(42*b*c^2*d + 25*b*e)*log(-c*x + sqrt(c^2*x^
2 - 1)) + (40*b*c^5*e*x^5 + 2*(42*b*c^5*d + 25*b*c^3*e)*x^3 + 3*(42*b*c^3*d
 + 25*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.98

$$\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{adx^5}{5}+\frac{aex^7}{7}+\frac{bdx^5\,\mathrm{acsc}\,(cx)}{5}+\frac{bex^7\,\mathrm{acsc}\,(cx)}{7} + \frac{bd\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\,\mathrm{acosh}\,(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\,\mathrm{asin}\,(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c} + \frac{be\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\,\mathrm{acosh}\,(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\,\mathrm{asin}\,(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

```
[In] integrate(x**4*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*acsc(c*x)/5 + b*e*x**7*acsc(c*x)/7 + b*d
*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1))
 - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2)
 > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 +
1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/
(5*c) + b*e*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1)) + x**5/(24*c*sqrt(c**
2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**
2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2) > 1), (-I*c*x**7/(6*s
qrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 + 1)) - 5*I*x**3/(48*c*
*3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(
c*x)/(16*c**6), True))/(7*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.44

$$\int x^4\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{7}aex^7+\frac{1}{5}adx^5$$

$$+\frac{1}{80}\left(16\,x^5\operatorname{arccsc}(cx)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bd$$

$$+\frac{1}{672}\left(96\,x^7\operatorname{arccsc}(cx)+\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right.$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/80*(16*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2)
+ 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/
(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(
-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d + 1/672*(96*x^7*arccsc(c*x) + (2*(15*(-1
/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2)
+ 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*
x^2) - 1) + c^6) + 15*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/
(c^2*x^2) + 1) - 1)/c^6)/c)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1166 vs. 2(178) = 356.

Time = 1.66 (sec) , antiderivative size = 1166, normalized size of antiderivative = 5.66

$$\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*b*e*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))/c + 15*a
*e*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7/c + 5*b*e*x^6*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^6/c^2 + 84*b*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c +
 84*a*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 105*b*e*x^5*(sqrt(-1/(c^2*x^
2) + 1) + 1)^5*arcsin(1/(c*x))/c^3 + 105*a*e*x^5*(sqrt(-1/(c^2*x^2) + 1) +
1)^5/c^3 + 42*b*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^2 + 45*b*e*x^4*(sqrt
(-1/(c^2*x^2) + 1) + 1)^4/c^4 + 420*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*
arcsin(1/(c*x))/c^3 + 420*a*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^3 + 315*
b*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^5 + 315*a*e*x^3*(s
qrt(-1/(c^2*x^2) + 1) + 1)^3/c^5 + 336*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2/c^4 + 225*b*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^6 + 840*b*d*x*(sqrt(-
1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 840*a*d*x*(sqrt(-1/(c^2*x^2) +
1) + 1)/c^5 + 525*b*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^7 +
525*a*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^7 + 1008*b*d*log(sqrt(-1/(c^2*x^2)
 + 1) + 1)/c^6 - 1008*b*d*log(1/(abs(c)*abs(x)))/c^6 + 600*b*e*log(sqrt(-1/
(c^2*x^2) + 1) + 1)/c^8 - 600*b*e*log(1/(abs(c)*abs(x)))/c^8 + 840*b*d*arcs
in(1/(c*x))/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 840*a*d/(c^7*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) + 525*b*e*arcsin(1/(c*x))/(c^9*x*(sqrt(-1/(c^2*x^2) +
1) + 1)) + 525*a*e/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 336*b*d/(c^8*x^2*
(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 225*b*e/(c^10*x^2*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^2) + 420*b*d*arcsin(1/(c*x))/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 + 420*a*d/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 315*b*e*arcsin(1/(c*x
))/(c^11*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 315*a*e/(c^11*x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3) - 42*b*d/(c^10*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) -
45*b*e/(c^12*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 84*b*d*arcsin(1/(c*x))/(
c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 84*a*d/(c^11*x^5*(sqrt(-1/(c^2*x
^2) + 1) + 1)^5) + 105*b*e*arcsin(1/(c*x))/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1
) + 1)^5) + 105*a*e/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 5*b*e/(c^14
*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 15*b*e*arcsin(1/(c*x))/(c^15*x^7*(sq
rt(-1/(c^2*x^2) + 1) + 1)^7) + 15*a*e/(c^15*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1
)^7))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^4\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^4*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^4*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.77 $\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{b(20c^2d+9e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}$$

```
[Out] 1/3*d*x^3*(a+b*arccsc(c*x))+1/5*e*x^5*(a+b*arccsc(c*x))+1/120*b*(20*c^2*d+9
*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^4/(c^2*x^2)^(1/2)+1/120*b*(20*c^2*d+
9*e)*x^2*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/20*b*e*x^4*(c^2*x^2-1)^(1/
2)/c/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {14, 5347, 12, 470, 327, 223, 212}

$$\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{bx\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(20c^2d+9e)}{120c^4\sqrt{c^2x^2}}+\frac{bex^4\sqrt{c^2x^2-1}}{20c\sqrt{c^2x^2}}+\frac{bx^2\sqrt{c^2x^2-1}(20c^2d+9e)}{120c^3\sqrt{c^2x^2}}$$

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(20*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(120*c^3*Sqrt[c^2*x^2]) + (b*e*
x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + (d*x^3*(a + b*ArcCsc[c*x]))/
3 + (e*x^5*(a + b*ArcCsc[c*x]))/5 + (b*(20*c^2*d + 9*e)*x*ArcTanh[(c*x)/Sqr
t[-1 + c^2*x^2]])/(120*c^4*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^2\left(5d+3ex^2\right)}{15\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^2\left(5d+3ex^2\right)}{\sqrt{-1+c^2x^2}}\,dx}{15\sqrt{c^2x^2}}\\
&= \frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)-\frac{\left(bc\left(-20d-\frac{9e}{c^2}\right)x\right)\int\frac{x^2}{\sqrt{-1+c^2x^2}}\,dx}{60\sqrt{c^2x^2}}\\
&= \frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)-\frac{\left(b\left(-20d-\frac{9e}{c^2}\right)x\right)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{120c\sqrt{c^2x^2}}\\
&= \frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)-\frac{\left(b\left(-20d-\frac{9e}{c^2}\right)x\right)\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{120c\sqrt{c^2x^2}}\\
&= \frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{b(20c^2d+9e)\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int x^2 (d+ex^2) \left(a+b\csc^{-1}(cx)\right) \, dx$$
$$= \frac{c^2x^2\left(8ac^3x(5d+3ex^2)+b\sqrt{1-\frac{1}{c^2x^2}}(9e+c^2(20d+6ex^2))\right)+8bc^5x^3(5d+3ex^2)\csc^{-1}(cx)+b(20c^2d+9\cdots}{120c^5}$$

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (c^2*x^2*(8*a*c^3*x*(5*d + 3*e*x^2) + b*Sqrt[1 - 1/(c^2*x^2)]*(9*e + c^2*(2
0*d + 6*e*x^2))) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcCsc[c*x] + b*(20*c^2*d +
9*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(120*c^5)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.58

method	result
parts	$a\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{b\ \mathrm{arccsc}(cx)e\,x^5}{5}+\frac{b\ \mathrm{arccsc}(cx)x^3d}{3}+\frac{b(c^2x^2-1)x^2e}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b(c^2x^2-1)d}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b(c^2x^2-1)e}{40c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\cdots$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\ \mathrm{arccsc}(cx)d\,c^3x^3}{3}+\frac{b\,c^3\ \mathrm{arccsc}(cx)e\,x^5}{5}+\frac{b\left(c^2x^2-1\right)d}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\left(c^2x^2-1\right)x^2e}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\cdots}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\ \mathrm{arccsc}(cx)d\,c^3x^3}{3}+\frac{b\,c^3\ \mathrm{arccsc}(cx)e\,x^5}{5}+\frac{b\left(c^2x^2-1\right)d}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\left(c^2x^2-1\right)x^2e}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\cdots}{c^3}$

```
[In] int(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*e*x^5+1/3*d*x^3)+1/5*b*arccsc(c*x)*e*x^5+1/3*b*arccsc(c*x)*x^3*d+1/2
0*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e+1/6*b/c^3*(c^2*x^2-1)
/((c^2*x^2-1)/c^2/x^2)^(1/2)*d+3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)
^(1/2)*e+1/6*b/c^4*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x
+(c^2*x^2-1)^(1/2))+3/40*b/c^6*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2
)/x*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{24\,ac^5ex^5+40\,ac^5dx^3+8\,(3\,bc^5ex^5+5\,bc^5dx^3-5\,bc^5d-3\,bc^5e)\,\mathrm{arccsc}\,(cx)-16\,(5\,bc^5d+3\,bc^5e)\arctan}{120}$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3 -
 5*b*c^5*d - 3*b*c^5*e)*arccsc(c*x) - 16*(5*b*c^5*d + 3*b*c^5*e)*arctan(-c*
x + sqrt(c^2*x^2 - 1)) - (20*b*c^2*d + 9*b*e)*log(-c*x + sqrt(c^2*x^2 - 1))
 + (6*b*c^3*e*x^3 + (20*b*c^3*d + 9*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.83

$$\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{adx^3}{3}+\frac{aex^5}{5}+\frac{bdx^3\,\mathrm{acsc}\,(cx)}{3}+\frac{bex^5\,\mathrm{acsc}\,(cx)}{5}$$
$$+\frac{bd\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for}\ |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$
$$+\frac{be\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\,\mathrm{acosh}\,(cx)}{8c^4} & \text{for}\ |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\,\mathrm{asin}\,(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

```
[In] integrate(x**2*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*acsc(c*x)/3 + b*e*x**5*acsc(c*x)/5 + b*d
*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**
2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1
)) - I*asin(c*x)/(2*c**2), True))/(3*c) + b*e*Piecewise((c*x**5/(4*sqrt(c**
2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2
- 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**
2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2
*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.44

$$\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{5}aex^5+\frac{1}{3}adx^3 + \frac{1}{12}\left(4x^3\operatorname{arccsc}(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd + \frac{1}{80}\left(16x^5\operatorname{arccsc}(cx)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2)
+ 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 -
log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arccsc(c*x) - (2
*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2)
- 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)
/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 822 vs. 2(139) = 278.

Time = 1.16 (sec) , antiderivative size = 822, normalized size of antiderivative = 5.11

$$\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{960}\left(\frac{6bex^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{6aex^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5}{c}+\frac{3bex^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c^2}+\right.$$

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c + 6*a*e*x
^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 3*b*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1
)^4/c^2 + 40*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + 40*
```

```
a*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + 30*b*e*x^3*(sqrt(-1/(c^2*x^2) +
1) + 1)^3*arcsin(1/(c*x))/c^3 + 30*a*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c
^3 + 40*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 24*b*e*x^2*(sqrt(-1/(c
^2*x^2) + 1) + 1)^2/c^4 + 120*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(
c*x))/c^3 + 120*a*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 60*b*e*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 60*a*e*x*(sqrt(-1/(c^2*x^2) + 1)
+ 1)/c^5 + 160*b*d*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 160*b*d*log(1/(abs
(c)*abs(x)))/c^4 + 72*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 72*b*e*log(
1/(abs(c)*abs(x)))/c^6 + 120*b*d*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2)
+ 1) + 1)) + 120*a*d/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 60*b*e*arcsin(1
/(c*x))/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 60*a*e/(c^7*x*(sqrt(-1/(c^2*
x^2) + 1) + 1)) - 40*b*d/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 24*b*e/
(c^8*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 40*b*d*arcsin(1/(c*x))/(c^7*x^3*
(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 40*a*d/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) +
 1)^3) + 30*b*e*arcsin(1/(c*x))/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) +
30*a*e/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 3*b*e/(c^10*x^4*(sqrt(-1/
(c^2*x^2) + 1) + 1)^4) + 6*b*e*arcsin(1/(c*x))/(c^11*x^5*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^5) + 6*a*e/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d + ex^2) \left(a + b \csc^{-1}(cx)\right) dx = \int x^2 \left(e\,x^2 + d\right) \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) dx$$

```
[In] int(x^2*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.78 $\int (d+ex^2)(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 109

$$\int (d+ex^2)\left(a+b\csc^{-1}(cx)\right)dx=\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{b(6c^2d+e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

```
[Out] d*x*(a+b*arccsc(c*x))+1/3*e*x^3*(a+b*arccsc(c*x))+1/6*b*(6*c^2*d+e)*x*arcta
nh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)^(1/2)+1/6*b*e*x^2*(c^2*x^2-1)^(1/2)
/c/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5337, 12, 396, 223, 212}

$$\int (d+ex^2)\left(a+b\csc^{-1}(cx)\right)dx=dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{bx\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(6c^2d+e)}{6c^2\sqrt{c^2x^2}}+\frac{bex^2\sqrt{c^2x^2-1}}{6c\sqrt{c^2x^2}}$$

```
[In] Int[(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*x^2*Sqrt[-1 + c^2*x^2])/(6*c*Sqrt[c^2*x^2]) + d*x*(a + b*ArcCsc[c*x])
+ (e*x^3*(a + b*ArcCsc[c*x]))/3 + (b*(6*c^2*d + e)*x*ArcTanh[(c*x)/Sqrt[-1
+ c^2*x^2]])/(6*c^2*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5337

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{3d+ex^2}{3\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{3d+ex^2}{\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}}\\
&= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)-\frac{\left(b\left(-6c^2d-e\right)x\right)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{6c\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx\left(a+b\csc^{-1}(cx)\right) + \frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right) - \frac{(b(-6c^2d-e)\,x)\,\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, \frac{x}{\sqrt{-1+c^2x^2}}\right)}{6c\sqrt{c^2x^2}}$$

$$= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} + dx\left(a+b\csc^{-1}(cx)\right) + \frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right) + \frac{b(6c^2d+e)\,x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.37

$$\int \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx = adx + \frac{1}{3}aex^3 + \frac{bex^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c} + bdx\csc^{-1}(cx) + \frac{1}{3}bex^3\csc^{-1}(cx) + \frac{bd\sqrt{1-\frac{1}{c^2x^2}}x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}} + \frac{be\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (b*e*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d*
x*ArcCsc[c*x] + (b*e*x^3*ArcCsc[c*x])/3 + (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcT
anh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2] + (b*e*Log[x*(1 + Sqrt[(-
1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.21

method	result
parts	$a\left(\frac{1}{3}x^3e+dx\right)+\frac{b\left(\frac{c\,\mathrm{arccsc}(cx)x^3e}{3}+\mathrm{arccsc}(cx)dxc+\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2}\right.\right.}{6c^3x\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c}$
derivativedivides	$\frac{\frac{a\left(c^3dx+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\mathrm{arccsc}(cx)d\,c^3x+\frac{\mathrm{arccsc}(cx)e\,c^3x^3}{3}+\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)\right.}{6cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^2}}{c}$
default	$\frac{\frac{a\left(c^3dx+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\mathrm{arccsc}(cx)d\,c^3x+\frac{\mathrm{arccsc}(cx)e\,c^3x^3}{3}+\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)\right.}{6cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^2}}{c}$

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*x^3*e+d*x)+b/c*(1/3*c*arccsc(c*x)*x^3*e+arccsc(c*x)*d*x*c+1/6/c^3*(c
^2*x^2-1)^(1/2)*(6*d*c^2*ln(c*x+(c^2*x^2-1)^(1/2))+e*c*x*(c^2*x^2-1)^(1/2)+
e*ln(c*x+(c^2*x^2-1)^(1/2)))/x/((c^2*x^2-1)/c^2/x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.29

$$\int \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{2\,ac^3ex^3+6\,ac^3dx+\sqrt{c^2x^2-1}bcex+2\left(bc^3ex^3+3\,bc^3dx-3\,bc^3d-bc^3e\right)\mathrm{arccsc}\left(cx\right)-4\left(3\,bc^3d+bc^3e\right)}{6\,c^3}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e*x^3 + 6*a*c^3*d*x + sqrt(c^2*x^2 - 1)*b*c*e*x + 2*(b*c^3*e*x
^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*arccsc(c*x) - 4*(3*b*c^3*d + b*c^3*
e)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (6*b*c^2*d + b*e)*log(-c*x + sqrt(c^2
*x^2 - 1)))/c^3
```

Sympy [A] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int (d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$= adx + \frac{aex^3}{3} + bdx\,\mathrm{acsc}\,(cx) + \frac{bex^3\,\mathrm{acsc}\,(cx)}{3} + \frac{bd\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c} + \frac{be\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x + a*e*x**3/3 + b*d*x*acsc(c*x) + b*e*x**3*acsc(c*x)/3 + b*d*Piecewise
((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + b*e*Piecewise(
(x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-
I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin
(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int (d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$= \frac{1}{3}aex^3 + \frac{1}{12}\left(4x^3\,\mathrm{arccsc}\,(cx) + \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be + adx + \frac{\left(2\,cx\,\mathrm{arccsc}\,(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2\,c}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(
c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c
^2*x^2) + 1) - 1)/c^2)/c)*b*e + a*d*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-
1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(95) = 190$.

Time = 0.94 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.34

$$\int \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{24}\left(\frac{bex^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{aex^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c}+\frac{bex^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^2}+\frac{12\,bc\right.$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/24*(b*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + a*e*x^3*(s
qrt(-1/(c^2*x^2) + 1) + 1)^3/c + b*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2
 + 12*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c + 12*a*d*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)/c + 3*b*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(
c*x))/c^3 + 3*a*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 24*b*d*log(sqrt(-1/(
c^2*x^2) + 1) + 1)/c^2 - 24*b*d*log(1/(abs(c)*abs(x)))/c^2 + 4*b*e*log(sqrt
(-1/(c^2*x^2) + 1) + 1)/c^4 - 4*b*e*log(1/(abs(c)*abs(x)))/c^4 + 12*b*d*arc
sin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 12*a*d/(c^3*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) + 3*b*e*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1)
 + 1)) + 3*a*e/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - b*e/(c^6*x^2*(sqrt(-1
/(c^2*x^2) + 1) + 1)^2) + b*e*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3) + a*e/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.79 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^2}\,dx = -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{x} + ex(a+b\csc^{-1}(cx)) + \frac{bex\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

```
[Out] -d*(a+b*arccsc(c*x))/x+e*x*(a+b*arccsc(c*x))+b*e*x*arctanh(c*x/(c^2*x^2-1)^
(1/2))/(c^2*x^2)^(1/2)-b*c*d*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5347, 462, 223, 212}

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^2}\,dx = -\frac{d(a+b\csc^{-1}(cx))}{x} + ex(a+b\csc^{-1}(cx)) + \frac{bex\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}} - \frac{bcd\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] -((b*c*d*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/x + e
*x*(a + b*ArcCsc[c*x]) + (b*e*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[c^2
*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d\left(a+b\csc^{-1}(cx)\right)}{x}+ex\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{-d+ex^2}{x^2\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}-\frac{d\left(a+b\csc^{-1}(cx)\right)}{x}+ex\left(a+b\csc^{-1}(cx)\right)+\frac{(bcex)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}-\frac{d\left(a+b\csc^{-1}(cx)\right)}{x}+ex\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{(bcex)\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}
\end{aligned}
$$

$$=-\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{x}+ex\left(a+b\csc^{-1}(cx)\right)+\frac{bex\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx=-\frac{ad}{x}+aex-bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{bd\csc^{-1}(cx)}{x}+bex\csc^{-1}(cx)+\frac{be\sqrt{1-\frac{1}{c^2x^2}}x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*e*x - b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*d*ArcCsc[c*x
])/x + b*e*x*ArcCsc[c*x] + (b*e*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[
-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

method	result	size
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(-\frac{\operatorname{arccsc}(cx)d}{xc}+\frac{\operatorname{arccsc}(cx)ex}{c}-\frac{\sqrt{c^2x^2-1}\left(d\,c^2\sqrt{c^2x^2-1}-e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^4x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)$	116
derivativedivides	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arccsc}(cx)xe-\frac{\operatorname{arccsc}(cx)dc}{x}+\frac{\sqrt{c^2x^2-1}\left(-d\,c^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2}\right)$	120
default	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arccsc}(cx)xe-\frac{\operatorname{arccsc}(cx)dc}{x}+\frac{\sqrt{c^2x^2-1}\left(-d\,c^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2}\right)$	120

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(e*x-d/x)+b*c*(-arccsc(c*x)*d/x/c+1/c*arccsc(c*x)*e*x-1/c^4*(c^2*x^2-1)^(
1/2)*(d*c^2*(c^2*x^2-1)^(1/2)-e*ln(c*x+(c^2*x^2-1)^(1/2))*c*x)/x^2/((c^2*x^
2-1)/c^2/x^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^2}\,dx = -\frac{bc^2dx-acex^2+bex\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}bcd+acd-2\,(bcd-bce)x\arctan\left(-cx+\sqrt{c}\right.}{cx}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*c^2*d*x - a*c*e*x^2 + b*e*x*log(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^
2 - 1)*b*c*d + a*c*d - 2*(b*c*d - b*c*e)*x*arctan(-c*x + sqrt(c^2*x^2 - 1))
 - (b*c*e*x^2 - b*c*d + (b*c*d - b*c*e)*x)*arccsc(c*x))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^2}\,dx = -\frac{ad}{x}+aex-bcd\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd\,\mathrm{acsc}\,(cx)}{x} + bex\,\mathrm{acsc}\,(cx)+\frac{be\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c}$$

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**2,x)
```

```
[Out] -a*d/x + a*e*x - b*c*d*sqrt(1 - 1/(c**2*x**2)) - b*d*acsc(c*x)/x + b*e*x*ac
sc(c*x) + b*e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), Tr
ue))/c
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^2}\,dx = -\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)bd+aex + \frac{\left(2\,cx\operatorname{arccsc}(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be}{2\,c}-\frac{ad}{x}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d + a*e*x + 1/2*(2*c*x*arccsc
(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))
*b*e/c - a*d/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. 2(79) = 158.

Time = 0.57 (sec) , antiderivative size = 1052, normalized size of antiderivative = 12.09

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*e*arcsin(1/(c*x))/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sq
rt(-1/(c^2*x^2) + 1) + 1)^3)) + a*e/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1
/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)) - 2*b*c*d/(x*(sqrt(-1/(c^2*x^2) +
1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3))) + 2*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c*x*(sqrt(-1/(c^2*x^
2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x
^2) + 1) + 1)^3))) - 2*b*e*log(1/(abs(c)*abs(x)))/(c*x*(sqrt(-1/(c^2*x^2) +
 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^3))) - 4*b*d*arcsin(1/(c*x))/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*
(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)
^3))) - 4*a*d/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2)
+ 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))) + 2*b*e*arcsin(1/(c
*x))/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1)
+ 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))) + 2*a*e/(c^2*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqr
t(-1/(c^2*x^2) + 1) + 1)^3))) + 2*b*d/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3
*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1
)^3))) + 2*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c^3*x^3*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^
2) + 1) + 1)^3))) - 2*b*e*log(1/(abs(c)*abs(x)))/(c^3*x^3*(sqrt(-1/(c^2*x^2
) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*
x^2) + 1) + 1)^3))) + b*e*arcsin(1/(c*x))/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) +
1) + 1)^3))) + a*e/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(
c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))))*c
```

Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^2}\,dx = a\,e\,x-\frac{a\,d}{x}+\frac{b\,e\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}-b\,c\,d\,\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{b\,d\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x}+b\,e\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)$$

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^2,x)
```

```
[Out] a*e*x - (a*d)/x + (b*e*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c - b*c*d*(1 - 1/(
c^2*x^2))^(1/2) - (b*d*asin(1/(c*x)))/x + b*e*x*asin(1/(c*x))
```

3.80 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4} dx = -\frac{bc(2c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} - \frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{3x^3} - \frac{e(a+b\csc^{-1}(cx))}{x}$$

```
[Out] -1/3*d*(a+b*arccsc(c*x))/x^3-e*(a+b*arccsc(c*x))/x-1/9*b*c*(2*c^2*d+9*e)*(c
^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/9*b*c*d*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5347, 12, 464, 270}

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4} dx = -\frac{d(a+b\csc^{-1}(cx))}{3x^3} - \frac{e(a+b\csc^{-1}(cx))}{x} - \frac{bc\sqrt{c^2x^2-1}(2c^2d+9e)}{9\sqrt{c^2x^2}} - \frac{bcd\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] -1/9*(b*c*(2*c^2*d + 9*e)*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*x^2] - (b*c*d*Sqrt[-
1 + c^2*x^2])/(9*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/(3*x^3) - (e*
(a + b*ArcCsc[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(a+b\csc^{-1}(cx))}{3x^3}-\frac{e(a+b\csc^{-1}(cx))}{x}+\frac{(bcx)\int\frac{-d-3ex^2}{3x^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{d(a+b\csc^{-1}(cx))}{3x^3}-\frac{e(a+b\csc^{-1}(cx))}{x}+\frac{(bcx)\int\frac{-d-3ex^2}{x^4\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}}\\
&= -\frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{3x^3}\\
&\quad -\frac{e(a+b\csc^{-1}(cx))}{x}+\frac{(bc(-2c^2d-9e)\,x)\int\frac{1}{x^2\sqrt{-1+c^2x^2}}\,dx}{9\sqrt{c^2x^2}}
\end{aligned}
$$

$$= -\frac{bc(2c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} - \frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{3x^3} - \frac{e(a+b\csc^{-1}(cx))}{x}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^4}\,dx$$
$$= -\frac{3a(d+3ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+9ex^2)+3b(d+3ex^2)\csc^{-1}(cx)}{9x^3}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] -1/9*(3*a*(d + 3*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 9*
e*x^2) + 3*b*(d + 3*e*x^2)*ArcCsc[c*x])/x^3
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03

method	result	size
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^3}\right)+b\,c^3\left(-\frac{\operatorname{arccsc}(cx)e}{c^3x}-\frac{\operatorname{arccsc}(cx)d}{3x^3c^3}-\frac{(c^2x^2-1)\left(2c^4d\,x^2+9c^2e\,x^2+c^2d\right)}{9c^6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^4}\right)$	108
derivativedivides	$c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{3c\,x^3}-\frac{\operatorname{arccsc}(cx)e}{cx}-\frac{\left(c^2x^2-1\right)\left(2c^4d\,x^2+9c^2e\,x^2+c^2d\right)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	121
default	$c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{3c\,x^3}-\frac{\operatorname{arccsc}(cx)e}{cx}-\frac{\left(c^2x^2-1\right)\left(2c^4d\,x^2+9c^2e\,x^2+c^2d\right)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	121

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-e/x-1/3*d/x^3)+b*c^3*(-1/c^3*arccsc(c*x)*e/x-1/3*arccsc(c*x)*d/x^3/c^3-
1/9/c^6*(c^2*x^2-1)*(2*c^4*d*x^2+9*c^2*e*x^2+c^2*d)/((c^2*x^2-1)/c^2/x^2)^(
1/2)/x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{9\,aex^2+3\,ad+3\,(3\,bex^2+bd)\,\mathrm{arccsc}\,(cx)+\sqrt{c^2x^2-1}((2\,bc^2d+9\,be)x^2+bd)}{9\,x^3}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*arccsc(c*x) + sqrt(c^2*x^2 -
1)*((2*b*c^2*d + 9*b*e)*x^2 + b*d))/x^3
```

Sympy [A] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{ad}{3x^3}-\frac{ae}{x}-bce\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd\,\mathrm{acsc}\,(cx)}{3x^3}-\frac{be\,\mathrm{acsc}\,(cx)}{x} - \frac{bd\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**4,x)
```

```
[Out] -a*d/(3*x**3) - a*e/x - b*c*e*sqrt(1 - 1/(c**2*x**2)) - b*d*acsc(c*x)/(3*x*
*3) - b*e*acsc(c*x)/x - b*d*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c
*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x*
*2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^4}\,dx = -\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\mathrm{arccsc}\,(cx)}{x}\right)be + \frac{1}{9}\,bd\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\mathrm{arccsc}\,(cx)}{x^3}\right)-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*e + 1/9*b*d*((c^4*(-1/(c^2*x^
2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x)/x^3) - a*e/
x - 1/3*a*d/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4}\,dx$$
$$=\frac{1}{9}\left(bc^2d\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,bc^2d\sqrt{-\frac{1}{c^2x^2}+1}-\frac{3\,bcd\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-\frac{3\,bcd\arcsin\left(\frac{1}{cx}\right)}{x}-9\,be\sqrt{-\frac{}{c}}\right.$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2) - 3*b*c^2*d*sqrt(-1/(c^2*x^2) + 1) -
3*b*c*d*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 3*b*c*d*arcsin(1/(c*x))/x - 9
*b*e*sqrt(-1/(c^2*x^2) + 1) - 9*b*e*arcsin(1/(c*x))/(c*x) - 9*a*e/(c*x) - 3
*a*d/(c*x^3))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4}\,dx=\int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^4, x)
```

3.81 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6} dx$

Optimal result

Integrand size = 19, antiderivative size = 152

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6} dx = -\frac{2bc^3(12c^2d+25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} - \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{5x^5} - \frac{e(a+b\csc^{-1}(cx))}{3x^3}$$

```
[Out] -1/5*d*(a+b*arccsc(c*x))/x^5-1/3*e*(a+b*arccsc(c*x))/x^3-2/225*b*c^3*(12*c^
2*d+25*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/25*b*c*d*(c^2*x^2-1)^(1/2)/x^
4/(c^2*x^2)^(1/2)-1/225*b*c*(12*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 5347, 12, 464, 277, 270}

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6} dx = -\frac{d(a+b\csc^{-1}(cx))}{5x^5} - \frac{e(a+b\csc^{-1}(cx))}{3x^3} - \frac{bc\sqrt{c^2x^2-1}(12c^2d+25e)}{225x^2\sqrt{c^2x^2}} - \frac{bcd\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}} - \frac{2bc^3\sqrt{c^2x^2-1}(12c^2d+25e)}{225\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] (-2*b*c^3*(12*c^2*d + 25*e)*Sqrt[-1 + c^2*x^2])/(225*Sqrt[c^2*x^2]) - (b*c*
d*Sqrt[-1 + c^2*x^2])/(25*x^4*Sqrt[c^2*x^2]) - (b*c*(12*c^2*d + 25*e)*Sqrt[
-1 + c^2*x^2])/(225*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/(5*x^5) -
(e*(a + b*ArcCsc[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}+\frac{(bcx)\int\frac{-3d-5ex^2}{15x^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}+\frac{(bcx)\int\frac{-3d-5ex^2}{x^6\sqrt{-1+c^2x^2}}\,dx}{15\sqrt{c^2x^2}}\\
&= -\frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}\\
&\quad+\frac{(bc(-12c^2d-25e)\,x)\int\frac{1}{x^4\sqrt{-1+c^2x^2}}\,dx}{75\sqrt{c^2x^2}}\\
&= -\frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}-\frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{5x^5}\\
&\quad-\frac{e(a+b\csc^{-1}(cx))}{3x^3}+\frac{(2bc^3(-12c^2d-25e)\,x)\int\frac{1}{x^2\sqrt{-1+c^2x^2}}\,dx}{225\sqrt{c^2x^2}}\\
&= -\frac{2bc^3(12c^2d+25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}}-\frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}\\
&\quad-\frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.62

$$\int\frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{15a(3d+5ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}\,x(25ex^2(1+2c^2x^2)+3d(3+4c^2x^2+8c^4x^4))+15b(3d+5ex^2)\csc^{-1}(ca}{225x^5}$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(15*a*(3*d + 5*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(25*e*x^2*(1 + 2
*c^2*x^2) + 3*d*(3 + 4*c^2*x^2 + 8*c^4*x^4)) + 15*b*(3*d + 5*e*x^2)*ArcCsc[
c*x])/x^5
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

method	result
parts	$a\left(-\frac{d}{5x^5}-\frac{e}{3x^3}\right)+b\,c^5\left(-\frac{\operatorname{arccsc}(cx)d}{5x^5c^5}-\frac{\operatorname{arccsc}(cx)e}{3c^5x^3}-\frac{(c^2x^2-1)\left(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25c^2e\,x^2+9c\right.}{225c^8\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^6}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{5c^3x^5}-\frac{\operatorname{arccsc}(cx)e}{3c^3x^3}-\frac{(c^2x^2-1)\left(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25c^2e\,x^2+9c^2d\right)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$
default	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{5c^3x^5}-\frac{\operatorname{arccsc}(cx)e}{3c^3x^3}-\frac{(c^2x^2-1)\left(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25c^2e\,x^2+9c^2d\right)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/5*d/x^5-1/3*e/x^3)+b*c^5*(-1/5*arccsc(c*x)*d/x^5/c^5-1/3/c^5*arccsc(c
*x)*e/x^3-1/225/c^8*(c^2*x^2-1)*(24*c^6*d*x^4+50*c^4*e*x^4+12*c^4*d*x^2+25*
c^2*e*x^2+9*c^2*d)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{75\,aex^2+45\,ad+15\,(5\,bex^2+3\,bd)\operatorname{arccsc}(cx)+(2\,(12\,bc^4d+25\,bc^2e)x^4+(12\,bc^2d+25\,be)x^2+9\,bd)\sqrt{}}{225\,x^5}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*arccsc(c*x) + (2*(12*b
*c^4*d + 25*b*c^2*e)*x^4 + (12*b*c^2*d + 25*b*e)*x^2 + 9*b*d)*sqrt(c^2*x^2
- 1))/x^5
```

Sympy [A] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx$$
$$= -\frac{ad}{5x^5} - \frac{ae}{3x^3} - \frac{bd\,\mathrm{acsc}\,(cx)}{5x^5} - \frac{be\,\mathrm{acsc}\,(cx)}{3x^3} - \frac{bd\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c} - \frac{be\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**6,x)
```

```
[Out] -a*d/(5*x**5) - a*e/(3*x**3) - b*d*acsc(c*x)/(5*x**5) - b*e*acsc(c*x)/(3*x*
*3) - b*d*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x
**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (
8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x*
*3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) - b*e*Piecewise((2*c*
*3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**
2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3
*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx$$
$$= -\frac{1}{75}\,bd\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{15\,\mathrm{arccsc}\,(cx)}{x^5}\right) + \frac{1}{9}\,be\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{3\,\mathrm{arccsc}\,(cx)}{x^3}\right) - \frac{ae}{3\,x^3} - \frac{ad}{5\,x^5}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/75*b*d*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2
) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) + 1/9*b*e*((c^4*
(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x)/
x^3) - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{1}{225}\left(9\,bc^4d\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}-30\,bc^4d\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{45\,bc^3d\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+45\,bc$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/225*(9*b*c^4*d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 30*b*c^4*d*(
-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x
+ 45*b*c^4*d*sqrt(-1/(c^2*x^2) + 1) + 90*b*c^3*d*(1/(c^2*x^2) - 1)*arcsin(1
/(c*x))/x - 25*b*c^2*e*(-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d*arcsin(1/(c*x)
)/x + 75*b*c^2*e*sqrt(-1/(c^2*x^2) + 1) + 75*b*c*e*(1/(c^2*x^2) - 1)*arcsin
(1/(c*x))/x + 75*b*c*e*arcsin(1/(c*x))/x + 75*a*e/(c*x^3) + 45*a*d/(c*x^5))
*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^6, x)
```

3.82 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 197

$$\begin{aligned}\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx = &-\frac{8bc^5(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}}\\ &-\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}\\ &-\frac{4bc^3(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675x^2\sqrt{c^2x^2}}\\ &-\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}\end{aligned}$$

```
[Out] -1/7*d*(a+b*arccsc(c*x))/x^7-1/5*e*(a+b*arccsc(c*x))/x^5-8/3675*b*c^5*(30*c
^2*d+49*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/49*b*c*d*(c^2*x^2-1)^(1/2)/x
^6/(c^2*x^2)^(1/2)-1/1225*b*c*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^
2)^(1/2)-4/3675*b*c^3*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

$= \{14, 5347, 12, 464, 277, 270\}$

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^8}\,dx = -\frac{d(a+b\csc^{-1}(cx))}{7x^7} - \frac{e(a+b\csc^{-1}(cx))}{5x^5} - \frac{bc\sqrt{c^2x^2-1}(30c^2d+49e)}{1225x^4\sqrt{c^2x^2}} - \frac{bcd\sqrt{c^2x^2-1}}{49x^6\sqrt{c^2x^2}} - \frac{8bc^5\sqrt{c^2x^2-1}(30c^2d+49e)}{3675\sqrt{c^2x^2}} - \frac{4bc^3\sqrt{c^2x^2-1}(30c^2d+49e)}{3675x^2\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] (-8*b*c^5*(30*c^2*d + 49*e)*Sqrt[-1 + c^2*x^2])/(3675*Sqrt[c^2*x^2]) - (b*c
*d*Sqrt[-1 + c^2*x^2])/(49*x^6*Sqrt[c^2*x^2]) - (b*c*(30*c^2*d + 49*e)*Sqrt
[-1 + c^2*x^2])/(1225*x^4*Sqrt[c^2*x^2]) - (4*b*c^3*(30*c^2*d + 49*e)*Sqrt[
-1 + c^2*x^2])/(3675*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/(7*x^7) -
 (e*(a + b*ArcCsc[c*x]))/(5*x^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(a+b\csc^{-1}(cx))}{7x^7} - \frac{e(a+b\csc^{-1}(cx))}{5x^5} + \frac{(bcx)\int \frac{-5d-7ex^2}{35x^8\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d(a+b\csc^{-1}(cx))}{7x^7} - \frac{e(a+b\csc^{-1}(cx))}{5x^5} + \frac{(bcx)\int \frac{-5d-7ex^2}{x^8\sqrt{-1+c^2x^2}}\,dx}{35\sqrt{c^2x^2}} \\
&= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{7x^7} - \frac{e(a+b\csc^{-1}(cx))}{5x^5} \\
&\quad + \frac{(bc(-30c^2d-49e)\,x)\int \frac{1}{x^6\sqrt{-1+c^2x^2}}\,dx}{245\sqrt{c^2x^2}} \\
&= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{7x^7} \\
&\quad - \frac{e(a+b\csc^{-1}(cx))}{5x^5} + \frac{(4bc^3(-30c^2d-49e)\,x)\int \frac{1}{x^4\sqrt{-1+c^2x^2}}\,dx}{1225\sqrt{c^2x^2}} \\
&= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{4bc^3(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675x^2\sqrt{c^2x^2}} \\
&\quad - \frac{d(a+b\csc^{-1}(cx))}{7x^7} - \frac{e(a+b\csc^{-1}(cx))}{5x^5} + \frac{(8bc^5(-30c^2d-49e)\,x)\int \frac{1}{x^2\sqrt{-1+c^2x^2}}\,dx}{3675\sqrt{c^2x^2}} \\
&= -\frac{8bc^5(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}} - \frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} \\
&\quad - \frac{4bc^3(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675x^2\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{7x^7} - \frac{e(a+b\csc^{-1}(cx))}{5x^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^8}\,dx =$$

$$-\frac{105a(5d+7ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(49ex^2(3+4c^2x^2+8c^4x^4)+15d(5+6c^2x^2+8c^4x^4+16c^6x^6))+105b}{3675x^7}$$

[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^8,x]

[Out] -1/3675*(105*a*(5*d + 7*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(49*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) + 105*b*(5*d + 7*e*x^2)*ArcCsc[c*x])/x^7

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.74

method	result
parts	$a\left(-\frac{d}{7x^7}-\frac{e}{5x^5}\right)+b\,c^7\left(-\frac{\operatorname{arccsc}(cx)d}{7x^7c^7}-\frac{\operatorname{arccsc}(cx)e}{5c^7x^5}-\frac{(c^2x^2-1)\,(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4}{3675c^{10}\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^8}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{7c^5x^7}-\frac{\operatorname{arccsc}(cx)e}{5c^5x^5}-\frac{(c^2x^2-1)\left(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4+90c^4d\,x^2\right.}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$
default	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{7c^5x^7}-\frac{\operatorname{arccsc}(cx)e}{5c^5x^5}-\frac{(c^2x^2-1)\left(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4+90c^4d\,x^2\right.}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$

[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x,method=_RETURNVERBOSE)

[Out] a*(-1/7*d/x^7-1/5*e/x^5)+b*c^7*(-1/7*arccsc(c*x)*d/x^7/c^7-1/5/c^7*arccsc(c*x)*e/x^5-1/3675/c^10*(c^2*x^2-1)*(240*c^8*d*x^6+392*c^6*e*x^6+120*c^6*d*x^4+196*c^4*e*x^4+90*c^4*d*x^2+147*c^2*e*x^2+75*c^2*d)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^8)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^8}\,dx = -\frac{735\,aex^2+525\,ad+105\,(7\,bex^2+5\,bd)\,\mathrm{arccsc}\,(cx)+(8\,(30\,bc^6d+49\,bc^4e)x^6+4\,(30\,bc^4d+49\,bc^2e)x^4}{3675\,x^7}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*arccsc(c*x) + (8*(
30*b*c^6*d + 49*b*c^4*e)*x^6 + 4*(30*b*c^4*d + 49*b*c^2*e)*x^4 + 3*(30*b*c^
2*d + 49*b*e)*x^2 + 75*b*d)*sqrt(c^2*x^2 - 1))/x^7
```

Sympy [A] (verification not implemented)

Time = 29.20 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^8}\,dx = -\frac{ad}{7x^7}-\frac{ae}{5x^5}-\frac{bd\,\mathrm{acsc}\,(cx)}{7x^7}-\frac{be\,\mathrm{acsc}\,(cx)}{5x^5} - \frac{bd\left(\begin{cases}\frac{16c^7\sqrt{c^2x^2-1}}{35x}+\frac{8c^5\sqrt{c^2x^2-1}}{35x^3}+\frac{6c^3\sqrt{c^2x^2-1}}{35x^5}+\frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } |c^2x^2|>1 \\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x}+\frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3}+\frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5}+\frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise}\end{cases}\right)}{7c} - \frac{be\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1 \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**8,x)
```

```
[Out] -a*d/(7*x**7) - a*e/(5*x**5) - b*d*acsc(c*x)/(7*x**7) - b*e*acsc(c*x)/(5*x*
*5) - b*d*Piecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**2*
x**2 - 1)/(35*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2*x*
*2 - 1)/(7*x**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/(35*
x) + 8*I*c**5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**2 + 1
)/(35*x**5) + I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) - b*e*Piecewi
se((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3
) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c*
*2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c
**2*x**2 + 1)/(5*x**5), True))/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx$$
$$= \frac{1}{245}bd\left(\frac{5\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}} - 21\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 35\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 35\,c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{35\,\mathrm{arccsc}\,(cx)}{x^7}\right)$$
$$- \frac{1}{75}be\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{15\,\mathrm{arccsc}\,(cx)}{x^5}\right)$$
$$- \frac{ae}{5\,x^5} - \frac{ad}{7\,x^7}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(5/2
) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c - 35
*arccsc(c*x)/x^7) - 1/75*b*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/
(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^
5) - 1/5*a*e/x^5 - 1/7*a*d/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(169) = 338.

Time = 0.29 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{1}{3675}\left(75\,bc^6d\left(\frac{1}{c^2x^2}-1\right)^3\sqrt{-\frac{1}{c^2x^2}+1} + 315\,bc^6d\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1} + \frac{525\,bc^5d\left(\frac{1}{c^2x^2}-1\right)^3\mathrm{arc}}{x}\right.$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

```
[Out] -1/3675*(75*b*c^6*d*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1) + 315*b*c^6*
d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) + 525*b*c^5*d*(1/(c^2*x^2) - 1
)^3*arcsin(1/(c*x))/x - 525*b*c^6*d*(-1/(c^2*x^2) + 1)^(3/2) + 1575*b*c^5*d
*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 525*b*c^6*d*sqrt(-1/(c^2*x^2) + 1)
 + 147*b*c^4*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) + 1575*b*c^5*d*(1
/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 490*b*c^4*e*(-1/(c^2*x^2) + 1)^(3/2) +
525*b*c^5*d*arcsin(1/(c*x))/x + 735*b*c^3*e*(1/(c^2*x^2) - 1)^2*arcsin(1/(c
*x))/x + 735*b*c^4*e*sqrt(-1/(c^2*x^2) + 1) + 1470*b*c^3*e*(1/(c^2*x^2) - 1
)*arcsin(1/(c*x))/x + 735*b*c^3*e*arcsin(1/(c*x))/x + 735*a*e/(c*x^5) + 525
*a*d/(c*x^7))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^8}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^8, x)
```

3.83 $\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 196

$$\int x^5\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b(4c^2d+3e)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}+\frac{b(8c^2d+9e)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}+\frac{b(4c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}}+\frac{bex(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)$$

```
[Out] 1/6*d*x^6*(a+b*arccsc(c*x))+1/8*e*x^8*(a+b*arccsc(c*x))+1/72*b*(8*c^2*d+9*e
)*x*(c^2*x^2-1)^(3/2)/c^7/(c^2*x^2)^(1/2)+1/120*b*(4*c^2*d+9*e)*x*(c^2*x^2-
1)^(5/2)/c^7/(c^2*x^2)^(1/2)+1/56*b*e*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^(1/
2)+1/24*b*(4*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used

= {14, 5347, 12, 457, 78}

$$\int x^5 (d+ex^2) \left(a+b\csc^{-1}(cx)\right) dx = \frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right) + \frac{bx(c^2x^2-1)^{5/2}(4c^2d+9e)}{120c^7\sqrt{c^2x^2}} + \frac{bx(c^2x^2-1)^{3/2}(8c^2d+9e)}{72c^7\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2-1}(4c^2d+3e)}{24c^7\sqrt{c^2x^2}} + \frac{bex(c^2x^2-1)^{7/2}}{56c^7\sqrt{c^2x^2}}$$

```
[In] Int[x^5*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(4*c^2*d + 3*e)*x*Sqrt[-1 + c^2*x^2])/(24*c^7*Sqrt[c^2*x^2]) + (b*(8*c^2
*d + 9*e)*x*(-1 + c^2*x^2)^(3/2))/(72*c^7*Sqrt[c^2*x^2]) + (b*(4*c^2*d + 9*
e)*x*(-1 + c^2*x^2)^(5/2))/(120*c^7*Sqrt[c^2*x^2]) + (b*e*x*(-1 + c^2*x^2)^
(7/2))/(56*c^7*Sqrt[c^2*x^2]) + (d*x^6*(a + b*ArcCsc[c*x]))/6 + (e*x^8*(a +
 b*ArcCsc[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^5(4d+3ex^2)}{24\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^5(4d+3ex^2)}{\sqrt{-1+c^2x^2}}\,dx}{24\sqrt{c^2x^2}}\\
&= \frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\text{Subst}\left(\int\frac{x^2(4d+3ex)}{\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{48\sqrt{c^2x^2}}\\
&= \frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{(bcx)\text{Subst}\left(\int\left(\frac{4c^2d+3e}{c^6\sqrt{-1+c^2x}}+\frac{(8c^2d+9e)\sqrt{-1+c^2x}}{c^6}+\frac{(4c^2d+9e)(-1+c^2x)^{3/2}}{c^6}+\frac{3e(-1+c^2x)^{5/2}}{c^6}\right)dx,x,x^2\right)}{48\sqrt{c^2x^2}}\\
&= \frac{b(4c^2d+3e)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}+\frac{b(8c^2d+9e)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}\\
&\quad+\frac{b(4c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}}+\frac{bex(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}}\\
&\quad+\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.59

$$\int x^5\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$=\frac{x\left(105ax^5(4d+3ex^2)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(144e+8c^2(28d+9ex^2)+2c^4(56dx^2+27ex^4)+c^6(84dx^4+45ex^6)\right)}{c^7}+105bx^5(4d+3ex^2)\csc^{-1}\right.}{2520}$$

```
[In] Integrate[x^5*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

[Out] (x*(105*a*x^5*(4*d + 3*e*x^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*(144*e + 8*c^2*(28 *d + 9*e*x^2) + 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45*e*x^6)))/c ^7 + 105*b*x^5*(4*d + 3*e*x^2)*ArcCsc[c*x]))/2520

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

method	result
parts	$a\left(\frac{1}{8}e\,x^{8}+\frac{1}{6}x^{6}d\right)+\frac{b\left(\frac{c^{6}\operatorname{arccsc}(cx)e\,x^{8}}{8}+\frac{\operatorname{arccsc}(cx)d\,x^{6}c^{6}}{6}+\frac{\left(c^{2}x^{2}-1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}+72c^{2}e\right.}{2520c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,x}\right.}{c^{6}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{6}c^{8}d\,x^{6}+\frac{1}{8}e\,c^{8}x^{8}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arccsc}(cx)d\,c^{8}x^{6}}{6}+\frac{\operatorname{arccsc}(cx)e\,c^{8}x^{8}}{8}+\frac{\left(c^{2}x^{2}-1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}+72c^{2}e\,x^{2}+2\right.}{2520\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right.}{c^{2}}}{c^{6}}$
default	$\frac{\frac{a\left(\frac{1}{6}c^{8}d\,x^{6}+\frac{1}{8}e\,c^{8}x^{8}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arccsc}(cx)d\,c^{8}x^{6}}{6}+\frac{\operatorname{arccsc}(cx)e\,c^{8}x^{8}}{8}+\frac{\left(c^{2}x^{2}-1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}+72c^{2}e\,x^{2}+2\right.}{2520\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right.}{c^{2}}}{c^{6}}$

[In] int(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)

[Out] a*(1/8*e*x^8+1/6*x^6*d)+b/c^6*(1/8*c^6*arccsc(c*x)*e*x^8+1/6*arccsc(c*x)*d* x^6*c^6+1/2520/c^3*(c^2*x^2-1)*(45*c^6*e*x^6+84*c^6*d*x^4+54*c^4*e*x^4+112* c^4*d*x^2+72*c^2*e*x^2+224*c^2*d+144*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

$$\int x^5\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{315\,ac^8ex^8+420\,ac^8dx^6+105\,(3\,bc^8ex^8+4\,bc^8dx^6)\operatorname{arccsc}(cx)+(45\,bc^6ex^6+6\,(14\,bc^6d+9\,bc^4e)x^4+22}{2520\,c^8}$$

[In] integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")

[Out] 1/2520*(315*a*c^8*e*x^8 + 420*a*c^8*d*x^6 + 105*(3*b*c^8*e*x^8 + 4*b*c^8*d* x^6)*arccsc(c*x) + (45*b*c^6*e*x^6 + 6*(14*b*c^6*d + 9*b*c^4*e)*x^4 + 224*b *c^2*d + 8*(14*b*c^4*d + 9*b*c^2*e)*x^2 + 144*b*e)*sqrt(c^2*x^2 - 1))/c^8

Sympy [A] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.86

$$\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{adx^6}{6}+\frac{aex^8}{8}+\frac{bdx^6\operatorname{acsc}(cx)}{6}+\frac{bex^8\operatorname{acsc}(cx)}{8} + \frac{bd\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c} + \frac{be\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c}+\frac{6x^4\sqrt{c^2x^2-1}}{35c^3}+\frac{8x^2\sqrt{c^2x^2-1}}{35c^5}+\frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for } |c^2x^2|>1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c}+\frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3}+\frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5}+\frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}$$

```
[In] integrate(x**5*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**6/6 + a*e*x**8/8 + b*d*x**6*acsc(c*x)/6 + b*e*x**8*acsc(c*x)/8 + b*d
*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15
*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt
(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt
(-c**2*x**2 + 1)/(15*c**5), True))/(6*c) + b*e*Piecewise((x**6*sqrt(c**2*x*
*2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3) + 8*x**2*sqrt(c**2*x**
2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7), Abs(c**2*x**2) > 1), (
I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c**2*x**2 + 1)/(35*c**3)
 + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sqrt(-c**2*x**2 + 1)/(35*
c**7), True))/(8*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.93

$$\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{8}aex^8+\frac{1}{6}adx^6 + \frac{1}{90}\left(15\,x^6\operatorname{arccsc}(cx)+\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)bd + \frac{1}{280}\left(35\,x^8\operatorname{arccsc}(cx)+\frac{5\,c^6x^7\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}+21\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+35\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^7}\right.$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*
x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x
^2) + 1))/c^5)*b*d + 1/280*(35*x^8*arccsc(c*x) + (5*c^6*x^7*(-1/(c^2*x^2) +
 1)^(7/2) + 21*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x^2)
+ 1)^(3/2) + 35*x*sqrt(-1/(c^2*x^2) + 1))/c^7)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(168) = 336$.

Time = 0.43 (sec) , antiderivative size = 1244, normalized size of antiderivative = 6.35

$$\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/645120*(315*b*e*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*arcsin(1/(c*x))/c + 31
5*a*e*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8/c + 90*b*e*x^7*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^7/c^2 + 1680*b*d*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x
))/c + 1680*a*d*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 2520*b*e*x^6*(sqrt(-
1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c^3 + 2520*a*e*x^6*(sqrt(-1/(c^2*x^
2) + 1) + 1)^6/c^3 + 672*b*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^2 + 882*b
*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^4 + 10080*b*d*x^4*(sqrt(-1/(c^2*x^2
) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 10080*a*d*x^4*(sqrt(-1/(c^2*x^2) + 1) +
 1)^4/c^3 + 8820*b*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^5
 + 8820*a*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^5 + 5600*b*d*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3/c^4 + 4410*b*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^6
 + 25200*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^5 + 25200
*a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 17640*b*e*x^2*(sqrt(-1/(c^2*x
^2) + 1) + 1)^2*arcsin(1/(c*x))/c^7 + 17640*a*e*x^2*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^2/c^7 + 33600*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 + 22050*b*e*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)/c^8 + 33600*b*d*arcsin(1/(c*x))/c^7 + 33600*a*d/
c^7 + 22050*b*e*arcsin(1/(c*x))/c^9 + 22050*a*e/c^9 - 33600*b*d/(c^8*x*(sqr
t(-1/(c^2*x^2) + 1) + 1)) - 22050*b*e/(c^10*x*(sqrt(-1/(c^2*x^2) + 1) + 1))
 + 25200*b*d*arcsin(1/(c*x))/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 252
00*a*d/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 17640*b*e*arcsin(1/(c*x))
/(c^11*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 17640*a*e/(c^11*x^2*(sqrt(-1/(
c^2*x^2) + 1) + 1)^2) - 5600*b*d/(c^10*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
- 4410*b*e/(c^12*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 10080*b*d*arcsin(1/(
c*x))/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 10080*a*d/(c^11*x^4*(sqrt
(-1/(c^2*x^2) + 1) + 1)^4) + 8820*b*e*arcsin(1/(c*x))/(c^13*x^4*(sqrt(-1/(c
^2*x^2) + 1) + 1)^4) + 8820*a*e/(c^13*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) -
 672*b*d/(c^12*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 882*b*e/(c^14*x^5*(sqr
```

```
t(-1/(c^2*x^2) + 1) + 1)^5) + 1680*b*d*arcsin(1/(c*x))/(c^13*x^6*(sqrt(-1/(
c^2*x^2) + 1) + 1)^6) + 1680*a*d/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6)
+ 2520*b*e*arcsin(1/(c*x))/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 2520
*a*e/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) - 90*b*e/(c^16*x^7*(sqrt(-1/
(c^2*x^2) + 1) + 1)^7) + 315*b*e*arcsin(1/(c*x))/(c^17*x^8*(sqrt(-1/(c^2*x^
2) + 1) + 1)^8) + 315*a*e/(c^17*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^5\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^5*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^5*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.84 $\int x^3(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int x^3\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b(3c^2d+2e)\,x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}}+\frac{b(3c^2d+4e)\,x(-1+c^2x^2)^{3/2}}{36c^5\sqrt{c^2x^2}}+\frac{bex(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)$$

```
[Out] 1/4*d*x^4*(a+b*arccsc(c*x))+1/6*e*x^6*(a+b*arccsc(c*x))+1/36*b*(3*c^2*d+4*e
)*x*(c^2*x^2-1)^(3/2)/c^5/(c^2*x^2)^(1/2)+1/30*b*e*x*(c^2*x^2-1)^(5/2)/c^5/
(c^2*x^2)^(1/2)+1/12*b*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5347, 12, 457, 78}

$$\int x^3\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{bx(c^2x^2-1)^{3/2}\,(3c^2d+4e)}{36c^5\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}(3c^2d+2e)}{12c^5\sqrt{c^2x^2}}+\frac{bex(c^2x^2-1)^{5/2}}{30c^5\sqrt{c^2x^2}}$$

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(3*c^2*d + 2*e)*x*Sqrt[-1 + c^2*x^2])/(12*c^5*Sqrt[c^2*x^2]) + (b*(3*c^2
*d + 4*e)*x*(-1 + c^2*x^2)^(3/2))/(36*c^5*Sqrt[c^2*x^2]) + (b*e*x*(-1 + c^2
*x^2)^(5/2))/(30*c^5*Sqrt[c^2*x^2]) + (d*x^4*(a + b*ArcCsc[c*x]))/4 + (e*x^
6*(a + b*ArcCsc[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^3(3d+2ex^2)}{12\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^3(3d+2ex^2)}{\sqrt{-1+c^2x^2}}\,dx}{12\sqrt{c^2x^2}}$$

$$= \frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\text{Subst}\left(\int\frac{x(3d+2ex)}{\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{24\sqrt{c^2x^2}}$$

$$= \frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)$$
$$+\frac{(bcx)\text{Subst}\left(\int\left(\frac{3c^2d+2e}{c^4\sqrt{-1+c^2x}}+\frac{(3c^2d+4e)\sqrt{-1+c^2x}}{c^4}+\frac{2e(-1+c^2x)^{3/2}}{c^4}\right)\,dx,x,x^2\right)}{24\sqrt{c^2x^2}}$$

$$= \frac{b(3c^2d+2e)\,x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}}+\frac{b(3c^2d+4e)\,x(-1+c^2x^2)^{3/2}}{36c^5\sqrt{c^2x^2}}$$
$$+\frac{bex(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63

$$\int x^3\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{180}x\left(15ax^3\left(3d+2ex^2\right)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}(16e+c^2(30d+8ex^2)+3c^4(5dx^2+2ex^4))}{c^5}\right.$$
$$\left.+15bx^3\left(3d+2ex^2\right)\csc^{-1}(cx)\right)$$

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(15*a*x^3*(3*d + 2*e*x^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*(16*e + c^2*(30*d +
 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*ArcC
sc[c*x]))/180
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

method	result
parts	$a\left(\frac{1}{6}e\,x^{6}+\frac{1}{4}d\,x^{4}\right)+\frac{b\left(\frac{c^{4}\operatorname{arccsc}(cx)e\,x^{6}}{6}+\frac{\operatorname{arccsc}(cx)c^{4}d\,x^{4}}{4}+\frac{\left(c^{2}x^{2}-1\right)\left(6c^{4}e\,x^{4}+15c^{4}d\,x^{2}+8c^{2}e\,x^{2}+30c^{2}d+16e\right)}{180c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,x}\right)}{c^{4}}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^{2}d\left(c^{2}e\,x^{2}+c^{2}d\right)^{2}}{2}-\frac{\left(c^{2}e\,x^{2}+c^{2}d\right)^{3}}{3}\right)}{2c^{2}e^{2}}-\frac{b\,c^{4}\operatorname{arccsc}(cx)d^{3}}{12e^{2}}+\frac{b\,\operatorname{arccsc}(cx)d\,c^{4}x^{4}}{4}+\frac{b\,c^{4}e\,\operatorname{arccsc}(cx)x^{6}}{6}+\frac{b\,c^{3}\sqrt{c^{2}x^{2}-1}\,d^{3}\arctan\left(\right.}{12e^{2}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}}{c^{4}}$
default	$\frac{-\frac{a\left(\frac{c^{2}d\left(c^{2}e\,x^{2}+c^{2}d\right)^{2}}{2}-\frac{\left(c^{2}e\,x^{2}+c^{2}d\right)^{3}}{3}\right)}{2c^{2}e^{2}}-\frac{b\,c^{4}\operatorname{arccsc}(cx)d^{3}}{12e^{2}}+\frac{b\,\operatorname{arccsc}(cx)d\,c^{4}x^{4}}{4}+\frac{b\,c^{4}e\,\operatorname{arccsc}(cx)x^{6}}{6}+\frac{b\,c^{3}\sqrt{c^{2}x^{2}-1}\,d^{3}\arctan\left(\right.}{12e^{2}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}}{c^{4}}$

```
[In] int(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arccsc(c*x)*e*x^6+1/4*arccsc(c*x)*c^
4*d*x^4+1/180/c^3*(c^2*x^2-1)*(6*c^4*e*x^4+15*c^4*d*x^2+8*c^2*e*x^2+30*c^2*
d+16*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.69

$$\int x^{3}\left(d+ex^{2}\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{30\,ac^{6}ex^{6}+45\,ac^{6}dx^{4}+15\,(2\,bc^{6}ex^{6}+3\,bc^{6}dx^{4})\operatorname{arccsc}(cx)+(6\,bc^{4}ex^{4}+30\,bc^{2}d+(15\,bc^{4}d+8\,bc^{2}e)x^{2}+}{180\,c^{6}}$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*e*x^6 + 45*a*c^6*d*x^4 + 15*(2*b*c^6*e*x^6 + 3*b*c^6*d*x^4)
*arccsc(c*x) + (6*b*c^4*e*x^4 + 30*b*c^2*d + (15*b*c^4*d + 8*b*c^2*e)*x^2 +
 16*b*e)*sqrt(c^2*x^2 - 1))/c^6
```

Sympy [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.78

$$\int x^3(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\operatorname{acsc}(cx)}{4}+\frac{bex^6\operatorname{acsc}(cx)}{6}+\frac{bd\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}+\frac{be\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

```
[In] integrate(x**3*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*acsc(c*x)/4 + b*e*x**6*acsc(c*x)/6 + b*d
*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3)
, Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*
x**2 + 1)/(3*c**3), True))/(4*c) + b*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/
(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c*
*5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqr
t(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6
*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93

$$\int x^3(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}aex^6+\frac{1}{4}adx^4+\frac{1}{12}\left(3x^4\operatorname{arccsc}(cx)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd+\frac{1}{90}\left(15x^6\operatorname{arccsc}(cx)+\frac{3c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2
) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d + 1/90*(15*x^6*arccsc(c
*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(
3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(131) = 262$.

Time = 0.34 (sec) , antiderivative size = 900, normalized size of antiderivative = 5.88

$$\int x^3(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/5760*(15*b*e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c + 15*a*
e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 6*b*e*x^5*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^5/c^2 + 90*b*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c +
90*a*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 90*b*e*x^4*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 90*a*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^
4/c^3 + 60*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 50*b*e*x^3*(sqrt(-1
/(c^2*x^2) + 1) + 1)^3/c^4 + 360*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arc
sin(1/(c*x))/c^3 + 360*a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 225*b*e
*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^5 + 225*a*e*x^2*(sqrt
(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 540*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4
+ 300*b*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 + 540*b*d*arcsin(1/(c*x))/c^5
+ 540*a*d/c^5 + 300*b*e*arcsin(1/(c*x))/c^7 + 300*a*e/c^7 - 540*b*d/(c^6*x*
(sqrt(-1/(c^2*x^2) + 1) + 1)) - 300*b*e/(c^8*x*(sqrt(-1/(c^2*x^2) + 1) + 1)
) + 360*b*d*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 360*
a*d/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 225*b*e*arcsin(1/(c*x))/(c^9
*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 225*a*e/(c^9*x^2*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^2) - 60*b*d/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 50*b*e/(c^
10*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 90*b*d*arcsin(1/(c*x))/(c^9*x^4*(s
qrt(-1/(c^2*x^2) + 1) + 1)^4) + 90*a*d/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1
)^4) + 90*b*e*arcsin(1/(c*x))/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 9
0*a*e/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 6*b*e/(c^12*x^5*(sqrt(-1/
(c^2*x^2) + 1) + 1)^5) + 15*b*e*arcsin(1/(c*x))/(c^13*x^6*(sqrt(-1/(c^2*x^2
) + 1) + 1)^6) + 15*a*e/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^3\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.85 $\int x(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 138

$$\int x\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}}+\frac{bex\left(-1+c^2x^2\right)^{3/2}}{12c^3\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{bcd^2x\arctan\left(\sqrt{-1+c^2x^2}\right)}{4e\sqrt{c^2x^2}}$$

```
[Out] 1/4*(e*x^2+d)^2*(a+b*arccsc(c*x))/e+1/12*b*e*x*(c^2*x^2-1)^(3/2)/c^3/(c^2*x
^2)^(1/2)+1/4*b*c*d^2*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/4*b*(
2*c^2*d+e)*x*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5345, 457, 90, 65, 211}

$$\int x\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{bcd^2x\arctan\left(\sqrt{c^2x^2-1}\right)}{4e\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}(2c^2d+e)}{4c^3\sqrt{c^2x^2}}+\frac{bex\left(c^2x^2-1\right)^{3/2}}{12c^3\sqrt{c^2x^2}}$$

```
[In] Int[x*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(2*c^2*d + e)*x*Sqrt[-1 + c^2*x^2])/(4*c^3*Sqrt[c^2*x^2]) + (b*e*x*(-1 +
 c^2*x^2)^(3/2))/(12*c^3*Sqrt[c^2*x^2]) + ((d + e*x^2)^2*(a + b*ArcCsc[c*x]
))/(4*e) + (b*c*d^2*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(4*e*Sqrt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{4e} + \frac{(bcx)\int \frac{(d+ex^2)^2}{x\sqrt{-1+c^2x^2}}\,dx}{4e\sqrt{c^2x^2}}$$

$$= \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{4e} + \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^2}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{8e\sqrt{c^2x^2}}$$

$$= \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{4e} + \frac{(bcx)\text{Subst}\left(\int \left(\frac{e(2c^2d+e)}{c^2\sqrt{-1+c^2x}} + \frac{d^2}{x\sqrt{-1+c^2x}} + \frac{e^2\sqrt{-1+c^2x}}{c^2}\right)\,dx, x, x^2\right)}{8e\sqrt{c^2x^2}}$$

$$= \frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}} + \frac{bex(-1+c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{4e} + \frac{(bcd^2x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{8e\sqrt{c^2x^2}}$$

$$= \frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}} + \frac{bex(-1+c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{4e} + \frac{(bd^2x)\,\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{4ce\sqrt{c^2x^2}}$$

$$= \frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}} + \frac{bex(-1+c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}} + \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{4e} + \frac{bcd^2x\arctan\left(\sqrt{-1+c^2x^2}\right)}{4e\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int x\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{x\left(3ac^3x(2d+ex^2)+b\sqrt{1-\frac{1}{c^2x^2}}(2e+c^2(6d+ex^2))+3bc^3x(2d+ex^2)\csc^{-1}(cx)\right)}{12c^3}$$

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(3*a*c^3*x*(2*d + e*x^2) + b*Sqrt[1 - 1/(c^2*x^2)]*(2*e + c^2*(6*d + e*x
^2)) + 3*b*c^3*x*(2*d + e*x^2)*ArcCsc[c*x]))/(12*c^3)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.57

method	result
parts	$\frac{a(e\,x^2+d)^2}{4e}+\frac{b\,\operatorname{arccsc}(cx)e\,x^4}{4}+\frac{b\,\operatorname{arccsc}(cx)x^2d}{2}+\frac{b\,d^2\operatorname{arccsc}(cx)}{4e}+\frac{b(c^2x^2-1)xe}{12c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\sqrt{c^2x^2-1}\,d^2\arctan\left(\right.}{4ce\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$
derivativedivides	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^2}{4c^2e}+\frac{b\,c^2\operatorname{arccsc}(cx)d^2}{4e}+\frac{b\,\operatorname{arccsc}(cx)d\,c^2x^2}{2}+\frac{b\,c^2e\,\operatorname{arccsc}(cx)x^4}{4}-\frac{bc\sqrt{c^2x^2-1}\,d^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}+\frac{b\left(c^2x^2-1\right)}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^2}$
default	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^2}{4c^2e}+\frac{b\,c^2\operatorname{arccsc}(cx)d^2}{4e}+\frac{b\,\operatorname{arccsc}(cx)d\,c^2x^2}{2}+\frac{b\,c^2e\,\operatorname{arccsc}(cx)x^4}{4}-\frac{bc\sqrt{c^2x^2-1}\,d^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}+\frac{b\left(c^2x^2-1\right)}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^2}$

```
[In] int(x*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*(e*x^2+d)^2/e+1/4*b*arccsc(c*x)*e*x^4+1/2*b*arccsc(c*x)*x^2*d+1/4*b*d
^2*arccsc(c*x)/e+1/12*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x*e-1/4
*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2*arctan(1/(c^2*x^
2-1)^(1/2))+1/2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d+1/6*b/c^5
*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int x\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{3\,ac^4ex^4+6\,ac^4dx^2+3\,(bc^4ex^4+2\,bc^4dx^2)\operatorname{arccsc}(cx)+(bc^2ex^2+6\,bc^2d+2\,be)\sqrt{c^2x^2-1}}{12\,c^4}$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*e*x^4 + 6*a*c^4*d*x^2 + 3*(b*c^4*e*x^4 + 2*b*c^4*d*x^2)*arccs
c(c*x) + (b*c^2*e*x^2 + 6*b*c^2*d + 2*b*e)*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.28

$$\int x(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{adx^2}{2} + \frac{aex^4}{4} + \frac{bdx^2\operatorname{acsc}(cx)}{2} + \frac{bex^4\operatorname{acsc}(cx)}{4} + \frac{bd\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c} + \frac{be\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

```
[In] integrate(x*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*acsc(c*x)/2 + b*e*x**4*acsc(c*x)/4 + b*d
*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2
+ 1)/c, True))/(2*c) + b*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sq
rt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 +
1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.71

$$\int x(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{4}aex^4 + \frac{1}{2}adx^2 + \frac{1}{2}\left(x^2\operatorname{arccsc}(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd + \frac{1}{12}\left(3x^4\operatorname{arccsc}(cx) + \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)be$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)
/c)*b*d + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x
*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(118) = 236$.

Time = 0.33 (sec) , antiderivative size = 556, normalized size of antiderivative = 4.03

$$\int x(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= \frac{1}{192}\left(\frac{3\,bex^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{3\,aex^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c}+\frac{2\,bex^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c^2}+\right.$$

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a*e*x
^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 2*b*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1
)^3/c^2 + 24*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + 24*
a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c + 12*b*e*x^2*(sqrt(-1/(c^2*x^2) +
1) + 1)^2*arcsin(1/(c*x))/c^3 + 12*a*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c
^3 + 48*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 18*b*e*x*(sqrt(-1/(c^2*x^2
) + 1) + 1)/c^4 + 48*b*d*arcsin(1/(c*x))/c^3 + 48*a*d/c^3 + 18*b*e*arcsin(1
/(c*x))/c^5 + 18*a*e/c^5 - 48*b*d/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 18
*b*e/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 24*b*d*arcsin(1/(c*x))/(c^5*x^2
*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 24*a*d/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^2) + 12*b*e*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) +
 12*a*e/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 2*b*e/(c^8*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3) + 3*b*e*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^4) + 3*a*e/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\int x\left(e\,x^2+d\right)\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.86 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x}\,dx = \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right) - bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{1}{2}ibd\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

```
[Out] 1/2*I*b*d*arccsc(c*x)^2+1/2*e*x^2*(a+b*arccsc(c*x))-b*d*arccsc(c*x)*ln(1-(I
/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*d*arccsc(c*x)*ln(1/x)-d*(a+b*arccsc(c*x))*ln
(1/x)+1/2*I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+1/2*b*e*x*(1-1/c^2
/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules

used = {5349, 14, 4815, 6874, 270, 2363, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x}\,dx = -d\log\left(\frac{1}{x}\right)(a+b\csc^{-1}(cx)) + \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) + \frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c} + \frac{1}{2}ibd\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right) + \frac{1}{2}ibd\csc^{-1}(cx)^2 - bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + bd\log\left(\frac{1}{x}\right)\csc^{-1}(cx)$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] (b*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c) + (I/2)*b*d*ArcCsc[c*x]^2 + (e*x^2*(a +
 b*ArcCsc[c*x]))/2 - b*d*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + b*d*A
rcCsc[c*x]*Log[x^(-1)] - d*(a + b*ArcCsc[c*x])*Log[x^(-1)] + (I/2)*b*d*Poly
Log[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4815

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$=\frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right)-d\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{b\text{Subst}\left(\int\frac{-\frac{e}{2x^2}+d\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right)-d\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{b\text{Subst}\left(\int\left(-\frac{e}{2x^2\sqrt{1-\frac{x^2}{c^2}}}+\frac{d\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\right)\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right)-d\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{(bd)\text{Subst}\left(\int\frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{c}-\frac{(be)\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2c}$$

$$=\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right)+bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right)-d\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-(bd)\text{Subst}\left(\int\frac{\arcsin\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right)+bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right)-d\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-(bd)\text{Subst}\left(\int x\cot(x)\,dx,x,\csc^{-1}(cx)\right)$$

$$=\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}ibd\csc^{-1}(cx)^2+\frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right)+bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right)-d\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+(2ibd)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx,x,\csc^{-1}(cx)\right)$$

$$=\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}ibd\csc^{-1}(cx)^2+\frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right)-bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right)-d\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+(bd)\text{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx,x,\csc^{-1}(cx)\right)$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) - bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) - \frac{1}{2}(ibd)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2(a+b\csc^{-1}(cx)) - bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{1}{2}ibd\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int\frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx = \frac{1}{2}aex^2 + \frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c} + \frac{1}{2}bex^2\csc^{-1}(cx) - bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + ad\log(x) + \frac{1}{2}ibd\left(\csc^{-1}(cx)^2 + \text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\right)$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] (a*e*x^2)/2 + (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*e*x^2*ArcCs
c[c*x])/2 - b*d*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + a*d*Log[x] + (
I/2)*b*d*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.53

method	result
parts	$\frac{ae\,x^2}{2}+ad\ln(x)+b\left(\frac{i\,\mathrm{arccsc}(cx)^2 d}{2}+\frac{e\left(c^2x^2\,\mathrm{arccsc}(cx)+xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2c^2}-d\,\mathrm{arccsc}(cx)\ln\left(1+\right.\right.$
derivativedivides	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{ic^2 d\,\mathrm{arccsc}(cx)^2}{2}+\frac{e\left(c^2x^2\,\mathrm{arccsc}(cx)+xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2}-\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)c^2 d\,\mathrm{arccsc}(cx\right.}{}$
default	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{ic^2 d\,\mathrm{arccsc}(cx)^2}{2}+\frac{e\left(c^2x^2\,\mathrm{arccsc}(cx)+xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2}-\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)c^2 d\,\mathrm{arccsc}(cx\right.}{}$

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x,x,method=_RETURNVERBOSE)
[Out] 1/2*a*e*x^2+a*d*ln(x)+b*(1/2*I*arccsc(c*x)^2*d+1/2*e*(c^2*x^2*arccsc(c*x)+x
*c*((c^2*x^2-1)/c^2/x^2)^(1/2)-I)/c^2-d*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2
)^(1/2))-d*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*d*polylog(2,-I/c/x
-(1-1/c^2/x^2)^(1/2))+I*d*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x}\,dx=\int \frac{(ex^2+d)(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x}\,dx=\int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)}{x}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x,x)
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + a*d*log(x) + 1/4*(2*I*b*c^2*d*log(-c*x + 1)*log(x) + 2*I*b*c^
2*d*log(x)^2 + 2*I*b*c^2*d*dilog(c*x) + 2*I*b*c^2*d*dilog(-c*x) + 2*(b*c^2*
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^2*log(c))*e*x^2 - I*(b*e*(l
og(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 8*b*d*integrate(1/2*log(x)/(c^2*x^3 -
 x), x))*c^2 + 4*c^2*integrate(1/2*(b*e*x^2 + 2*b*d*log(x))*sqrt(c*x + 1)*s
qrt(c*x - 1)/(c^2*x^3 - x), x) + I*b*e*log(c*x - 1) + (-I*b*c^2*e*x^2 - 2*I
*b*c^2*d*log(x))*log(c^2*x^2) + (2*I*b*c^2*d*log(x) + I*b*e)*log(c*x + 1) -
 2*(-I*b*c^2*e*x^2 - 2*(b*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b
*c^2*log(c))*d)*log(x))/c^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expansi
on Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx = \frac{a\,e\,x^2}{2} - a\,d\,\ln\left(\frac{1}{x}\right) - b\,d\,\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right) + \frac{b\,e\,x\,\left(\sqrt{1-\frac{1}{c^2\,x^2}}+c\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{2\,c} + \frac{b\,d\,\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,1\mathrm{i}}{2} + \frac{b\,d\,{\mathrm{asin}\left(\frac{1}{c\,x}\right)}^2\,1\mathrm{i}}{2}$$

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x,x)
```

```
[Out] (b*d*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 - a*d*log(1/x) + (b*d*asin(1/(
c*x))^2*1i)/2 + (a*e*x^2)/2 - b*d*log(1 - exp(asin(1/(c*x))*2i))*asin(1/(c*
x)) + (b*e*x*((1 - 1/(c^2*x^2))^(1/2) + c*x*asin(1/(c*x))))/(2*c)
```

3.87 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^3}\,dx = -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2d\csc^{-1}(cx)+\frac{1}{2}ibe\csc^{-1}(cx)^2 -\frac{d(a+b\csc^{-1}(cx))}{2x^2}-be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) +be\csc^{-1}(cx)\log\left(\frac{1}{x}\right)-e(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) +\frac{1}{2}ibe\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

```
[Out] 1/4*b*c^2*d*arccsc(c*x)+1/2*I*b*e*arccsc(c*x)^2-1/2*d*(a+b*arccsc(c*x))/x^2
-b*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*e*arccsc(c*x)*ln(1/x
)-e*(a+b*arccsc(c*x))*ln(1/x)+1/2*I*b*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2
))^2)-1/4*b*c*d*(1-1/c^2/x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules

used = {5349, 14, 4815, 12, 6874, 327, 222, 2363, 4721, 3798, 2221, 2317, 2438}

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^3}\,dx = -\frac{d(a+b\csc^{-1}(cx))}{2x^2} - e\log\left(\frac{1}{x}\right)(a+b\csc^{-1}(cx)) - \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + be\log\left(\frac{1}{x}\right)\csc^{-1}(cx)$$

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] -1/4*(b*c*d*Sqrt[1 - 1/(c^2*x^2)])/x + (b*c^2*d*ArcCsc[c*x])/4 + (I/2)*b*e*
ArcCsc[c*x]^2 - (d*(a + b*ArcCsc[c*x]))/(2*x^2) - b*e*ArcCsc[c*x]*Log[1 - E
^((2*I)*ArcCsc[c*x])] + b*e*ArcCsc[c*x]*Log[x^(-1)] - e*(a + b*ArcCsc[c*x])
*Log[x^(-1)] + (I/2)*b*e*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4815

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
```

```
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{2x^2} - e\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{dx^2+2e\log(x)}{2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{2x^2} - e\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{dx^2+2e\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{2x^2} - e\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \left(\frac{dx^2}{\sqrt{1-\frac{x^2}{c^2}}} + \frac{2e\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\right)dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{2x^2} - e\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c} + \frac{(be)\text{Subst}\left(\int \frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{d(a+b\csc^{-1}(cx))}{2x^2} + be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{1}{4}(bcd)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right) - (be)\text{Subst}\left(\int \frac{\arcsin\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{d(a+b\csc^{-1}(cx))}{2x^2} + be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - (be)\text{Subst}\left(\int x\cot(x)\,dx, x, \csc^{-1}(cx)\right)$$

$$
\begin{aligned}
&= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\csc^{-1}(cx))}{2x^2} \\
&\quad + be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) \\
&\quad + (2ibe)\text{Subst}\left(\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right) \\
&= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\csc^{-1}(cx))}{2x^2} \\
&\quad - be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - e\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + (be)\text{Subst}\left(\int \log\left(1-e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right) \\
&= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\csc^{-1}(cx))}{2x^2} \\
&\quad - be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - e\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) - \frac{1}{2}(ibe)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right) \\
&= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\csc^{-1}(cx))}{2x^2} \\
&\quad - be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - e\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{1}{2}ibe\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$
\begin{aligned}
\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx &= -\frac{ad}{2x^2} - \frac{bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x} \\
&\quad - \frac{bd\csc^{-1}(cx)}{2x^2} + \frac{1}{4}bc^2d\arcsin\left(\frac{1}{cx}\right) \\
&\quad - be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + ae\log(x) \\
&\quad + \frac{1}{2}ibe\left(\csc^{-1}(cx)^2 + \text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\right)
\end{aligned}
$$

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^3,x]
```

[Out] -1/2*(a*d)/x^2 - (b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*d*ArcCsc[c*x])/(2*x^2) + (b*c^2*d*ArcSin[1/(c*x)])/4 - b*e*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + a*e*Log[x] + (I/2)*b*e*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])])

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.39

method	result
parts	$-\frac{ad}{2x^2}+ae\ln(x)+b\,c^2\left(\frac{i\,\mathrm{arccsc}(cx)^2 e}{2c^2}-\frac{e\,\mathrm{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}+\frac{ie\,\mathrm{polylog}\left(2,-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}\right.$
derivativedivides	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{b\left(\frac{i\,\mathrm{arccsc}(cx)^2e}{2}-e\,\mathrm{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+ie\,\mathrm{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-e\right.}{}\right.$
default	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{b\left(\frac{i\,\mathrm{arccsc}(cx)^2e}{2}-e\,\mathrm{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+ie\,\mathrm{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-e\right.}{}\right.$

[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*a*d/x^2+a*e*ln(x)+b*c^2*(1/2*I/c^2*arccsc(c*x)^2*e-1/c^2*e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I/c^2*e*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-1/c^2*e*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I/c^2*e*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+1/4*arccsc(c*x)*d*cos(2*arccsc(c*x))-1/8*d*sin(2*arccsc(c*x)))

Fricas [F]

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx=\int \frac{(ex^2+d)(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")

[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))/x^3, x)

Sympy [F]

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)}{x^3}\,dx$$

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**3,x)
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
[Out] (c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) + a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b*e + 1/4*b*d*((c^4*x*sqrt(-
1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(
c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) + a*e*log(x) - 1/2*a*d/x^2
```

Giac [F]

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsc(c*x) + a)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx = -a\,e\,\ln\left(\frac{1}{x}\right)-\frac{a\,d}{2\,x^2}-b\,e\,\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2{}\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right) - \frac{b\,c\,d\,\sqrt{1-\frac{1}{c^2\,x^2}}}{4\,x}-\frac{b\,c^2\,d\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\,\left(\frac{2}{c^2\,x^2}-1\right)}{4} + \frac{b\,e\,\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{2}+\frac{b\,e\,{\mathrm{asin}\left(\frac{1}{c\,x}\right)}^2\,1{}\mathrm{i}}{2}$$

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^3,x)
```

```
[Out] (b*e*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 - a*e*log(1/x) + (b*e*asin(1/(
c*x))^2*1i)/2 - (a*d)/(2*x^2) - b*e*log(1 - exp(asin(1/(c*x))*2i))*asin(1/(
c*x)) - (b*c*d*(1 - 1/(c^2*x^2))^(1/2))/(4*x) - (b*c^2*d*asin(1/(c*x))*(2/(
c^2*x^2) - 1))/4
```

3.88 $\int x^2(d+ex^2)^2 (a+b\csc^{-1}(cx)) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 252

$$\int x^2(d+ex^2)^2 (a+b\csc^{-1}(cx)) \, dx = \frac{b(280c^4d^2+252c^2de+75e^2)\, x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} + \frac{be(84c^2d+25e)\, x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{3}d^2x^3(a+b\csc^{-1}(cx)) + \frac{2}{5}dex^5(a+b\csc^{-1}(cx)) + \frac{1}{7}e^2x^7(a+b\csc^{-1}(cx)) + \frac{b(280c^4d^2+252c^2de+75e^2)\, x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{1680c^6\sqrt{c^2x^2}}$$

```
[Out] 1/3*d^2*x^3*(a+b*arccsc(c*x))+2/5*d*e*x^5*(a+b*arccsc(c*x))+1/7*e^2*x^7*(a+
b*arccsc(c*x))+1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*x*arctanh(c*x/(c^2
*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)+1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2
)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)+1/840*b*e*(84*c^2*d+25*e)*x^4*(
c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/42*b*e^2*x^6*(c^2*x^2-1)^(1/2)/c/(c^
2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {276, 5347, 12, 1281, 470, 327, 223, 212}

$$\begin{aligned}\int x^2\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx = {} & \frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)\\ & +\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)\\ & +\frac{bx\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(280c^4d^2+252c^2de+75e^2\right)}{1680c^6\sqrt{c^2x^2}}\\ & +\frac{be^2x^6\sqrt{c^2x^2-1}}{42c\sqrt{c^2x^2}}+\frac{bex^4\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{840c^3\sqrt{c^2x^2}}\\ & +\frac{bx^2\sqrt{c^2x^2-1}\left(280c^4d^2+252c^2de+75e^2\right)}{1680c^5\sqrt{c^2x^2}}\end{aligned}$$

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x^2*Sqrt[-1 + c^2*x^2])/(1680*c^5*S
qrt[c^2*x^2]) + (b*e*(84*c^2*d + 25*e)*x^4*Sqrt[-1 + c^2*x^2])/(840*c^3*Sqr
t[c^2*x^2]) + (b*e^2*x^6*Sqrt[-1 + c^2*x^2])/(42*c*Sqrt[c^2*x^2]) + (d^2*x^
3*(a + b*ArcCsc[c*x]))/3 + (2*d*e*x^5*(a + b*ArcCsc[c*x]))/5 + (e^2*x^7*(a
+ b*ArcCsc[c*x]))/7 + (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x*ArcTanh[(c*
x)/Sqrt[-1 + c^2*x^2]])/(1680*c^6*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^2\left(35d^2+42dex^2+15e^2x^4\right)}{105\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^2(35d^2+42dex^2+15e^2x^4)}{\sqrt{-1+c^2x^2}}\,dx}{105\sqrt{c^2x^2}}$$

$$= \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)+\frac{(bx)\int\frac{x^2(210c^2d^2+3e(84c^2d+25e)x^2)}{\sqrt{-1+c^2x^2}}\,dx}{630c\sqrt{c^2x^2}}$$

$$= \frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right) + \frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right) + -\frac{(b(-840c^4d^2-9e(84c^2d+25e))\,x)\int\frac{x^2}{\sqrt{-1+c^2x^2}}\,dx}{2520c^3\sqrt{c^2x^2}}$$

$$= \frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}+\frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)+-\frac{(b(-840c^4d^2-9e(84c^2d+25e))\,x)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{5040c^5\sqrt{c^2x^2}}$$

$$= \frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}+\frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right) + \frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)+ -\frac{(b(-840c^4d^2-9e(84c^2d+25e))\,x)\,\mathrm{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{5040c^5\sqrt{c^2x^2}}$$

$$= \frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}+\frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)+\frac{b(280c^4d^2+252c^2de+75e^2)\,x\,\mathrm{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{1680c^6\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.73

$$\int x^2(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= \frac{c^2x^2\Big(16ac^5x(35d^2+42dex^2+15e^2x^4)+b\sqrt{1-\frac{1}{c^2x^2}}(75e^2+2c^2e(126d+25ex^2)+8c^4(35d^2+21dex^2+\cdots}{\cdots}$$

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (c^2*x^2*(16*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[1 - 1/(c^2
*x^2)]*(75*e^2 + 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2 +
5*e^2*x^4))) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCsc[c*x]
+ b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x]
)/(1680*c^7)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(222) = 444.

Time = 0.97 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.82

method	result
parts	$a\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}x^3d^2\right)+\frac{b\,\mathrm{arccsc}(cx)e^2x^7}{7}+\frac{2b\,\mathrm{arccsc}(cx)de\,x^5}{5}+\frac{b\,\mathrm{arccsc}(cx)d^2x^3}{3}+\cdots$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arccsc}(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\,\mathrm{arccsc}(cx)de\,x^5}{5}+\frac{b\,c^3\,\mathrm{arccsc}(cx)e^2x^7}{7}+\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\cdots}{\cdots}$
default	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arccsc}(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\,\mathrm{arccsc}(cx)de\,x^5}{5}+\frac{b\,c^3\,\mathrm{arccsc}(cx)e^2x^7}{7}+\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\cdots}{\cdots}$

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*x^3*d^2)+1/7*b*arccsc(c*x)*e^2*x^7+2/5*b*arc
csc(c*x)*d*e*x^5+1/3*b*arccsc(c*x)*d^2*x^3+1/42*b/c^3*(c^2*x^2-1)/((c^2*x^2
-1)/c^2/x^2)^(1/2)*x^4*e^2+1/10*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/
2)*x^2*d*e+5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e^2+1/6*
b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2+3/20*b/c^5*(c^2*x^2-1)/((
c^2*x^2-1)/c^2/x^2)^(1/2)*d*e+1/6*b/c^4*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/
x^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))+5/112*b/c^7*(c^2*x^2-1)/((c^2*x^
2-1)/c^2/x^2)^(1/2)*e^2+3/20*b/c^6*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/x*d*e*ln(c*x+(c^2*x^2-1)^(1/2))+5/112*b/c^8*(c^2*x^2-1)^(1/2)/((c^2*x
^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.08

$$\int x^2(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{240\,ac^7e^2x^7+672\,ac^7dex^5+560\,ac^7d^2x^3+16\,(15\,bc^7e^2x^7+42\,bc^7dex^5+35\,bc^7d^2x^3-35\,bc^7d^2-42\,bc^7de$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 + 16*(15*
b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b*c
^7*d*e - 15*b*c^7*e^2)*arccsc(c*x) - 32*(35*b*c^7*d^2 + 42*b*c^7*d*e + 15*b
*c^7*e^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (280*b*c^4*d^2 + 252*b*c^2*d*e
 + 75*b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + (40*b*c^5*e^2*x^5 + 2*(84*b*c^
5*d*e + 25*b*c^3*e^2)*x^3 + (280*b*c^5*d^2 + 252*b*c^3*d*e + 75*b*c*e^2)*x)
*sqrt(c^2*x^2 - 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 11.86 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.15

$$\int x^2(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{ad^2x^3}{3}+\frac{2adex^5}{5}+\frac{ae^2x^7}{7}+\frac{bd^2x^3\operatorname{acsc}(cx)}{3}+\frac{2bdex^5\operatorname{acsc}(cx)}{5}+\frac{be^2x^7\operatorname{acsc}(cx)}{7}$$
$$+\frac{bd^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$
$$+\frac{2bde\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$
$$+\frac{be^2\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*acsc(c*x)/3 +
2*b*d*e*x**5*acsc(c*x)/5 + b*e**2*x**7*acsc(c*x)/7 + b*d**2*Piecewise((x*sq
```

```
rt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x
**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)
/(2*c**2), True))/(3*c) + 2*b*d*e*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1))
 + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*ac
osh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1))
 - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1))
- 3*I*asin(c*x)/(8*c**4), True))/(5*c) + b*e**2*Piecewise((c*x**7/(6*sqrt(c
**2*x**2 - 1)) + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**
2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6),
Abs(c**2*x**2) > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqr
t(-c**2*x**2 + 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c*
*5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.60

$$\int x^2\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{7}\,ae^2x^7+\frac{2}{5}\,adex^5+\frac{1}{3}\,ad^2x^3$$

$$+\frac{1}{12}\left(4\,x^3\operatorname{arccsc}\left(cx\right)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd^2$$

$$+\frac{1}{40}\left(16\,x^5\operatorname{arccsc}\left(cx\right)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bde$$

$$+\frac{1}{672}\left(96\,x^7\operatorname{arccsc}\left(cx\right)+\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\,\log\left(\sqrt{-\ldots\right.}}{c^6}}{c}\right.$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/12*(4*x^3*arccsc(c*x) + (
2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d^2 + 1/40*(1
6*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) +
1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(
```

```
-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d*
e + 1/672*(96*x^7*arccsc(c*x) + (2*(15*(-1/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c
^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) - 1)^3 +
3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*log(sqrt(-1
/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^6)/c)*b*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. $2(222) = 444$.

Time = 5.13 (sec) , antiderivative size = 1579, normalized size of antiderivative = 6.27

$$\int x^2(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*b*e^2*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))/c + 15
*a*e^2*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7/c + 5*b*e^2*x^6*(sqrt(-1/(c^2*x^2
) + 1) + 1)^6/c^2 + 168*b*d*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(
c*x))/c + 168*a*d*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 105*b*e^2*x^5*(s
qrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c^3 + 105*a*e^2*x^5*(sqrt(-1/(
c^2*x^2) + 1) + 1)^5/c^3 + 84*b*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^2
+ 560*b*d^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + 560*a*d^
2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + 45*b*e^2*x^4*(sqrt(-1/(c^2*x^2) +
1) + 1)^4/c^4 + 840*b*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x)
)/c^3 + 840*a*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^3 + 560*b*d^2*x^2*(s
qrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 315*b*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1) +
1)^3*arcsin(1/(c*x))/c^5 + 315*a*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^5
 + 672*b*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^4 + 1680*b*d^2*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 1680*a*d^2*x*(sqrt(-1/(c^2*x^2)
+ 1) + 1)/c^3 + 225*b*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^6 + 1680*b*d
*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 1680*a*d*e*x*(sqrt(
-1/(c^2*x^2) + 1) + 1)/c^5 + 2240*b*d^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4
 - 2240*b*d^2*log(1/(abs(c)*abs(x)))/c^4 + 525*b*e^2*x*(sqrt(-1/(c^2*x^2) +
 1) + 1)*arcsin(1/(c*x))/c^7 + 525*a*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^7
 + 2016*b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 2016*b*d*e*log(1/(abs(c
)*abs(x)))/c^6 + 1680*b*d^2*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1)
+ 1)) + 1680*a*d^2/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 600*b*e^2*log(sqr
t(-1/(c^2*x^2) + 1) + 1)/c^8 - 600*b*e^2*log(1/(abs(c)*abs(x)))/c^8 + 1680*
b*d*e*arcsin(1/(c*x))/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1680*a*d*e/(c^
7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 560*b*d^2/(c^6*x^2*(sqrt(-1/(c^2*x^2) +
 1) + 1)^2) + 525*b*e^2*arcsin(1/(c*x))/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)
) + 525*a*e^2/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 672*b*d*e/(c^8*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2) + 560*b*d^2*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3) + 560*a*d^2/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
```

```
 - 225*b*e^2/(c^10*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 840*b*d*e*arcsin(1
/(c*x))/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 840*a*d*e/(c^9*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 315*b*e^2*arcsin(1/(c*x))/(c^11*x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3) + 315*a*e^2/(c^11*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 - 84*b*d*e/(c^10*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 45*b*e^2/(c^12*x^4*
(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 168*b*d*e*arcsin(1/(c*x))/(c^11*x^5*(sqrt
(-1/(c^2*x^2) + 1) + 1)^5) + 168*a*d*e/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) +
1)^5) + 105*b*e^2*arcsin(1/(c*x))/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)
 + 105*a*e^2/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 5*b*e^2/(c^14*x^6*
(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 15*b*e^2*arcsin(1/(c*x))/(c^15*x^7*(sqrt(
-1/(c^2*x^2) + 1) + 1)^7) + 15*a*e^2/(c^15*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)
^7))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

3.89 $\int (d+ex^2)^2 (a+b\csc^{-1}(cx)) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 191

$$\int (d+ex^2)^2 (a+b\csc^{-1}(cx)) \, dx = \frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} + \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x\left(a+b\csc^{-1}(cx)\right) + \frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right) + \frac{b(120c^4d^2+40c^2de+9e^2)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}$$

```
[Out] d^2*x*(a+b*arccsc(c*x))+2/3*d*e*x^3*(a+b*arccsc(c*x))+1/5*e^2*x^5*(a+b*arcc
sc(c*x))+1/120*b*(120*c^4*d^2+40*c^2*d*e+9*e^2)*x*arctanh(c*x/(c^2*x^2-1)^(
1/2))/c^4/(c^2*x^2)^(1/2)+1/120*b*e*(40*c^2*d+9*e)*x^2*(c^2*x^2-1)^(1/2)/c^
3/(c^2*x^2)^(1/2)+1/20*b*e^2*x^4*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used

= {200, 5337, 12, 1173, 396, 223, 212}

$$\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)+\frac{bx\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(120c^4d^2+40c^2de+9e^2\right)}{120c^4\sqrt{c^2x^2}}+\frac{be^2x^4\sqrt{c^2x^2-1}}{20c\sqrt{c^2x^2}}+\frac{bex^2\sqrt{c^2x^2-1}(40c^2d+9e)}{120c^3\sqrt{c^2x^2}}$$

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(40*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(120*c^3*Sqrt[c^2*x^2]) + (b*
e^2*x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + d^2*x*(a + b*ArcCsc[c*x]
) + (2*d*e*x^3*(a + b*ArcCsc[c*x]))/3 + (e^2*x^5*(a + b*ArcCsc[c*x]))/5 + (
b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(
120*c^4*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 5337

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right) \\
&\quad+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{15d^2+10dex^2+3e^2x^4}{15\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right) \\
&\quad+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{15d^2+10dex^2+3e^2x^4}{\sqrt{-1+c^2x^2}}\,dx}{15\sqrt{c^2x^2}} \\
&= \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right) \\
&\quad+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)+\frac{(bx)\int\frac{60c^2d^2+e\left(40c^2d+9e\right)x^2}{\sqrt{-1+c^2x^2}}\,dx}{60c\sqrt{c^2x^2}} \\
&= \frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}(cx)\right) \\
&\quad+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right) \\
&\quad-\frac{\left(b\left(-120c^4d^2-e(40c^2d+9e)\right)x\right)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{120c^3\sqrt{c^2x^2}}
\end{aligned}
$$

$$= \frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} + \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x\left(a+b\csc^{-1}(cx)\right)$$
$$+ \frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)$$
$$- \frac{\left(b(-120c^4d^2-e(40c^2d+9e))\,x\right)\text{Subst}\left(\int \frac{1}{1-c^2x^2}\,dx, x, \frac{x}{\sqrt{-1+c^2x^2}}\right)}{120c^3\sqrt{c^2x^2}}$$
$$= \frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} + \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x\left(a+b\csc^{-1}(cx)\right)$$
$$+ \frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)$$
$$+ \frac{b(120c^4d^2+40c^2de+9e^2)\,x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int \left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= \frac{c^2x\left(8ac^3(15d^2+10dex^2+3e^2x^4)+be\sqrt{1-\frac{1}{c^2x^2}}x(9e+c^2(40d+6ex^2))\right)+8bc^5x(15d^2+10dex^2+3e^2x}{120c^5}$$

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (c^2*x*(8*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 - 1/(c^2*x^2
)]*x*(9*e + c^2*(40*d + 6*e*x^2))) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2
*x^4)*ArcCsc[c*x] + b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Log[(1 + Sqrt[1 -
1/(c^2*x^2)])*x])/(120*c^5)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.77

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b\,\text{arccsc}(cx)e^2x^5}{5}+\frac{2b\,\text{arccsc}(cx)de\,x^3}{3}+b\,\text{arccsc}\,(cx)\,d^2x+\frac{b(c^2x^2-1}{20c^3\sqrt{c^2}}$
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\text{arccsc}(cx)d^2cx+\frac{2bc\,\text{arccsc}(cx)de\,x^3}{3}+\frac{bc\,\text{arccsc}(cx)e^2x^5}{5}+\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\text{arccsc}(cx)d^2cx+\frac{2bc\,\text{arccsc}(cx)de\,x^3}{3}+\frac{bc\,\text{arccsc}(cx)e^2x^5}{5}+\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c}$

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+1/5*b*arccsc(c*x)*e^2*x^5+2/3*b*arccsc(c*
x)*d*e*x^3+b*arccsc(c*x)*d^2*x+1/20*b/c^3*(c^2*x^2-1)*x^2/((c^2*x^2-1)/c^2/
x^2)^(1/2)*e^2+1/3*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e+b/c^2*
(c^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2*ln(c*x+(c^2*x^2-1)^(1/2
))+3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2+1/3*b/c^4*(c^2*x^
2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2))+3/40
*b/c^6*(c^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*ln(c*x+(c^2*x^2-
1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.24

$$\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= \frac{24\,ac^5e^2x^5 + 80\,ac^5dex^3 + 120\,ac^5d^2x + 8\,(3\,bc^5e^2x^5 + 10\,bc^5dex^3 + 15\,bc^5d^2x - 15\,bc^5d^2 - 10\,bc^5de - 3\,bc^5}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x + 8*(3*b*c^5*e
^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e -
3*b*c^5*e^2)*arccsc(c*x) - 16*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*a
rctan(-c*x + sqrt(c^2*x^2 - 1)) - (120*b*c^4*d^2 + 40*b*c^2*d*e + 9*b*e^2)*
log(-c*x + sqrt(c^2*x^2 - 1)) + (6*b*c^3*e^2*x^3 + (40*b*c^3*d*e + 9*b*c*e^
2)*x)*sqrt(c^2*x^2 - 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.86

$$\int \left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\operatorname{acsc}(cx) + \frac{2bdex^3\operatorname{acsc}(cx)}{3} + \frac{be^2x^5\operatorname{acsc}(cx)}{5} + \frac{bd^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$
$$+ \frac{2bde\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$
$$+ \frac{be^2\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*acsc(c*x) + 2*b*d*e*x*
*3*acsc(c*x)/3 + b*e**2*x**5*acsc(c*x)/5 + b*d**2*Piecewise((acosh(c*x), Ab
s(c**2*x**2) > 1), (-I*asin(c*x), True))/c + 2*b*d*e*Piecewise((x*sqrt(c**2
*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*
sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**
2), True))/(3*c) + b*e**2*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/
(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)
/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**
3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*as
in(c*x)/(8*c**4), True))/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.55

$$\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right) dx = \frac{1}{5}ae^2x^5 + \frac{2}{3}adex^3 + \frac{1}{6}\left(4x^3\operatorname{arccsc}(cx) + \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde + \frac{1}{80}\left(16x^5\operatorname{arccsc}(cx) - \frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4} - \frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4} + \frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be^2 + ad^2x + \frac{\left(2cx\operatorname{arccsc}(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2c}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/6*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^
2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2
 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/80*(16*x^5*arccsc(c*x)
 - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*
x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1)
 + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e^2 + a*d^2*x + 1/2
*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x
^2) + 1) + 1))*b*d^2/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. 2(169) = 338.

Time = 3.50 (sec) , antiderivative size = 1033, normalized size of antiderivative = 5.41

$$\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right) dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b*e^2*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c + 6*a*e
^2*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 3*b*e^2*x^4*(sqrt(-1/(c^2*x^2) +
```

```
1) + 1)^4/c^2 + 80*b*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))
/c + 80*a*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + 30*b*e^2*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^3 + 30*a*e^2*x^3*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^3/c^3 + 80*b*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 480*b*
d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c + 480*a*d^2*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)/c + 24*b*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^4 +
240*b*d*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 240*a*d*e*x*
(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 960*b*d^2*log(sqrt(-1/(c^2*x^2) + 1) + 1
)/c^2 - 960*b*d^2*log(1/(abs(c)*abs(x)))/c^2 + 60*b*e^2*x*(sqrt(-1/(c^2*x^2
) + 1) + 1)*arcsin(1/(c*x))/c^5 + 60*a*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c
^5 + 320*b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 320*b*d*e*log(1/(abs(c
)*abs(x)))/c^4 + 480*b*d^2*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) +
 1)) + 480*a*d^2/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 72*b*e^2*log(sqrt(-
1/(c^2*x^2) + 1) + 1)/c^6 - 72*b*e^2*log(1/(abs(c)*abs(x)))/c^6 + 240*b*d*e
*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 240*a*d*e/(c^5*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)) + 60*b*e^2*arcsin(1/(c*x))/(c^7*x*(sqrt(-1/(c^2
*x^2) + 1) + 1)) + 60*a*e^2/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 80*b*d*e
/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 24*b*e^2/(c^8*x^2*(sqrt(-1/(c^2
*x^2) + 1) + 1)^2) + 80*b*d*e*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3) + 80*a*d*e/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 30*b*e^2*
arcsin(1/(c*x))/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 30*a*e^2/(c^9*x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 3*b*e^2/(c^10*x^4*(sqrt(-1/(c^2*x^2) +
1) + 1)^4) + 6*b*e^2*arcsin(1/(c*x))/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)
^5) + 6*a*e^2/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))*c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
[Out] int((d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

3.90 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^2}\,dx = & -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}+\frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}\\ & -\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex\left(a+b\csc^{-1}(cx)\right)\\ & +\frac{1}{3}e^2x^3\left(a+b\csc^{-1}(cx)\right)\\ & +\frac{be(12c^2d+e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}\end{aligned}$$

```
[Out] -d^2*(a+b*arccsc(c*x))/x+2*d*e*x*(a+b*arccsc(c*x))+1/3*e^2*x^3*(a+b*arccsc(
c*x))+1/6*b*e*(12*c^2*d+e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)^(
1/2)-b*c*d^2*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/6*b*e^2*x^2*(c^2*x^2-1)^(1
/2)/c/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{276, 5347, 12, 1279, 396, 223, 212\}$

$$\int \frac{\left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{x} + 2dex\left(a+b\csc^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\csc^{-1}(cx)\right) + \frac{bex\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(12c^2d+e\right)}{6c^2\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{c^2x^2-1}}{6c\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] -((b*c*d^2*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*x^2]) + (b*e^2*x^2*Sqrt[-1 + c^2*x^
2])/(6*c*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcCsc[c*x]))/x + 2*d*e*x*(a + b*ArcC
sc[c*x]) + (e^2*x^3*(a + b*ArcCsc[c*x]))/3 + (b*e*(12*c^2*d + e)*x*ArcTanh[
(c*x)/Sqrt[-1 + c^2*x^2]])/(6*c^2*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{1}{3}e^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{-3d^2+6dex^2+e^2x^4}{3x^2\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{1}{3}e^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{-3d^2+6dex^2+e^2x^4}{x^2\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}}\\
&= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex\left(a+b\csc^{-1}(cx)\right)\\
&\quad+\frac{1}{3}e^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{6de+e^2x^2}{\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}}\\
&= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}+\frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{x}\\
&\quad+2dex\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\csc^{-1}(cx)\right)\\
&\quad+-\frac{(b(-12c^2de-e^2)\,x)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{6c\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{x} + 2dex(a+b\csc^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\csc^{-1}(cx)) + -\frac{(b(-12c^2de-e^2)\,x)\,\mathrm{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx, x, \frac{x}{\sqrt{-1+c^2x^2}}\right)}{6c\sqrt{c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{x} + 2dex(a+b\csc^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\csc^{-1}(cx)) + \frac{be(12c^2d+e)\,x\mathrm{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.82

$$\int\frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \frac{c^2\left(b\sqrt{1-\frac{1}{c^2x^2}}x(-6c^2d^2+e^2x^2)+2ac(-3d^2+6dex^2+e^2x^4)\right)+2bc^3(-3d^2+6dex^2+e^2x^4)\csc^{-1}(cx)+}{6c^3x}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] (c^2*(b*Sqrt[1 - 1/(c^2*x^2)]*x*(-6*c^2*d^2 + e^2*x^2) + 2*a*c*(-3*d^2 + 6*
d*e*x^2 + e^2*x^4)) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcCsc[c*x] +
b*e*(12*c^2*d + e)*x*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3*x)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.53

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+\frac{b\,\mathrm{arccsc}(cx)e^2x^3}{3}+2be\,\mathrm{arccsc}\,(cx)\,xd-\frac{b\,\mathrm{arccsc}(cx)d^2}{x}+\frac{b(c^2x^2-1)e^2}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-$
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b\,\mathrm{arccsc}(cx)dex}{c}+\frac{b\,\mathrm{arccsc}(cx)e^2x^3}{3c}-\frac{b\,\mathrm{arccsc}(cx)d^2}{cx}-\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\right.$
default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b\,\mathrm{arccsc}(cx)dex}{c}+\frac{b\,\mathrm{arccsc}(cx)e^2x^3}{3c}-\frac{b\,\mathrm{arccsc}(cx)d^2}{cx}-\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\right.$

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+1/3*b*arccsc(c*x)*e^2*x^3+2*b*e*arccsc(c*x)*x
*d-b*arccsc(c*x)*d^2/x+1/6*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^
2-b/c*(c^2*x^2-1)/x^2/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2+2*b/c^2*(c^2*x^2-1)^(
1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2))+1/6*b/c^4*
e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2
))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2}\,dx = \frac{2\,ac^3e^2x^4 - 6\,bc^4d^2x + 12\,ac^3dex^2 - 6\,ac^3d^2 + 4\,(3\,bc^3d^2 - 6\,bc^3de - bc^3e^2)x\arctan\left(-cx+\sqrt{c^2x^2-1}\right) - (\cdots}{\cdots}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^4 - 6*b*c^4*d^2*x + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 + 4*(
3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) -
 (12*b*c^2*d*e + b*e^2)*x*log(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^3*e^2*x^4
+ 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x
)*arccsc(c*x) - (6*b*c^3*d^2 - b*c*e^2*x^2)*sqrt(c^2*x^2 - 1))/(c^3*x)
```

Sympy [A] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2}\,dx &= -\frac{ad^2}{x} + 2adex + \frac{ae^2x^3}{3} - bcd^2\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd^2\operatorname{acsc}(cx)}{x} + 2bdex\operatorname{acsc}(cx) \\ &+ \frac{be^2x^3\operatorname{acsc}(cx)}{3} + \frac{2bde\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1 \\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} \\ &+ \frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1 \\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}\end{aligned}$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**2,x)
```

```
[Out] -a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 - b*c*d**2*sqrt(1 - 1/(c**2*x**2)) -
b*d**2*acsc(c*x)/x + 2*b*d*e*x*acsc(c*x) + b*e**2*x**3*acsc(c*x)/3 + 2*b*d*
e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + b*e
**2*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*
x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2
+ 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21

$$
\begin{aligned}
&\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2}\,dx \\
&= \frac{1}{3}ae^2x^3 - \left(c\sqrt{-\frac{1}{c^2x^2}+1} + \frac{\operatorname{arccsc}(cx)}{x}\right)bd^2 \\
&\quad + \frac{1}{12}\left(4\,x^3\operatorname{arccsc}(cx) + \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 \\
&\quad + 2\,adex \\
&\quad + \frac{\left(2\,cx\operatorname{arccsc}(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bde}{c} - \frac{ad^2}{x}
\end{aligned}
$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 - (c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d^2 + 1/12*(4*
x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) +
 log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)
/c)*b*e^2 + 2*a*d*e*x + (2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1
) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d*e/c - a*d^2/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2502 vs. $2(145) = 290$.

Time = 2.32 (sec) , antiderivative size = 2502, normalized size of antiderivative = 15.35

$$
\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2}\,dx = \text{Too large to display}
$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/24*(b*e^2*arcsin(1/(c*x))/(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*
x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)) + a*e^2/(c/(x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)) + b*e^2/(c*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sq
rt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*b*d*e*arcsin(1/(c*x))/(x^2*(sqrt(-1/(c^
2*x^2) + 1) + 1)^2*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt
(-1/(c^2*x^2) + 1) + 1)^5))) + 24*a*d*e/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2
*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^5))) - 24*b*c*d^2/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^3*(sqrt(-
1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b
*e^2*arcsin(1/(c*x))/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x^3*(sqrt(
-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*
a*e^2/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x^3*(sqrt(-1/(c^2*x^2) +
1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 48*b*d*e*log(sqrt
(-1/(c^2*x^2) + 1) + 1)/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) - 4
8*b*d*e*log(1/(abs(c)*abs(x)))/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5
))) - 48*b*d^2*arcsin(1/(c*x))/(x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*
(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))
) - 48*a*d^2/(x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b*e^2*log(sq
rt(-1/(c^2*x^2) + 1) + 1)/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^3*(
sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)))
 - 4*b*e^2*log(1/(abs(c)*abs(x)))/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(
c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) +
 1)^5))) + b*e^2/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 48*b*d
*e*arcsin(1/(c*x))/(c^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1
/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 48*a
*d*e/(c^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*b*d^2/(c*x^5*(s
qrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c
*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 6*b*e^2*arcsin(1/(c*x))/(c^4*x^4*(
sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(
c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 6*a*e^2/(c^4*x^4*(sqrt(-1/(c^2*x^
2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/
(c^2*x^2) + 1) + 1)^5))) + 48*b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c^3*x^
5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) +
1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) - 48*b*d*e*log(1/(abs(c)*abs(x))
)/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) +
 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b*e^2*log(sqrt(-1/(
c^2*x^2) + 1) + 1)/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1
/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) - 4*b*
e^2*log(1/(abs(c)*abs(x)))/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*
```

```
(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))
) - b*e^2/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(c^2*x^2
) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*b*d*e*arcs
in(1/(c*x))/(c^4*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x
^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*a*d*e/(c
^4*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^
3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b*e^2*arcsin(1/(c*x))/(
c^6*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)
^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*a*e^2/(c^6*x^6*(sqrt(-
1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*
(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) - b*e^2/(c^7*x^7*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^7*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2
) + 1) + 1)^5))) + b*e^2*arcsin(1/(c*x))/(c^8*x^8*(sqrt(-1/(c^2*x^2) + 1) +
 1)^8*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^5))) + a*e^2/(c^8*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*(c/(x^3*(sq
rt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))))*
c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x^2}\,dx=\int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^2, x)
```

3.91 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x(a+b\csc^{-1}(cx))+\frac{be^2x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

```
[Out] -1/3*d^2*(a+b*arccsc(c*x))/x^3-2*d*e*(a+b*arccsc(c*x))/x+e^2*x*(a+b*arccsc(
c*x))+b*e^2*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/(c^2*x^2)^(1/2)-2/9*b*c*d*(c^2
*d+9*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/9*b*c*d^2*(c^2*x^2-1)^(1/2)/x^2
/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {276, 5347, 12, 1279, 462, 223, 212}

$$\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x(a+b\csc^{-1}(cx))+\frac{be^2x\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}-\frac{2bcd\sqrt{c^2x^2-1}(c^2d+9e)}{9\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] (-2*b*c*d*(c^2*d + 9*e)*Sqrt[-1 + c^2*x^2])/(9*Sqrt[c^2*x^2]) - (b*c*d^2*Sq
rt[-1 + c^2*x^2])/(9*x^2*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcCsc[c*x]))/(3*x^3)
 - (2*d*e*(a + b*ArcCsc[c*x]))/x + e^2*x*(a + b*ArcCsc[c*x]) + (b*e^2*x*Arc
Tanh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[c^2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}\\
&\quad+e^2x\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{-d^2-6dex^2+3e^2x^4}{3x^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}\\
&\quad+e^2x\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{-d^2-6dex^2+3e^2x^4}{x^4\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}}\\
&= -\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}\\
&\quad+e^2x\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{-2d(c^2d+9e)+9e^2x^2}{x^2\sqrt{-1+c^2x^2}}\,dx}{9\sqrt{c^2x^2}}\\
&= -\frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}\\
&\quad-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x\left(a+b\csc^{-1}(cx)\right)+\frac{(bce^2x)\int\frac{1}{\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}\\
&\quad-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}\\
&\quad+e^2x\left(a+b\csc^{-1}(cx)\right)+\frac{(bce^2x)\,\text{Subst}\left(\int\frac{1}{1-c^2x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}\\
&= -\frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}\\
&\quad-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x\left(a+b\csc^{-1}(cx)\right)+\frac{be^2x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x^4}\,dx$$
$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}\,x(d+2c^2dx^2+18ex^2)+3a(d^2+6dex^2-3e^2x^4)}{9x^3}$$
$$-\frac{b(d^2+6dex^2-3e^2x^4)\csc^{-1}(cx)}{3x^3}+\frac{be^2\log\left(\left(1+\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{c}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] -1/9*(b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 18*e*x^2) + 3*a*(d^2
 + 6*d*e*x^2 - 3*e^2*x^4))/x^3 - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCsc[c*
x])/(3*x^3) + (b*e^2*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/c
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.46

method	result
parts	$a\left(e^2x-\frac{2de}{x}-\frac{d^2}{3x^3}\right)+b\,\operatorname{arccsc}(cx)\,e^2x-\frac{2b\,\operatorname{arccsc}(cx)de}{x}-\frac{b\,\operatorname{arccsc}(cx)d^2}{3x^3}-\frac{2bc(c^2x^2-1)d^2}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^2}-\frac{2b\cdots}{c\cdots}$
derivativedivides	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\,\operatorname{arccsc}(cx)e^2x}{c^3}-\frac{b\,\operatorname{arccsc}(cx)d^2}{3c^3x^3}-\frac{2b\,\operatorname{arccsc}(cx)de}{c^3x}-\frac{2b(c^2x^2-1)d^2}{9c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b(c^2x\cdots}{9\sqrt{\frac{c^2x\cdots}{c^2\cdots}}}\right.$
default	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\,\operatorname{arccsc}(cx)e^2x}{c^3}-\frac{b\,\operatorname{arccsc}(cx)d^2}{3c^3x^3}-\frac{2b\,\operatorname{arccsc}(cx)de}{c^3x}-\frac{2b(c^2x^2-1)d^2}{9c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b(c^2x\cdots}{9\sqrt{\frac{c^2x\cdots}{c^2\cdots}}}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*(e^2*x-2*d*e/x-1/3*d^2/x^3)+b*arccsc(c*x)*e^2*x-2*b*arccsc(c*x)*d*e/x-1/3
*b*arccsc(c*x)*d^2/x^3-2/9*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^2*
d^2-2*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^2*d*e+b/c^2*(c^2*x^2-1)
^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))-1/9*b/c*
(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^4*d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4}\,dx = \frac{9\,ace^2x^4 - 9\,be^2x^3\log\left(-cx+\sqrt{c^2x^2-1}\right) - 18\,acdex^2 + 6\,(bcd^2+6\,bcde-3\,bce^2)x^3\arctan\left(-cx+\sqrt{c^2x^2\right.}}{}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a*c*e^2*x^4 - 9*b*e^2*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) - 18*a*c*d*e
*x^2 + 6*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*arctan(-c*x + sqrt(c^2*x^2 -
 1)) - 3*a*c*d^2 - 2*(b*c^4*d^2 + 9*b*c^2*d*e)*x^3 + 3*(3*b*c*e^2*x^4 - 6*b
*c*d*e*x^2 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*arccsc(c*x) -
 (b*c*d^2 + 2*(b*c^3*d^2 + 9*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))/(c*x^3)
```

Sympy [A] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{ad^2}{3x^3} - \frac{2ade}{x} + ae^2x - 2bcde\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd^2\operatorname{acsc}(cx)}{3x^3} - \frac{2bde\operatorname{acsc}(cx)}{x} + be^2x\operatorname{acsc}(cx) - \frac{bd^2\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c} + \frac{be^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**4,x)
```

```
[Out] -a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x - 2*b*c*d*e*sqrt(1 - 1/(c**2*x**2))
 - b*d**2*acsc(c*x)/(3*x**3) - 2*b*d*e*acsc(c*x)/x + b*e**2*x*acsc(c*x) - b
*d**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(
3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sq
rt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c) + b*e**2*Piecewise((acosh(c*x), A
bs(c**2*x**2) > 1), (-I*asin(c*x), True))/c
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4} dx$$
$$= -2\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)bde + ae^2x + \frac{1}{9}bd^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\operatorname{arccsc}(cx)}{x^3}\right) + \frac{\left(2cx\operatorname{arccsc}(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be^2}{2c} - \frac{2ade}{x} - \frac{ad^2}{3x^3}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -2*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d*e + a*e^2*x + 1/9*b*d^2*(
(c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(
c*x)/x^3) + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(
-sqrt(-1/(c^2*x^2) + 1) + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4288 vs. $2(139) = 278$.

Time = 98.89 (sec) , antiderivative size = 4288, normalized size of antiderivative = 27.31

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4} dx = \text{Too large to display}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

```
[Out] -1/18*(4*b*c^3*d^2/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2)
 + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-
1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 9
*b*e^2*arcsin(1/(c*x))/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1
/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7)) - 9*a*e^2/(c/(x*(sqrt(-1/(c^2*x^
2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt
(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7)) +
```

36*b*c*d*e/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 18*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 18*b*e^2*log(1/(abs(c)*abs(x)))/(c*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 72*b*d*e*arcsin(1/(c*x))/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 72*a*d*e/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 12*b*c*d^2/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 36*b*e^2*arcsin(1/(c*x))/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 36*a*e^2/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 36*b*d*e/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 48*b*d^2*arcsin(1/(c*x))/(x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 48*a*d^2/(x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 54*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 54*b*e^2*log(1/(abs(c)*abs(x)))/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 144*b*d*e*arcsin(1/(c*x))/(c^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 144*a*d*e/(c^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) +

3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 12*b*d^2/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 54*b*e^2*arcsin(1/(c*x))/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 54*a*e^2/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 36*b*d*e/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 54*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 54*b*e^2*log(1/(abs(c)*abs(x)))/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 72*b*d*e*arcsin(1/(c*x))/(c^4*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 72*a*d*e/(c^4*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 4*b*d^2/(c^3*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 36*b*e^2*arcsin(1/(c*x))/(c^6*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 36*a*e^2/(c^6*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 36*b*d*e/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 18*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c^7*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 18*b*e^2*log(1/(abs(c)*abs(x)))/(c^7*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 9*b*e^2*arcsin(1/(c

```
*x))/(c^8*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*(c/(x*(sqrt(-1/(c^2*x^2) + 1)
+ 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2
*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 9*a*e^2
/(c^8*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)
) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2
) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^4, x)
```

3.92 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 183

$$\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{bc(24c^4d^2+100c^2de+225e^2)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{2bcd(6c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{5x^5} - \frac{2de(a+b\csc^{-1}(cx))}{3x^3} - \frac{e^2(a+b\csc^{-1}(cx))}{x}$$

```
[Out] -1/5*d^2*(a+b*arccsc(c*x))/x^5-2/3*d*e*(a+b*arccsc(c*x))/x^3-e^2*(a+b*arccs
c(c*x))/x-1/225*b*c*(24*c^4*d^2+100*c^2*d*e+225*e^2)*(c^2*x^2-1)^(1/2)/(c^2
*x^2)^(1/2)-1/25*b*c*d^2*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-2/225*b*c*d*
(6*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{276, 5347, 12, 1279, 464, 270\}$

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{d^2(a+b\csc^{-1}(cx))}{5x^5} - \frac{2de(a+b\csc^{-1}(cx))}{3x^3} - \frac{e^2(a+b\csc^{-1}(cx))}{x} - \frac{bcd^2\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}} - \frac{2bcd\sqrt{c^2x^2-1}(6c^2d+25e)}{225x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{c^2x^2-1}(24c^4d^2+100c^2de+225e^2)}{225\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(b*c*(24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*Sqrt[-1 + c^2*x^2])/Sqrt[c
^2*x^2] - (b*c*d^2*Sqrt[-1 + c^2*x^2])/(25*x^4*Sqrt[c^2*x^2]) - (2*b*c*d*(6
*c^2*d + 25*e)*Sqrt[-1 + c^2*x^2])/(225*x^2*Sqrt[c^2*x^2]) - (d^2*(a + b*Ar
cCsc[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcCsc[c*x]))/(3*x^3) - (e^2*(a + b*Arc
Csc[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\csc^{-1}(cx))}{5x^5}-\frac{2de(a+b\csc^{-1}(cx))}{3x^3}-\frac{e^2(a+b\csc^{-1}(cx))}{x}+\frac{(bcx)\int\frac{-3d^2-10dex^2-15e^2x^4}{15x^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{d^2(a+b\csc^{-1}(cx))}{5x^5}-\frac{2de(a+b\csc^{-1}(cx))}{3x^3}-\frac{e^2(a+b\csc^{-1}(cx))}{x}+\frac{(bcx)\int\frac{-3d^2-10dex^2-15e^2x^4}{x^6\sqrt{-1+c^2x^2}}\,dx}{15\sqrt{c^2x^2}}\\
&= -\frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{5x^5}-\frac{2de(a+b\csc^{-1}(cx))}{3x^3}-\frac{e^2(a+b\csc^{-1}(cx))}{x}+\frac{(bcx)\int\frac{-2d(6c^2d+25e)-75e^2x^2}{x^4\sqrt{-1+c^2x^2}}\,dx}{75\sqrt{c^2x^2}}\\
&= -\frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}-\frac{2bcd(6c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{5x^5}-\frac{2de(a+b\csc^{-1}(cx))}{3x^3}-\frac{e^2(a+b\csc^{-1}(cx))}{x}\\
&\quad+\frac{(bc(-225e^2-4c^2d(6c^2d+25e))x)\int\frac{1}{x^2\sqrt{-1+c^2x^2}}\,dx}{225\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{bc(225e^2 + 4c^2d(6c^2d + 25e))\sqrt{-1 + c^2x^2}}{225\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1 + c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{2bcd(6c^2d + 25e)\sqrt{-1 + c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d^2(a + b\csc^{-1}(cx))}{5x^5} - \frac{2de(a + b\csc^{-1}(cx))}{3x^3} - \frac{e^2(a + b\csc^{-1}(cx))}{x}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{(d + ex^2)^2 (a + b\csc^{-1}(cx))}{x^6} dx = -\frac{15a(3d^2 + 10dex^2 + 15e^2x^4) + bc\sqrt{1 - \frac{1}{c^2x^2}}x(225e^2x^4 + 50dex^2(1 + 2c^2x^2) + 3d^2(3 + 4c^2x^2 + 8c^4x^4)) +}{225x^5}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^2)]*
x*(225*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*c^4*
x^4)) + 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcCsc[c*x])/x^5
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96

method	result
parts	$a\left(-\frac{e^2}{x} - \frac{d^2}{5x^5} - \frac{2de}{3x^3}\right) + b\,c^5\left(-\frac{\operatorname{arccsc}(cx)e^2}{c^5x} - \frac{\operatorname{arccsc}(cx)d^2}{5x^5c^5} - \frac{2\operatorname{arccsc}(cx)de}{3c^5x^3} - \frac{(c^2x^2-1)(24c^8d^2x^4+100}{}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e^2}{cx} - \frac{d^2}{5c\,x^5} - \frac{2de}{3c\,x^3}\right)}{c^4} + \frac{b\left(-\frac{\operatorname{arccsc}(cx)e^2}{cx} - \frac{\operatorname{arccsc}(cx)d^2}{5c\,x^5} - \frac{2\operatorname{arccsc}(cx)de}{3c\,x^3} - \frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6d^2x}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^4}\right.$
default	$c^5\left(\frac{a\left(-\frac{e^2}{cx} - \frac{d^2}{5c\,x^5} - \frac{2de}{3c\,x^3}\right)}{c^4} + \frac{b\left(-\frac{\operatorname{arccsc}(cx)e^2}{cx} - \frac{\operatorname{arccsc}(cx)d^2}{5c\,x^5} - \frac{2\operatorname{arccsc}(cx)de}{3c\,x^3} - \frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6d^2x}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^4}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-e^2/x-1/5*d^2/x^5-2/3*d*e/x^3)+b*c^5*(-1/c^5*arccsc(c*x)*e^2/x-1/5*arcc
sc(c*x)*d^2/x^5/c^5-2/3/c^5*arccsc(c*x)*d*e/x^3-1/225/c^10*(c^2*x^2-1)*(24*
```

```
c^8*d^2*x^4+100*c^6*d*e*x^4+12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^2+9
*c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{225\,ae^2x^4+150\,adex^2+45\,ad^2+15\,(15\,be^2x^4+10\,bdex^2+3\,bd^2)\,\mathrm{arccsc}\,(cx)+((24\,bc^4d^2+100\,bc^2de}{225\,x^5}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b*
d*e*x^2 + 3*b*d^2)*arccsc(c*x) + ((24*b*c^4*d^2 + 100*b*c^2*d*e + 225*b*e^2
)*x^4 + 9*b*d^2 + 2*(6*b*c^2*d^2 + 25*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^5
```

Sympy [A] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.83

$$\begin{aligned}
&\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6}\,dx \\
&= -\frac{ad^2}{5x^5}-\frac{2ade}{3x^3}-\frac{ae^2}{x}-bce^2\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd^2\,\mathrm{acsc}\,(cx)}{5x^5}-\frac{2bde\,\mathrm{acsc}\,(cx)}{3x^3} \\
&\quad -\frac{be^2\,\mathrm{acsc}\,(cx)}{x}-\frac{bd^2\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c} \\
&\quad -\frac{2bde\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}
\end{aligned}$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**6,x)
```

```
[Out] -a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x - b*c*e**2*sqrt(1 - 1/(c**2*
x**2)) - b*d**2*acsc(c*x)/(5*x**5) - 2*b*d*e*acsc(c*x)/(3*x**3) - b*e**2*ac
sc(c*x)/x - b*d**2*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sq
rt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2
) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 +
1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) - 2*b*d*e*Pi
```

```
ecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3),
 Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2
*x**2 + 1)/(3*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6}\,dx$$
$$= -\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)be^2 - \frac{1}{75}bd^2\left(\frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{15\operatorname{arccsc}(cx)}{x^5}\right) + \frac{2}{9}bde\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{3\operatorname{arccsc}(cx)}{x^3}\right) - \frac{ae^2}{x} - \frac{2ade}{3x^3} - \frac{ad^2}{5x^5}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*e^2 - 1/75*b*d^2*((3*c^6*(-1/
(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^
2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) + 2/9*b*d*e*((c^4*(-1/(c^2*x^2) + 1)^(
3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x)/x^3) - a*e^2/x - 2/3
*a*d*e/x^3 - 1/5*a*d^2/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{1}{225}\left(9bc^4d^2\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1} - 30bc^4d^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + \frac{45bc^3d^2\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x} + 45\right.$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/225*(9*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 30*b*c^4*d
^2*(-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d^2*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*
```

```
x))/x + 45*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1) + 90*b*c^3*d^2*(1/(c^2*x^2) - 1
)*arcsin(1/(c*x))/x - 50*b*c^2*d*e*(-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d^2*
arcsin(1/(c*x))/x + 150*b*c^2*d*e*sqrt(-1/(c^2*x^2) + 1) + 150*b*c*d*e*(1/(
c^2*x^2) - 1)*arcsin(1/(c*x))/x + 150*b*c*d*e*arcsin(1/(c*x))/x + 225*b*e^2
*sqrt(-1/(c^2*x^2) + 1) + 225*b*e^2*arcsin(1/(c*x))/(c*x) + 225*a*e^2/(c*x)
 + 150*a*d*e/(c*x^3) + 45*a*d^2/(c*x^5))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^6,x)
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^6, x)
```

3.93 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^8}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 241

$$\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^8}\,dx = -\frac{2bc^3(360c^4d^2+1176c^2de+1225e^2)\sqrt{-1+c^2x^2}}{11025\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{bc(360c^4d^2+1176c^2de+1225e^2)\sqrt{-1+c^2x^2}}{11025x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{7x^7} - \frac{2de(a+b\csc^{-1}(cx))}{5x^5} - \frac{e^2(a+b\csc^{-1}(cx))}{3x^3}$$

```
[Out] -1/7*d^2*(a+b*arccsc(c*x))/x^7-2/5*d*e*(a+b*arccsc(c*x))/x^5-1/3*e^2*(a+b*a
rccsc(c*x))/x^3-2/11025*b*c^3*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-
1)^(1/2)/(c^2*x^2)^(1/2)-1/49*b*c*d^2*(c^2*x^2-1)^(1/2)/x^6/(c^2*x^2)^(1/2)
-2/1225*b*c*d*(15*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-1/11025
*b*c*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {276, 5347, 12, 1279, 464, 277, 270}

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^8}\,dx = -\frac{d^2(a+b\csc^{-1}(cx))}{7x^7}-\frac{2de(a+b\csc^{-1}(cx))}{5x^5} -\frac{e^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{bcd^2\sqrt{c^2x^2-1}}{49x^6\sqrt{c^2x^2}} -\frac{2bcd\sqrt{c^2x^2-1}(15c^2d+49e)}{1225x^4\sqrt{c^2x^2}} -\frac{bc\sqrt{c^2x^2-1}(360c^4d^2+1176c^2de+1225e^2)}{11025x^2\sqrt{c^2x^2}} -\frac{2bc^3\sqrt{c^2x^2-1}(360c^4d^2+1176c^2de+1225e^2)}{11025\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] (-2*b*c^3*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 + c^2*x^2])/(1102
5*Sqrt[c^2*x^2]) - (b*c*d^2*Sqrt[-1 + c^2*x^2])/(49*x^6*Sqrt[c^2*x^2]) - (2
*b*c*d*(15*c^2*d + 49*e)*Sqrt[-1 + c^2*x^2])/(1225*x^4*Sqrt[c^2*x^2]) - (b*
c*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 + c^2*x^2])/(11025*x^2*Sq
rt[c^2*x^2]) - (d^2*(a + b*ArcCsc[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcCsc[c*x
]))/(5*x^5) - (e^2*(a + b*ArcCsc[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\csc^{-1}(cx))}{7x^7}-\frac{2de(a+b\csc^{-1}(cx))}{5x^5}\\
&\quad -\frac{e^2(a+b\csc^{-1}(cx))}{3x^3}+\frac{(bcx)\int\frac{-15d^2-42dex^2-35e^2x^4}{105x^8\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{d^2(a+b\csc^{-1}(cx))}{7x^7}-\frac{2de(a+b\csc^{-1}(cx))}{5x^5}\\
&\quad -\frac{e^2(a+b\csc^{-1}(cx))}{3x^3}+\frac{(bcx)\int\frac{-15d^2-42dex^2-35e^2x^4}{x^8\sqrt{-1+c^2x^2}}\,dx}{105\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{7x^7} - \frac{2de(a+b\csc^{-1}(cx))}{5x^5} - \frac{e^2(a+b\csc^{-1}(cx))}{3x^3} + \frac{(bcx)\int\frac{-6d(15c^2d+49e)-245e^2x^2}{x^6\sqrt{-1+c^2x^2}}\,dx}{735\sqrt{c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{7x^7} - \frac{2de(a+b\csc^{-1}(cx))}{5x^5} - \frac{e^2(a+b\csc^{-1}(cx))}{3x^3} + \frac{(bc(-1225e^2-24c^2d(15c^2d+49e))x)\int\frac{1}{x^4\sqrt{-1+c^2x^2}}\,dx}{3675\sqrt{c^2x^2}}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{bc(1225e^2+24c^2d(15c^2d+49e))\sqrt{-1+c^2x^2}}{11025x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{7x^7} - \frac{2de(a+b\csc^{-1}(cx))}{5x^5} - \frac{e^2(a+b\csc^{-1}(cx))}{3x^3} + \frac{(2bc^3(-1225e^2-24c^2d(15c^2d+49e))x)\int\frac{1}{x^2\sqrt{-1+c^2x^2}}\,dx}{11025\sqrt{c^2x^2}}$$

$$= -\frac{2bc^3(1225e^2+24c^2d(15c^2d+49e))\sqrt{-1+c^2x^2}}{11025\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{bc(1225e^2+24c^2d(15c^2d+49e))\sqrt{-1+c^2x^2}}{11025x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{7x^7} - \frac{2de(a+b\csc^{-1}(cx))}{5x^5} - \frac{e^2(a+b\csc^{-1}(cx))}{3x^3}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

$$\int\frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^8}\,dx = -\frac{105a(15d^2+42dex^2+35e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(1225e^2x^4(1+2c^2x^2)+294dex^2(3+4c^2x^2+8c^4x^4)+}{11025x^7}$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] -1/11025*(105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^
2)]*x*(1225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^
4) + 45*d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) + 105*b*(15*d^2 + 42*
d*e*x^2 + 35*e^2*x^4)*ArcCsc[c*x])/x^7
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86

method	result
parts	$a\left(-\frac{d^2}{7x^7}-\frac{2de}{5x^5}-\frac{e^2}{3x^3}\right)+b\,c^7\left(-\frac{\operatorname{arccsc}(cx)d^2}{7x^7c^7}-\frac{2\operatorname{arccsc}(cx)de}{5c^7x^5}-\frac{\operatorname{arccsc}(cx)e^2}{3c^7x^3}-\frac{(c^2x^2-1)(720c^{10}d^2x^6+\cdots}{\cdots}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{d^2}{7c^3x^7}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)e^2}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)d^2}{7c^3x^7}-\frac{2\operatorname{arccsc}(cx)de}{5c^3x^5}-\frac{(c^2x^2-1)(720c^{10}d^2x^6+2352c^8de\,x^6+\cdots}{\cdots}\right.}{c^4\cdots}\right.$
default	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{d^2}{7c^3x^7}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)e^2}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)d^2}{7c^3x^7}-\frac{2\operatorname{arccsc}(cx)de}{5c^3x^5}-\frac{(c^2x^2-1)(720c^{10}d^2x^6+2352c^8de\,x^6+\cdots}{\cdots}\right.}{c^4\cdots}\right.$

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/7*d^2/x^7-2/5*d*e/x^5-1/3/x^3*e^2)+b*c^7*(-1/7*arccsc(c*x)*d^2/x^7/c^
7-2/5/c^7*arccsc(c*x)*d*e/x^5-1/3/c^7*arccsc(c*x)/x^3*e^2-1/11025/c^12*(c^2
*x^2-1)*(720*c^10*d^2*x^6+2352*c^8*d*e*x^6+360*c^8*d^2*x^4+2450*c^6*e^2*x^6
+1176*c^6*d*e*x^4+270*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*d*e*x^2+225*c^4*
d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^8)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{3675\,ae^2x^4+4410\,adex^2+1575\,ad^2+105\,(35\,be^2x^4+42\,bdex^2+15\,bd^2)\operatorname{arccsc}(cx)+(2\,(360\,bc^6d^2+1\cdots}{\cdots}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
+ 42*b*d*e*x^2 + 15*b*d^2)*arccsc(c*x) + (2*(360*b*c^6*d^2 + 1176*b*c^4*d*e
 + 1225*b*c^2*e^2)*x^6 + (360*b*c^4*d^2 + 1176*b*c^2*d*e + 1225*b*e^2)*x^4
+ 225*b*d^2 + 18*(15*b*c^2*d^2 + 49*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^7
```

Sympy [A] (verification not implemented)

Time = 30.57 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.12

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^8} dx$$
$$= -\frac{ad^2}{7x^7} - \frac{2ade}{5x^5} - \frac{ae^2}{3x^3} - \frac{bd^2 \operatorname{acsc}(cx)}{7x^7} - \frac{2bde \operatorname{acsc}(cx)}{5x^5} - \frac{be^2 \operatorname{acsc}(cx)}{3x^3}$$
$$- \frac{bd^2 \left(\begin{cases} \frac{16c^7\sqrt{c^2x^2-1}}{35x} + \frac{8c^5\sqrt{c^2x^2-1}}{35x^3} + \frac{6c^3\sqrt{c^2x^2-1}}{35x^5} + \frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } |c^2x^2| > 1 \\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x} + \frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3} + \frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5} + \frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise} \end{cases}\right)}{7c}$$
$$- \frac{2bde \left(\begin{cases} \frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2| > 1 \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise} \end{cases}\right)}{5c}$$
$$- \frac{be^2 \left(\begin{cases} \frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2| > 1 \\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise} \end{cases}\right)}{3c}$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**8,x)
```

```
[Out] -a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*d**2*acsc(c*x)/(7
*x**7) - 2*b*d*e*acsc(c*x)/(5*x**5) - b*e**2*acsc(c*x)/(3*x**3) - b*d**2*Pi
ecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**2*x**2 - 1)/(3
5*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2*x**2 - 1)/(7*x
**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/(35*x) + 8*I*c**
5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**2 + 1)/(35*x**5)
+ I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) - 2*b*d*e*Piecewise((8*c*
*5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sq
rt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2
+ 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2
 + 1)/(5*x**5), True))/(5*c) - b*e**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)
/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqr
t(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^8}\,dx$$
$$= \frac{1}{245}bd^2\left(\frac{5c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{35\,\mathrm{arccsc}\,(cx)}{x^7}\right)$$
$$-\frac{2}{75}bde\left(\frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\mathrm{arccsc}\,(cx)}{x^5}\right)$$
$$+\frac{1}{9}be^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\mathrm{arccsc}\,(cx)}{x^3}\right)-\frac{ae^2}{3x^3}-\frac{2ade}{5x^5}-\frac{ad^2}{7x^7}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d^2*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(5
/2) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c -
35*arccsc(c*x)/x^7) - 2/75*b*d*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*
(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x
)/x^5) + 1/9*b*e^2*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2)
 + 1))/c - 3*arccsc(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x
^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. 2(211) = 422.

Time = 0.30 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{1}{11025}\left(225\,bc^6d^2\left(\frac{1}{c^2x^2}-1\right)^3\sqrt{-\frac{1}{c^2x^2}+1}+945\,bc^6d^2\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{1575\,bc^5d^2\left(\frac{1}{c^2x^2}-\right.}{x}\right.$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

```
[Out] -1/11025*(225*b*c^6*d^2*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1) + 945*b*
c^6*d^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) + 1575*b*c^5*d^2*(1/(c^2
*x^2) - 1)^3*arcsin(1/(c*x))/x - 1575*b*c^6*d^2*(-1/(c^2*x^2) + 1)^(3/2) +
```

```
4725*b*c^5*d^2*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 1575*b*c^6*d^2*sqrt(
-1/(c^2*x^2) + 1) + 882*b*c^4*d*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1
) + 4725*b*c^5*d^2*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 2940*b*c^4*d*e*(-1
/(c^2*x^2) + 1)^(3/2) + 1575*b*c^5*d^2*arcsin(1/(c*x))/x + 4410*b*c^3*d*e*(
1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 4410*b*c^4*d*e*sqrt(-1/(c^2*x^2) + 1
) + 8820*b*c^3*d*e*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 1225*b*c^2*e^2*(-1
/(c^2*x^2) + 1)^(3/2) + 4410*b*c^3*d*e*arcsin(1/(c*x))/x + 3675*b*c^2*e^2*s
qrt(-1/(c^2*x^2) + 1) + 3675*b*c*e^2*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x +
3675*b*c*e^2*arcsin(1/(c*x))/x + 3675*a*e^2/(c*x^3) + 4410*a*d*e/(c*x^5) +
1575*a*d^2/(c*x^7))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x^8}\,dx=\int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^8, x)
```

3.94 $\int x^3(d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 242

$$\int x^3(d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx = \frac{b(6c^4d^2+8c^2de+3e^2)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}} + \frac{b(6c^4d^2+16c^2de+9e^2)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}} + \frac{be(8c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}} + \frac{be^2x(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}} + \frac{1}{4}d^2x^4(a+b\csc^{-1}(cx)) + \frac{1}{3}dex^6(a+b\csc^{-1}(cx)) + \frac{1}{8}e^2x^8(a+b\csc^{-1}(cx))$$

```
[Out] 1/4*d^2*x^4*(a+b*arccsc(c*x))+1/3*d*e*x^6*(a+b*arccsc(c*x))+1/8*e^2*x^8*(a+
b*arccsc(c*x))+1/72*b*(6*c^4*d^2+16*c^2*d*e+9*e^2)*x*(c^2*x^2-1)^(3/2)/c^7/
(c^2*x^2)^(1/2)+1/120*b*e*(8*c^2*d+9*e)*x*(c^2*x^2-1)^(5/2)/c^7/(c^2*x^2)^(
1/2)+1/56*b*e^2*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^(1/2)+1/24*b*(6*c^4*d^2+8
*c^2*d*e+3*e^2)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {272, 45, 5347, 12, 1265, 785}

$$\int x^3\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}(cx)\right)+\frac{bex(c^2x^2-1)^{5/2}\left(8c^2d+9e\right)}{120c^7\sqrt{c^2x^2}}+\frac{be^2x(c^2x^2-1)^{7/2}}{56c^7\sqrt{c^2x^2}}+\frac{bx(c^2x^2-1)^{3/2}\left(6c^4d^2+16c^2de+9e^2\right)}{72c^7\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}(6c^4d^2+8c^2de+3e^2)}{24c^7\sqrt{c^2x^2}}$$

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*x*Sqrt[-1 + c^2*x^2])/(24*c^7*Sqrt[c^2*x
^2]) + (b*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*x*(-1 + c^2*x^2)^(3/2))/(72*c^7*
Sqrt[c^2*x^2]) + (b*e*(8*c^2*d + 9*e)*x*(-1 + c^2*x^2)^(5/2))/(120*c^7*Sqrt
[c^2*x^2]) + (b*e^2*x*(-1 + c^2*x^2)^(7/2))/(56*c^7*Sqrt[c^2*x^2]) + (d^2*x
^4*(a + b*ArcCsc[c*x]))/4 + (d*e*x^6*(a + b*ArcCsc[c*x]))/3 + (e^2*x^8*(a +
 b*ArcCsc[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right) \\
&\quad+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^3(6d^2+8dex^2+3e^2x^4)}{24\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= \frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right) \\
&\quad+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^3(6d^2+8dex^2+3e^2x^4)}{\sqrt{-1+c^2x^2}}\,dx}{24\sqrt{c^2x^2}} \\
&= \frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right) \\
&\quad+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\text{Subst}\left(\int\frac{x(6d^2+8dex+3e^2x^2)}{\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{48\sqrt{c^2x^2}} \\
&= \frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}(cx)\right) \\
&\quad+\frac{(bcx)\text{Subst}\left(\int\left(\frac{6c^4d^2+8c^2de+3e^2}{c^6\sqrt{-1+c^2x}}+\frac{(6c^4d^2+16c^2de+9e^2)\sqrt{-1+c^2x}}{c^6}+\frac{e(8c^2d+9e)(-1+c^2x)^{3/2}}{c^6}+\frac{3e^2(-1+c^2x)^{5/2}}{c^6}\right)\right.}{48\sqrt{c^2x^2}} \\
&= \frac{b(6c^4d^2+8c^2de+3e^2)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}+\frac{b(6c^4d^2+16c^2de+9e^2)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}} \\
&\quad+\frac{be(8c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}}+\frac{be^2x(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}} \\
&\quad+\frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}(cx)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.66

$$\int x^3\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{x\left(105ax^3(6d^2+8dex^2+3e^2x^4)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(144e^2+8c^2e(56d+9ex^2)+c^4(420d^2+224dex^2+54e^2x^4)+3c^6(70d^2x^2+56dex^4+15e\right.}{c^7}\right.}{2520}$$

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(105*a*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + (b*Sqrt[1 - 1/(c^2*x^2)]*(1
44*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 + 54*e^2*x^4
) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/c^7 + 105*b*x^3*(6*d^2 +
 8*d*e*x^2 + 3*e^2*x^4)*ArcCsc[c*x]))/2520
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.82

method	result
parts	$a\left(\frac{1}{8}e^2x^8+\frac{1}{3}de\,x^6+\frac{1}{4}x^4d^2\right)+\frac{b\left(\frac{c^4\,\mathrm{arccsc}(cx)e^2x^8}{8}+\frac{c^4\,\mathrm{arccsc}(cx)de\,x^6}{3}+\frac{\mathrm{arccsc}(cx)d^2x^4c^4}{4}+\frac{(c^2x^2-1)\left(45c^6e^2x^6\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^2d\left(c^2e\,x^2+c^2d\right)^3}{3}-\frac{\left(c^2e\,x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}-\frac{b\,c^4\,\mathrm{arccsc}(cx)d^4}{24e^2}+\frac{b\,\mathrm{arccsc}(cx)d^2c^4x^4}{4}+\frac{b\,c^4e\,\mathrm{arccsc}(cx)d\,x^6}{3}+\frac{b\,c^4e^2\,\mathrm{arccsc}(cx)x^8}{8}+}{}$
default	$\frac{-\frac{a\left(\frac{c^2d\left(c^2e\,x^2+c^2d\right)^3}{3}-\frac{\left(c^2e\,x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}-\frac{b\,c^4\,\mathrm{arccsc}(cx)d^4}{24e^2}+\frac{b\,\mathrm{arccsc}(cx)d^2c^4x^4}{4}+\frac{b\,c^4e\,\mathrm{arccsc}(cx)d\,x^6}{3}+\frac{b\,c^4e^2\,\mathrm{arccsc}(cx)x^8}{8}+}{}$

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*x^4*d^2)+b/c^4*(1/8*c^4*arccsc(c*x)*e^2*x^8+
1/3*c^4*arccsc(c*x)*d*e*x^6+1/4*arccsc(c*x)*d^2*x^4*c^4+1/2520/c^5*(c^2*x^2
-1)*(45*c^6*e^2*x^6+168*c^6*d*e*x^4+210*c^6*d^2*x^2+54*c^4*e^2*x^4+224*c^4*
d*e*x^2+420*c^4*d^2+72*c^2*e^2*x^2+448*c^2*d*e+144*e^2)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.77

$$\int x^3\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{315\,ac^8e^2x^8+840\,ac^8dex^6+630\,ac^8d^2x^4+105\left(3\,bc^8e^2x^8+8\,bc^8dex^6+6\,bc^8d^2x^4\right)\operatorname{arccsc}(cx)+\left(45\,bc^6e^2x\right.}{}$$

[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")

[Out] 1/2520*(315*a*c^8*e^2*x^8 + 840*a*c^8*d*e*x^6 + 630*a*c^8*d^2*x^4 + 105*(3*b*c^8*e^2*x^8 + 8*b*c^8*d*e*x^6 + 6*b*c^8*d^2*x^4)*arccsc(c*x) + (45*b*c^6*e^2*x^6 + 420*b*c^4*d^2 + 448*b*c^2*d*e + 6*(28*b*c^6*d*e + 9*b*c^4*e^2)*x^4 + 144*b*e^2 + 2*(105*b*c^6*d^2 + 112*b*c^4*d*e + 36*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))/c^8

Sympy [A] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.04

$$\begin{aligned}\int x^3\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx &= \frac{ad^2x^4}{4}+\frac{adex^6}{3}+\frac{ae^2x^8}{8}+\frac{bd^2x^4\operatorname{acsc}(cx)}{4}+\frac{bdex^6\operatorname{acsc}(cx)}{3}\\ &+\frac{be^2x^8\operatorname{acsc}(cx)}{8}+\frac{bd^2\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}\\ &+\frac{bde\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{3c}\\ &+\frac{be^2\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c}+\frac{6x^4\sqrt{c^2x^2-1}}{35c^3}+\frac{8x^2\sqrt{c^2x^2-1}}{35c^5}+\frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for } |c^2x^2|>1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c}+\frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3}+\frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5}+\frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}\end{aligned}$$

[In] integrate(x**3*(e*x**2+d)**2*(a+b*acsc(c*x)),x)

[Out] a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*acsc(c*x)/4 + b*d*e*x**6*acsc(c*x)/3 + b*e**2*x**8*acsc(c*x)/8 + b*d**2*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1

```
), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3),
True))/(4*c) + b*d*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqr
t(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2
) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/
(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(3*c) + b*e**2*Piece
wise((x**6*sqrt(c**2*x**2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3)
 + 8*x**2*sqrt(c**2*x**2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7),
 Abs(c**2*x**2) > 1), (I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c
**2*x**2 + 1)/(35*c**3) + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sq
rt(-c**2*x**2 + 1)/(35*c**7), True))/(8*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.05

$$\int x^3 \left(d + ex^2\right)^2 \left(a + b \csc^{-1}(cx)\right) \, dx = \frac{1}{8} a e^2 x^8 + \frac{1}{3} a d e x^6 + \frac{1}{4} a d^2 x^4 + \frac{1}{12} \left(3\, x^4 \operatorname{arccsc}(cx) + \frac{c^2 x^3 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{3}{2}} + 3\, x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^3}\right) b d^2 + \frac{1}{45} \left(15\, x^6 \operatorname{arccsc}(cx) + \frac{3\, c^4 x^5 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{5}{2}} + 10\, c^2 x^3 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{3}{2}} + 15\, x \sqrt{-\frac{1}{c^2 x^2} + 1}}{c^5}\right) b d e + \frac{1}{280} \left(35\, x^8 \operatorname{arccsc}(cx) + \frac{5\, c^6 x^7 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{7}{2}} + 21\, c^4 x^5 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{5}{2}} + 35\, c^2 x^3 \left(-\frac{1}{c^2 x^2} + 1\right)^{\frac{3}{2}} + 35\, x \sqrt{-}}{c^7}\right.$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arccsc(c*x) + (
c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d^2 +
 1/45*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^
3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*d*e + 1/28
0*(35*x^8*arccsc(c*x) + (5*c^6*x^7*(-1/(c^2*x^2) + 1)^(7/2) + 21*c^4*x^5*(-
1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 35*x*sqrt(-1
/(c^2*x^2) + 1))/c^7)*b*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1706 vs. $2(212) = 424$.

Time = 0.45 (sec) , antiderivative size = 1706, normalized size of antiderivative = 7.05

$$\int x^3(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] 1/645120*(315*b*e^2*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*arcsin(1/(c*x))/c +
315*a*e^2*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8/c + 90*b*e^2*x^7*(sqrt(-1/(c^2
*x^2) + 1) + 1)^7/c^2 + 3360*b*d*e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsi
n(1/(c*x))/c + 3360*a*d*e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 2520*b*e^2
*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c^3 + 2520*a*e^2*x^6*(s
qrt(-1/(c^2*x^2) + 1) + 1)^6/c^3 + 1344*b*d*e*x^5*(sqrt(-1/(c^2*x^2) + 1) +
 1)^5/c^2 + 10080*b*d^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/
c + 10080*a*d^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 882*b*e^2*x^5*(sqrt(
-1/(c^2*x^2) + 1) + 1)^5/c^4 + 20160*b*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)
^4*arcsin(1/(c*x))/c^3 + 20160*a*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^3
 + 6720*b*d^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 8820*b*e^2*x^4*(sqrt
(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^5 + 8820*a*e^2*x^4*(sqrt(-1/(c^
2*x^2) + 1) + 1)^4/c^5 + 11200*b*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^4
 + 40320*b*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 403
20*a*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 4410*b*e^2*x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3/c^6 + 50400*b*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*a
rcsin(1/(c*x))/c^5 + 50400*a*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 6
0480*b*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 17640*b*e^2*x^2*(sqrt(-1/(c
^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^7 + 17640*a*e^2*x^2*(sqrt(-1/(c^2*x^2
) + 1) + 1)^2/c^7 + 67200*b*d*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 + 60480*
b*d^2*arcsin(1/(c*x))/c^5 + 60480*a*d^2/c^5 + 22050*b*e^2*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^8 + 67200*b*d*e*arcsin(1/(c*x))/c^7 + 67200*a*d*e/c^7 - 604
80*b*d^2/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 22050*b*e^2*arcsin(1/(c*x))
/c^9 + 22050*a*e^2/c^9 - 67200*b*d*e/(c^8*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) +
 40320*b*d^2*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 403
20*a*d^2/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 22050*b*e^2/(c^10*x*(sq
rt(-1/(c^2*x^2) + 1) + 1)) + 50400*b*d*e*arcsin(1/(c*x))/(c^9*x^2*(sqrt(-1/
(c^2*x^2) + 1) + 1)^2) + 50400*a*d*e/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^
2) - 6720*b*d^2/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 17640*b*e^2*arcs
in(1/(c*x))/(c^11*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 17640*a*e^2/(c^11*x
^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 11200*b*d*e/(c^10*x^3*(sqrt(-1/(c^2*x^
2) + 1) + 1)^3) + 10080*b*d^2*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) +
 1) + 1)^4) + 10080*a*d^2/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 4410*b
*e^2/(c^12*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 20160*b*d*e*arcsin(1/(c*x)
)/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 20160*a*d*e/(c^11*x^4*(sqrt(-
```

```
1/(c^2*x^2) + 1) + 1)^4) + 8820*b*e^2*arcsin(1/(c*x))/(c^13*x^4*(sqrt(-1/(c
^2*x^2) + 1) + 1)^4) + 8820*a*e^2/(c^13*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4)
 - 1344*b*d*e/(c^12*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 882*b*e^2/(c^14*x
^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 3360*b*d*e*arcsin(1/(c*x))/(c^13*x^6*(
sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 3360*a*d*e/(c^13*x^6*(sqrt(-1/(c^2*x^2) +
1) + 1)^6) + 2520*b*e^2*arcsin(1/(c*x))/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) +
 1)^6) + 2520*a*e^2/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) - 90*b*e^2/(c
^16*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7) + 315*b*e^2*arcsin(1/(c*x))/(c^17*x
^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8) + 315*a*e^2/(c^17*x^8*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^8))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^3\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

3.95 $\int x(d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 195

$$\int x(d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx = \frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} + \frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} + \frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{(d+ex^2)^3\,(a+b\csc^{-1}(cx))}{6e} + \frac{bcd^3x\arctan\left(\sqrt{-1+c^2x^2}\right)}{6e\sqrt{c^2x^2}}$$

```
[Out] 1/6*(e*x^2+d)^3*(a+b*arccsc(c*x))/e+1/18*b*e*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(3
/2)/c^5/(c^2*x^2)^(1/2)+1/30*b*e^2*x*(c^2*x^2-1)^(5/2)/c^5/(c^2*x^2)^(1/2)+
1/6*b*c*d^3*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/6*b*(3*c^4*d^2+
3*c^2*d*e+e^2)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used

= {5345, 457, 90, 65, 211}

$$\int x\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)}{6e}+\frac{bcd^3x\arctan\left(\sqrt{c^2x^2-1}\right)}{6e\sqrt{c^2x^2}}+\frac{bex(c^2x^2-1)^{3/2}\left(3c^2d+2e\right)}{18c^5\sqrt{c^2x^2}}+\frac{be^2x(c^2x^2-1)^{5/2}}{30c^5\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}(3c^4d^2+3c^2de+e^2)}{6c^5\sqrt{c^2x^2}}$$

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 + 3*c^2*d*e + e^2)*x*Sqrt[-1 + c^2*x^2])/(6*c^5*Sqrt[c^2*x^2]
) + (b*e*(3*c^2*d + 2*e)*x*(-1 + c^2*x^2)^(3/2))/(18*c^5*Sqrt[c^2*x^2]) + (
b*e^2*x*(-1 + c^2*x^2)^(5/2))/(30*c^5*Sqrt[c^2*x^2]) + ((d + e*x^2)^3*(a +
b*ArcCsc[c*x]))/(6*e) + (b*c*d^3*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*e*Sqrt[c^
2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^3\,(a+b\csc^{-1}(cx))}{6e} + \frac{(bcx)\int \frac{(d+ex^2)^3}{x\sqrt{-1+c^2x^2}}\,dx}{6e\sqrt{c^2x^2}}$$

$$= \frac{(d+ex^2)^3\,(a+b\csc^{-1}(cx))}{6e} + \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^3}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{12e\sqrt{c^2x^2}}$$

$$= \frac{(d+ex^2)^3\,(a+b\csc^{-1}(cx))}{6e} + \frac{(bcx)\text{Subst}\left(\int \left(\frac{e(3c^4d^2+3c^2de+e^2)}{c^4\sqrt{-1+c^2x}} + \frac{d^3}{x\sqrt{-1+c^2x}} + \frac{e^2(3c^2d+2e)\sqrt{-1+c^2x}}{c^4} + \frac{e^3(-1+c^2x)^{3/2}}{c^4}\right)dx, x, x^2\right)}{12e\sqrt{c^2x^2}}$$

$$= \frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} + \frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} + \frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{(d+ex^2)^3\,(a+b\csc^{-1}(cx))}{6e} + \frac{(bcd^3x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{12e\sqrt{c^2x^2}}$$

$$= \frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} + \frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} + \frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{(d+ex^2)^3\,(a+b\csc^{-1}(cx))}{6e} + \frac{(bd^3x)\,\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{6ce\sqrt{c^2x^2}}$$

$$= \frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} + \frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} + \frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{(d+ex^2)^3\,(a+b\csc^{-1}(cx))}{6e} + \frac{bcd^3x\arctan\left(\sqrt{-1+c^2x^2}\right)}{6e\sqrt{c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.64

$$\int x\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{90}x\left(15ax\left(3d^2+3dex^2+e^2x^4\right)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}(8e^2+2c^2e(15d+2ex^2)+3c^4(15d^2+5dex^2+e^2x^4))}{c^5}+15bx\left(3d^2+3dex^2+e^2x^4\right)\csc^{-1}(cx)\right)$$

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + (b*Sqrt[1 - 1/(c^2*x^2)]*(8*e^2
+ 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)))/c^5 + 1
5*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCsc[c*x]))/90
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(169) = 338.

Time = 0.97 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.81

method	result
parts	$\frac{a(e\,x^2+d)^3}{6e}+\frac{b\,\operatorname{arccsc}(cx)e^2x^6}{6}+\frac{b\,\operatorname{arccsc}(cx)de\,x^4}{2}+\frac{b\,\operatorname{arccsc}(cx)d^2x^2}{2}+\frac{b\,d^3\operatorname{arccsc}(cx)}{6e}+\frac{b(c^2x^2-1)x^3e^2}{30c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+$
derivativedivides	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^3}{6c^4e}+\frac{b\,c^2\operatorname{arccsc}(cx)d^3}{6e}+\frac{b\,\operatorname{arccsc}(cx)d^2c^2x^2}{2}+\frac{b\,c^2e\,\operatorname{arccsc}(cx)d\,x^4}{2}+\frac{b\,c^2e^2\operatorname{arccsc}(cx)x^6}{6}-\frac{bc\sqrt{c^2x^2-1}\,d^3\arctan\left(\frac{}{\sqrt{c}}\right.}{6e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}}{c^2}$
default	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^3}{6c^4e}+\frac{b\,c^2\operatorname{arccsc}(cx)d^3}{6e}+\frac{b\,\operatorname{arccsc}(cx)d^2c^2x^2}{2}+\frac{b\,c^2e\,\operatorname{arccsc}(cx)d\,x^4}{2}+\frac{b\,c^2e^2\operatorname{arccsc}(cx)x^6}{6}-\frac{bc\sqrt{c^2x^2-1}\,d^3\arctan\left(\frac{}{\sqrt{c}}\right.}{6e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}}{c^2}$

```
[In] int(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*(e*x^2+d)^3/e+1/6*b*arccsc(c*x)*e^2*x^6+1/2*b*arccsc(c*x)*d*e*x^4+1/2
*b*arccsc(c*x)*d^2*x^2+1/6*b*d^3*arccsc(c*x)/e+1/30*b/c^3*(c^2*x^2-1)/((c^2
*x^2-1)/c^2/x^2)^(1/2)*x^3*e^2+1/6*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^
(1/2)*x*d*e-1/6*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*a
rctan(1/(c^2*x^2-1)^(1/2))+2/45*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/
```

```
2)*x*e^2+1/2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2+1/3*b/c^5*
e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d+4/45*b/c^7*e^2*(c^2*x^2-1)/((
c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.78

$$\int x(d+ex^2)^2(a+b\csc^{-1}(cx))\,dx = \frac{15\,ac^6e^2x^6+45\,ac^6dex^4+45\,ac^6d^2x^2+15\,(bc^6e^2x^6+3\,bc^6dex^4+3\,bc^6d^2x^2)\,\mathrm{arccsc}\,(cx)+(3\,bc^4e^2x^4+45\,b}{90\,c^6}$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^6*e^2*x^6 + 45*a*c^6*d*e*x^4 + 45*a*c^6*d^2*x^2 + 15*(b*c^6*e^
2*x^6 + 3*b*c^6*d*e*x^4 + 3*b*c^6*d^2*x^2)*arccsc(c*x) + (3*b*c^4*e^2*x^4 +
 45*b*c^4*d^2 + 30*b*c^2*d*e + 8*b*e^2 + (15*b*c^4*d*e + 4*b*c^2*e^2)*x^2)*
sqrt(c^2*x^2 - 1))/c^6
```

Sympy [A] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.81

$$\begin{aligned}\int x(d+ex^2)^2(a+b\csc^{-1}(cx))\,dx &= \frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\,\mathrm{acsc}\,(cx)}{2}+\frac{bdex^4\,\mathrm{acsc}\,(cx)}{2}\\ &+\frac{be^2x^6\,\mathrm{acsc}\,(cx)}{6}+\frac{bd^2\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}\\ &+\frac{bde\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{2c}\\ &+\frac{be^2\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}\end{aligned}$$

```
[In] integrate(x*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*acsc(c*x)/2 + b*
d*e*x**4*acsc(c*x)/2 + b*e**2*x**6*acsc(c*x)/6 + b*d**2*Piecewise((sqrt(c**
2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
 + b*d*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/
(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqr
t(-c**2*x**2 + 1)/(3*c**3), True))/(2*c) + b*e**2*Piecewise((x**4*sqrt(c**2
*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2
- 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4
*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5)
, True))/(6*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.97

$$\int x\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= \frac{1}{6}ae^2x^6 + \frac{1}{2}adex^4 + \frac{1}{2}ad^2x^2 + \frac{1}{2}\left(x^2\operatorname{arccsc}(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2$$
$$+ \frac{1}{6}\left(3\,x^4\operatorname{arccsc}(cx) + \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bde$$
$$+ \frac{1}{90}\left(15\,x^6\operatorname{arccsc}(cx) + \frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be^2$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arccsc(c*x) + x*sq
rt(-1/(c^2*x^2) + 1)/c)*b*d^2 + 1/6*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*
x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d*e + 1/90*(15*x^6*arc
csc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) +
 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(169) = 338$.

Time = 0.39 (sec) , antiderivative size = 1160, normalized size of antiderivative = 5.95

$$\int x(d+ex^2)^2 (a+b\csc^{-1}(cx)) \, dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/5760*(15*b*e^2*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c + 15*
a*e^2*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 6*b*e^2*x^5*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^5/c^2 + 180*b*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c
*x))/c + 180*a*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 90*b*e^2*x^4*(sqr
t(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 90*a*e^2*x^4*(sqrt(-1/(c^2
*x^2) + 1) + 1)^4/c^3 + 120*b*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 +
720*b*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + 720*a*d^2*
x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c + 50*b*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^3/c^4 + 720*b*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/
c^3 + 720*a*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 1440*b*d^2*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)/c^2 + 225*b*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*
arcsin(1/(c*x))/c^5 + 225*a*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 10
80*b*d*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 1440*b*d^2*arcsin(1/(c*x))/c^
3 + 1440*a*d^2/c^3 + 300*b*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 + 1080*b*
d*e*arcsin(1/(c*x))/c^5 + 1080*a*d*e/c^5 - 1440*b*d^2/(c^4*x*(sqrt(-1/(c^2*
x^2) + 1) + 1)) + 300*b*e^2*arcsin(1/(c*x))/c^7 + 300*a*e^2/c^7 - 1080*b*d*
e/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 720*b*d^2*arcsin(1/(c*x))/(c^5*x^2
*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 720*a*d^2/(c^5*x^2*(sqrt(-1/(c^2*x^2) +
1) + 1)^2) - 300*b*e^2/(c^8*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 720*b*d*e*arc
sin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 720*a*d*e/(c^7*x^2*
(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 225*b*e^2*arcsin(1/(c*x))/(c^9*x^2*(sqrt(
-1/(c^2*x^2) + 1) + 1)^2) + 225*a*e^2/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2) - 120*b*d*e/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 50*b*e^2/(c^10*x
^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 180*b*d*e*arcsin(1/(c*x))/(c^9*x^4*(sq
rt(-1/(c^2*x^2) + 1) + 1)^4) + 180*a*d*e/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) +
 1)^4) + 90*b*e^2*arcsin(1/(c*x))/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4)
 + 90*a*e^2/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 6*b*e^2/(c^12*x^5*(
sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 15*b*e^2*arcsin(1/(c*x))/(c^13*x^6*(sqrt(-
1/(c^2*x^2) + 1) + 1)^6) + 15*a*e^2/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^
6))*c
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx=\int x\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

3.96 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 186

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x}\,dx &= \frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}\\&+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+dex^2(a+b\csc^{-1}(cx))\\&+\frac{1}{4}e^2x^4(a+b\csc^{-1}(cx))\\&-bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\&+bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right)-d^2(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right)\\&+\frac{1}{2}ibd^2\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

```
[Out] 1/2*I*b*d^2*arccsc(c*x)^2+d*e*x^2*(a+b*arccsc(c*x))+1/4*e^2*x^4*(a+b*arccsc
(c*x))-b*d^2*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*d^2*arccsc(c
*x)*ln(1/x)-d^2*(a+b*arccsc(c*x))*ln(1/x)+1/2*I*b*d^2*polylog(2,(I/c/x+(1-1
/c^2/x^2)^(1/2))^2)+1/6*b*e*(6*c^2*d+e)*x*(1-1/c^2/x^2)^(1/2)/c^3+1/12*b*e^
2*x^3*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {5349, 272, 45, 4815, 6874, 464, 270, 2363, 4721, 3798, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x}\,dx = & -d^2\log\left(\frac{1}{x}\right)(a+b\csc^{-1}(cx)) \\ & + dex^2(a+b\csc^{-1}(cx)) + \frac{1}{4}e^2x^4(a+b\csc^{-1}(cx)) \\ & + \frac{be^2x^3\sqrt{1-\frac{1}{c^2x^2}}}{12c} + \frac{bex\sqrt{1-\frac{1}{c^2x^2}}(6c^2d+e)}{6c^3} \\ & + \frac{1}{2}ibd^2\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right) + \frac{1}{2}ibd^2\csc^{-1}(cx)^2 \\ & - bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) \\ & + bd^2\log\left(\frac{1}{x}\right)\csc^{-1}(cx)\end{aligned}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] (b*e*(6*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x)/(6*c^3) + (b*e^2*Sqrt[1 - 1/(c^
2*x^2)]*x^3)/(12*c) + (I/2)*b*d^2*ArcCsc[c*x]^2 + d*e*x^2*(a + b*ArcCsc[c*x
]) + (e^2*x^4*(a + b*ArcCsc[c*x]))/4 - b*d^2*ArcCsc[c*x]*Log[1 - E^((2*I)*A
rcCsc[c*x])] + b*d^2*ArcCsc[c*x]*Log[x^(-1)] - d^2*(a + b*ArcCsc[c*x])*Log[
x^(-1)] + (I/2)*b*d^2*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4815

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)^2\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right) - d^2\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\frac{b\text{Subst}\left(\int \frac{-\frac{e\left(e+4dx^2\right)}{4x^4}+d^2\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)-d^2\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \left(-\frac{e\left(e+4dx^2\right)}{4x^4\sqrt{1-\frac{x^2}{c^2}}}+\frac{d^2\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\right)dx, x, \frac{1}{x}\right)}{c}$$

$$= dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)-d^2\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd^2)\,\text{Subst}\left(\int \frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c} - \frac{(be)\text{Subst}\left(\int \frac{e+4dx^2}{x^4\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4c}$$

$$=\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c}+dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)$$
$$+bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right)-d^2\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)$$
$$-\left(bd^2\right)\text{Subst}\left(\int\frac{\arcsin\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right)$$
$$-\frac{\left(be(6c^2d+e)\right)\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{6c^3}$$

$$=\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c}+dex^2\left(a+b\csc^{-1}(cx)\right)$$
$$+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)+bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right)$$
$$-d^2\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-\left(bd^2\right)\text{Subst}\left(\int x\cot(x)\,dx,x,\csc^{-1}(cx)\right)$$

$$=\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2$$
$$+dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)+bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right)$$
$$-d^2\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\left(2ibd^2\right)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx,x,\csc^{-1}(cx)\right)$$

$$=\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+dex^2\left(a+b\csc^{-1}(cx)\right)$$
$$+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)-bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right)$$
$$-d^2\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)+\left(bd^2\right)\text{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx,x,\csc^{-1}(cx)\right)$$

$$=\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+dex^2\left(a+b\csc^{-1}(cx)\right)$$
$$+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)-bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right)$$
$$-d^2\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-\frac{1}{2}\left(ibd^2\right)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i\csc^{-1}(cx)}\right)$$

$$
\begin{aligned}
&= \frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c} + \frac{1}{2}ibd^2\csc^{-1}(cx)^2 \\
&\quad + dex^2\left(a+b\csc^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right) \\
&\quad - bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - d^2\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{1}{2}ibd^2\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84

$$
\begin{aligned}
\int \frac{(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)}{x}\,dx &= adex^2 + \frac{1}{4}ae^2x^4 + \frac{bdex\left(\sqrt{1-\frac{1}{c^2x^2}}+cx\csc^{-1}(cx)\right)}{c} \\
&\quad + \frac{be^2x\left(\sqrt{1-\frac{1}{c^2x^2}}(2+c^2x^2)+3c^3x^3\csc^{-1}(cx)\right)}{12c^3} \\
&\quad + ad^2\log(x) \\
&\quad + \frac{1}{2}ibd^2\left(\csc^{-1}(cx)\left(\csc^{-1}(cx)+2i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\right)\right. \\
&\quad \left. + \mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right)
\end{aligned}
$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 + (b*d*e*x*(Sqrt[1 - 1/(c^2*x^2)] + c*x*ArcCsc[c*
x]))/c + (b*e^2*x*(Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2) + 3*c^3*x^3*ArcCsc[c
*x]))/(12*c^3) + a*d^2*Log[x] + (I/2)*b*d^2*(ArcCsc[c*x]*(ArcCsc[c*x] + (2*
I)*Log[1 - E^((2*I)*ArcCsc[c*x])]) + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.55

method	result
parts	$a\left(\frac{e^2x^4}{4}+de\,x^2+d^2\ln\left(x\right)\right)+b\left(\frac{id^2\,\mathrm{arccsc}(cx)^2}{2}+\frac{e\left(12c^4d\,\mathrm{arccsc}(cx)x^2+3\,\mathrm{arccsc}(cx)e\,c^4x^4+12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right.}{12}\right.$
derivativedivides	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln\left(cx\right)+\frac{b\left(\frac{ic^4d^2\,\mathrm{arccsc}(cx)^2}{2}+\frac{e\left(12c^4d\,\mathrm{arccsc}(cx)x^2+3\,\mathrm{arccsc}(cx)e\,c^4x^4+12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx+\right.}{12}\right.}{}$
default	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln\left(cx\right)+\frac{b\left(\frac{ic^4d^2\,\mathrm{arccsc}(cx)^2}{2}+\frac{e\left(12c^4d\,\mathrm{arccsc}(cx)x^2+3\,\mathrm{arccsc}(cx)e\,c^4x^4+12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx+\right.}{12}\right.}{}$

[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x,method=_RETURNVERBOSE)

[Out] a*(1/4*e^2*x^4+d*e*x^2+d^2*ln(x))+b*(1/2*I*d^2*arccsc(c*x)^2+1/12/c^4*e*(12*c^4*d*arccsc(c*x)*x^2+3*arccsc(c*x)*e*c^4*x^4+12*((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*e*c^3*x^3-12*I*c^2*d+2*((c^2*x^2-1)/c^2/x^2)^(1/2)*e*c*x-2*I*e)-d^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-d^2*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*d^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+I*d^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2)))

Fricas [F]

$$\int\frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x}\,dx=\int\frac{(ex^2+d)^2(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="fricas")

[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arccsc(c*x))/x, x)

Sympy [F]

$$\int\frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x}\,dx=\int\frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)^2}{x}\,dx$$

[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x,x)

[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**2/x, x)

Maxima [F]

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(ex^2+d\right)^2\left(b\,\mathrm{arccsc}\,(cx)+a\right)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + 1/8*(4*I*b*c^4*d^2*log(-c*x + 1)
*log(x) + 4*I*b*c^4*d^2*log(x)^2 + 4*I*b*c^4*d^2*dilog(c*x) + 4*I*b*c^4*d^2
*dilog(-c*x) + 2*(b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^4*l
og(c))*e^2*x^4 - I*(b*e^2*(x^2/c^2 + log(c*x + 1)/c^4 + log(c*x - 1)/c^4) +
 4*b*d*e*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 32*b*d^2*integrate(1/4*log
(x)/(c^2*x^3 - x), x))*c^4 + 8*c^4*integrate(1/4*(b*e^2*x^4 + 4*b*d*e*x^2 +
 4*b*d^2*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) + (I*b*c^2*e
^2 + 8*(b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^4*log(c))*d*e
)*x^2 + (-I*b*c^4*e^2*x^4 - 4*I*b*c^4*d*e*x^2 - 4*I*b*c^4*d^2*log(x))*log(c
^2*x^2) + (4*I*b*c^4*d^2*log(x) + 4*I*b*c^2*d*e + I*b*e^2)*log(c*x + 1) + (
4*I*b*c^2*d*e + I*b*e^2)*log(c*x - 1) - 2*(-I*b*c^4*e^2*x^4 - 4*I*b*c^4*d*e
*x^2 - 4*(b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^4*log(c))*d
^2)*log(x))/c^4
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expansi
on Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{x}\,dx = \int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x, x)
```

3.97 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 189

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^3}\,dx &= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} \\ &\quad+\frac{1}{4}bc^2d^2\csc^{-1}(cx)+ibde\csc^{-1}(cx)^2 \\ &\quad-\frac{d^2(a+b\csc^{-1}(cx))}{2x^2}+\frac{1}{2}e^2x^2(a+b\csc^{-1}(cx)) \\ &\quad-2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) \\ &\quad+2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) \\ &\quad-2de(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) \\ &\quad+ibde\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

```
[Out] 1/4*b*c^2*d^2*arccsc(c*x)+I*b*d*e*arccsc(c*x)^2-1/2*d^2*(a+b*arccsc(c*x))/x
^2+1/2*e^2*x^2*(a+b*arccsc(c*x))-2*b*d*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x
^2)^(1/2))^2)+2*b*d*e*arccsc(c*x)*ln(1/x)-2*d*e*(a+b*arccsc(c*x))*ln(1/x)+I
*b*d*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-1/4*b*c*d^2*(1-1/c^2/x^2)^(
1/2)/x+1/2*b*e^2*x*(1-1/c^2/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5349, 272, 45, 4815, 12, 6874, 270, 327, 222, 2363, 4721, 3798, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3}\,dx = & -\frac{d^2(a+b\csc^{-1}(cx))}{2x^2} - 2de\log\left(\frac{1}{x}\right)(a+b\csc^{-1}(cx)) \\ & + \frac{1}{2}e^2x^2(a+b\csc^{-1}(cx)) - \frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} \\ & + \frac{1}{4}bc^2d^2\csc^{-1}(cx) + \frac{be^2x\sqrt{1-\frac{1}{c^2x^2}}}{2c} \\ & + ibde\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right) + ibde\csc^{-1}(cx)^2 \\ & - 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) \\ & + 2bde\log\left(\frac{1}{x}\right)\csc^{-1}(cx)\end{aligned}$$

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] -1/4*(b*c*d^2*Sqrt[1 - 1/(c^2*x^2)])/x + (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)/(2
*c) + (b*c^2*d^2*ArcCsc[c*x])/4 + I*b*d*e*ArcCsc[c*x]^2 - (d^2*(a + b*ArcCs
c[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcCsc[c*x]))/2 - 2*b*d*e*ArcCsc[c*x]*Lo
g[1 - E^((2*I)*ArcCsc[c*x])] + 2*b*d*e*ArcCsc[c*x]*Log[x^(-1)] - 2*d*e*(a +
 b*ArcCsc[c*x])*Log[x^(-1)] + I*b*d*e*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4815

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(e+dx^2)^2\,\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\big(a+b\csc^{-1}(cx)\big) - 2de\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{-\frac{e^2}{x^2}+d^2x^2+4de\log(x)}{2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\big(a+b\csc^{-1}(cx)\big) - 2de\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{-\frac{e^2}{x^2}+d^2x^2+4de\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$
\begin{aligned}
&= -\frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right) - 2de\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\
&\quad + \frac{b\text{Subst}\left(\int\left(-\frac{e^2}{x^2\sqrt{1-\frac{x^2}{c^2}}} + \frac{d^2x^2}{\sqrt{1-\frac{x^2}{c^2}}} + \frac{4de\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}\right)dx, x, \frac{1}{x}\right)}{2c} \\
&= -\frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right) \\
&\quad - 2de\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd^2)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{2c} \\
&\quad + \frac{(2bde)\text{Subst}\left(\int\frac{\log(x)}{\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{c} - \frac{(be^2)\,\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{2c} \\
&= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right) \\
&\quad + 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - 2de\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\
&\quad + \frac{1}{4}\left(bcd^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right) - (2bde)\text{Subst}\left(\int\frac{\arcsin\left(\frac{x}{c}\right)}{x}dx, x, \frac{1}{x}\right) \\
&= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{4}bc^2d^2\csc^{-1}(cx) \\
&\quad - \frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right) + 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - 2de\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) - (2bde)\text{Subst}\left(\int x\cot(x)\,dx, x, \csc^{-1}(cx)\right) \\
&= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{4}bc^2d^2\csc^{-1}(cx) + ibde\csc^{-1}(cx)^2 \\
&\quad - \frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right) + 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) \\
&\quad - 2de\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + (4ibde)\text{Subst}\left(\int\frac{e^{2ix}x}{1-e^{2ix}}dx, x, \csc^{-1}(cx)\right)
\end{aligned}
$$

$$= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{1}{4}bc^2d^2\csc^{-1}(cx)$$
$$+ ibde\csc^{-1}(cx)^2 - \frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\csc^{-1}(cx))$$
$$- 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right)$$
$$- 2de(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) + (2bde)\text{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)$$

$$= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{1}{4}bc^2d^2\csc^{-1}(cx)$$
$$+ ibde\csc^{-1}(cx)^2 - \frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\csc^{-1}(cx))$$
$$- 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right)$$
$$- 2de(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) - (ibde)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)$$

$$= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x}{2c} + \frac{1}{4}bc^2d^2\csc^{-1}(cx)$$
$$+ ibde\csc^{-1}(cx)^2 - \frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\csc^{-1}(cx))$$
$$- 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right)$$
$$- 2de(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) + ibde\,\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3} dx$$
$$= \frac{1}{4}\left(-\frac{2ad^2}{x^2}+2ae^2x^2-\frac{2bd^2\csc^{-1}(cx)}{x^2}+\frac{2be^2x\left(\sqrt{1-\frac{1}{c^2x^2}}+cx\csc^{-1}(cx)\right)}{c} - \frac{bd^2\left(-1+c^2x^2+c^2x^2\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)\right)}{c\sqrt{1-\frac{1}{c^2x^2}}x^3} - 8bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+8ade\log(x) + 4ibde\left(\csc^{-1}(cx)^2+\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right)\right)$$

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*d^2*ArcCsc[c*x])/x^2 + (2*b*e^2*x*(Sqr
t[1 - 1/(c^2*x^2)] + c*x*ArcCsc[c*x]))/c - (b*d^2*(-1 + c^2*x^2 + c^2*x^2*S
qrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]]))/(c*Sqrt[1 - 1/(c^2*x^2)]*x^3
) - 8*b*d*e*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + 8*a*d*e*Log[x] + (
4*I)*b*d*e*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/4
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.45

method	result
parts	$a\left(\frac{e^2x^2}{2}-\frac{d^2}{2x^2}+2de\ln(x)\right)+ibde\,\text{arccsc}(cx)^2-\frac{bc\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x}+\frac{b\,c^2d^2\,\text{arccsc}(cx)}{4}-\frac{b\,\text{arccsc}(cx)d}{2x^2}$
derivativedivides	$c^2\left(\frac{a\,x^2e^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{ibde\,\text{arccsc}(cx)^2}{c^2}-\frac{b\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{b\,\text{arccsc}(cx)d^2}{4}-\frac{b\,\text{arccsc}(cx)d^2}{2c^2x^2}+b\right.$
default	$c^2\left(\frac{a\,x^2e^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{ibde\,\text{arccsc}(cx)^2}{c^2}-\frac{b\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{b\,\text{arccsc}(cx)d^2}{4}-\frac{b\,\text{arccsc}(cx)d^2}{2c^2x^2}+b\right.$

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2*e^2*x^2-1/2*d^2/x^2+2*d*e*ln(x))+I*b*d*e*arccsc(c*x)^2-1/4*b*c*d^2/x
*((c^2*x^2-1)/c^2/x^2)^(1/2)+1/4*b*c^2*d^2*arccsc(c*x)-1/2*b*arccsc(c*x)*d^
2/x^2+1/2*b*e^2*arccsc(c*x)*x^2+1/2*b/c*e^2*((c^2*x^2-1)/c^2/x^2)^(1/2)*x-1
```

```
/2*I*b/c^2*e^2-2*b*d*e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+2*I*b*d*
e*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-2*b*d*e*arccsc(c*x)*ln(1-I/c/x-(1-1
/c^2/x^2)^(1/2))+2*I*b*d*e*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))
```

Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3} \, dx = \int \frac{(ex^2+d)^2 (b\,\mathrm{arccsc}\,(cx)+a)}{x^3} \, dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsc(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3} \, dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)^2}{x^3} \, dx$$

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3} \, dx = \int \frac{(ex^2+d)^2 (b\,\mathrm{arccsc}\,(cx)+a)}{x^3} \, dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*e^2*x^2 + 1/4*b*d^2*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x
^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x
^2) + 2*a*d*e*log(x) - 1/2*a*d^2/x^2 + 1/4*(4*I*b*c^2*d*e*log(-c*x + 1)*log
(x) + 4*I*b*c^2*d*e*log(x)^2 + 4*I*b*c^2*d*e*dilog(c*x) + 4*I*b*c^2*d*e*dil
og(-c*x) + 2*(b*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^2*log(c
))*e^2*x^2 + I*b*e^2*log(c*x - 1) - I*(b*e^2*(log(c*x + 1)/c^2 + log(c*x -
1)/c^2) + 16*b*d*e*integrate(1/2*log(x)/(c^2*x^3 - x), x))*c^2 + 4*c^2*inte
grate(1/2*(b*e^2*x^2 + 4*b*d*e*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3
 - x), x) + (-I*b*c^2*e^2*x^2 - 4*I*b*c^2*d*e*log(x))*log(c^2*x^2) + (4*I*b
*c^2*d*e*log(x) + I*b*e^2)*log(c*x + 1) - 2*(-I*b*c^2*e^2*x^2 - 4*(b*c^2*ar
ctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^2*log(c))*d*e)*log(x))/c^2
```

Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\operatorname{arccsc}(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsc(c*x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^3, x)
```

3.98 $\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 565

$$\begin{aligned}
\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx &= \frac{x(a+b\csc^{-1}(cx))}{e} + \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} \\
&- \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+ \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&- \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+ \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&- \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+ \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&- \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+ \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}
\end{aligned}$$

```
[Out] x*(a+b*arccsc(c*x))/e+b*arctanh((1-1/c^2/x^2)^(1/2))/c/e-1/2*(a+b*arccsc(c*
x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
```

```
)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*ar
ccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+
e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*
I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*I*b*polylo
g(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*
(-d)^(1/2)/e^(3/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5349, 4817, 4723, 272, 65, 214, 4757, 4825, 4615, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = &-\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}\\&+\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}\\&-\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}\\&+\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}\\&+\frac{x(a+b\csc^{-1}(cx))}{e}+\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce}\\&-\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}\\&+\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}\\&-\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}\\&+\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}\end{aligned}$$

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

```
[Out] (x*(a + b*ArcCsc[c*x]))/e + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e) - (Sqr
t[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 + (
I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) -
 (Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sq
rt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[
1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/
2)) - ((I/2)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] - Sqrt[c^2*d + e])])/e^(3/2) + ((I/2)*b*Sqrt[-d]*PolyLog[2, (I*c*Sqrt[
-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(3/2) - ((I/2)*b*Sqr
t[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d
+ e])])/e^(3/2) + ((I/2)*b*Sqrt[-d]*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*
x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
```

```
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{ex^2}-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e}+\frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}\\
&= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e}-\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce}\\
&\quad+\frac{d\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{e}\\
&= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e}+\frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{3/2}}\\
&\quad+\frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{3/2}}-\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ce}\\
&= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e}+\frac{d\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2e^{3/2}}\\
&\quad+\frac{d\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2e^{3/2}}\\
&\quad+\frac{(bc)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)}{e}
\end{aligned}
$$

$$=\frac{x(a+b\csc^{-1}(cx))}{e}+\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce}+\frac{d\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^{3/2}}+\frac{d\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^{3/2}}+\frac{d\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^{3/2}}+\frac{d\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^{3/2}}$$

$$=\frac{x(a+b\csc^{-1}(cx))}{e}+\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce}-\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{2e^{3/2}}-\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{2e^{3/2}}+\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{2e^{3/2}}-\frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{2e^{3/2}}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e} + \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^{3/2}} + \frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^{3/2}} - \frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^{3/2}} + \frac{(ib\sqrt{-d})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^{3/2}}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e} + \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1260 vs. 2(565) = 1130.

Time = 1.82 (sec) , antiderivative size = 1260, normalized size of antiderivative = 2.23

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$$

$$= \frac{i\left(-4iac\sqrt{e}x - 4ibc\sqrt{e}x\csc^{-1}(cx) + 4iac\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + 8ibc\sqrt{d}\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right.}{}\right.\right.}{}$$

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

```
[Out] ((I/4)*((-4*I)*a*c*Sqrt[e]*x - (4*I)*b*c*Sqrt[e]*x*ArcCsc[c*x] + (4*I)*a*c*
Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 - (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi +
 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2
*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + b*c*Sqrt[d]*Pi*Log[1 + (Sqrt[e] - Sqrt
[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*c*Sqrt[d]*ArcCsc[c*x]*Log
[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*c*Sqr
t[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*c*Sqrt[d]*Pi*Log[1 + (-S
```

```
qrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*b*c*Sqrt[d]*Ar
cCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])
)] - 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1
+ (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*c*Sqrt[d]
*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*
b*c*Sqrt[d]*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
 b*c*Sqrt[d]*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] - 2*b*c*Sqrt[d]*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] + b*c*Sqrt[d]*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - b*c*Sqrt[d]*Pi*L
og[Sqrt[e] + (I*Sqrt[d])/x] - (4*I)*b*Sqrt[e]*Log[Cos[ArcCsc[c*x]/2]] + (4*
I)*b*Sqrt[e]*Log[Sin[ArcCsc[c*x]/2]] + (2*I)*b*c*Sqrt[d]*PolyLog[2, (Sqrt[e
] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)*b*c*Sqrt[d]*Pol
yLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)
*b*c*Sqrt[d]*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x])))] + (2*I)*b*c*Sqrt[d]*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sq
rt[d]*E^(I*ArcCsc[c*x]))]))/(c*e^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 44.19 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.73

method	result
parts	$\frac{ax}{e}-\frac{ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+\frac{b\left(\frac{c^{3}\operatorname{arccsc}(cx)x}{e}-\frac{c^{2}\ln\left(-1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)}{e}+\frac{c^{2}\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)}{e}-\frac{c^{4}d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}\right.}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^{3}x}{e}-\frac{a\,c^{3}d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+b\,c^{2}\left(\frac{cx\,\operatorname{arccsc}(cx)}{e}-\frac{c^{2}d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}d\_Z^{4}+\left(-2c^{2}d-4e\right)\_Z^{2}+c^{2}d\right)}\frac{\left(\_R1^{2}c^{2}d-c^{2}d-4e\right)}{}\right.}{8}\right.}{}$
default	$\frac{\frac{a\,c^{3}x}{e}-\frac{a\,c^{3}d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+b\,c^{2}\left(\frac{cx\,\operatorname{arccsc}(cx)}{e}-\frac{c^{2}d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}d\_Z^{4}+\left(-2c^{2}d-4e\right)\_Z^{2}+c^{2}d\right)}\frac{\left(\_R1^{2}c^{2}d-c^{2}d-4e\right)}{}\right.}{8}\right.}{}$

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] a/e*x-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c^3*(c^3*arccsc(c*x)/e*x-
c^2/e*ln(-1+I/c/x+(1-1/c^2/x^2)^(1/2))+c^2/e*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2)
)-1/8*c^4/e^2*d*sum((_R1^2*c^2*d-c^2*d-4*e)/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*
arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c
^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/8*
c^4/e^2*d*sum((_R1^2*c^2*d+4*_R1^2*e-c^2*d)/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*
arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c
^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{d + ex^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^2}{ex^2 + d}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{d + ex^2}\,dx = \int \frac{x^2(a + b\,\mathrm{acsc}\,(cx))}{d + ex^2}\,dx$$

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d),x)
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{d + ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{d + ex^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d} \,dx$$

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

3.99 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 507

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} - \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e}$$

```
[Out] -(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e+1/2*(a+b*arccsc(c*
x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
```

```
)))/e+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsc(c*x))
*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
/e+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e-1/2*I*b*polylog(2,-I*
c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e-1/2*I
*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^
(1/2)))/e-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(c^2*d+e)^(1/2)))/e-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))
*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5349, 4817, 4721, 3798, 2221, 2317, 2438, 4825, 4615}

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e} - \frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{e} - \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e} - \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e} - \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e} - \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e} + \frac{ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e}$$

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
[Out] ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sq
rt[c^2*d + e])])/(2*e) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*Ar
```

```
cCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcCsc[c*x])*Log[
1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) +
((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(2*e) - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])
])/e - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - S
qrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] - Sqrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I
*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, (I*c*S
qrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e + ((I/2)*b*PolyL
og[2, E^((2*I)*ArcCsc[c*x])])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x\,(e+dx^2)}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{ex} - \frac{dx\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e\,(e+dx^2)}\right)dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}\\
&= -\frac{\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)}{e}\\
&\quad + \frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{e}\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be} + \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)}{e}\\
&\quad - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e} + \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e}
\end{aligned}
$$

$$=\frac{i(a+b\csc^{-1}(cx))^2}{2be}-\frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e}$$
$$+\frac{b\text{Subst}\left(\int\log\left(1-e^{2ix}\right)\,dx,x,\csc^{-1}(cx)\right)}{e}$$
$$-\frac{\sqrt{-d}\text{Subst}\left(\int\frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx,x,\csc^{-1}(cx)\right)}{2e}$$
$$+\frac{\sqrt{-d}\text{Subst}\left(\int\frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx,x,\csc^{-1}(cx)\right)}{2e}$$
$$=-\frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e}-\frac{(ib)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i\csc^{-1}(cx)}\right)}{2e}$$
$$+\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e}$$
$$+\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e}$$
$$-\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e}$$
$$-\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e}$$

$$= \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} - \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e} - \frac{b\mathrm{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e} - \frac{b\mathrm{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e} - \frac{b\mathrm{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e} - \frac{b\mathrm{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e}$$

$$= \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} - \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} + \frac{ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e} + \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e} + \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e} + \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e} + \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e}$$

$$
\begin{aligned}
&= \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&- \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\
&- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\
&- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1123 vs. 2(507) = 1014.

Time = 0.45 (sec) , antiderivative size = 1123, normalized size of antiderivative = 2.21

$$
\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx
$$

$$
= \frac{ib\pi^2 - 4ib\pi\csc^{-1}(cx) + 8ib\csc^{-1}(cx)^2 - 16ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}(\pi+2\csc^{-1}(cx))\right)}{\sqrt{c^2d+e}}\right) -
$$

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

```
[Out] (I*b*Pi^2 - (4*I)*b*Pi*ArcCsc[c*x] + (8*I)*b*ArcCsc[c*x]^2 - (16*I)*b*ArcSi
n[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt
[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*b*ArcSin[Sqrt[1
 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(P
i + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e]
 - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 - (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sq
rt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d
```

```
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*S
qrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] - 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Arc
Csc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2
]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b
*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*
b*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*
x]))] + 8*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcCsc[c*x]*Log
[1 - E^((2*I)*ArcCsc[c*x])] + 2*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 2*b*Pi*
Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*a*Log[d + e*x^2] + (4*I)*b*PolyLog[2, (Sqr
t[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2,
 (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*Poly
Log[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (4*I
)*b*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
 (4*I)*b*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(8*e)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.28 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.78

method	result
parts	$\frac{a\ln(e\,x^2+d)}{2e}-\frac{ib\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{\left(\_R1^2c^2d-c^2d-4e\right)\left(i\,\mathrm{arccsc}(cx)\ln\left(\frac{\_R1-\frac{i}{cx}-\sqrt{1}}{\_R1}\right.\right.}{\_R1^2c^2d-c^2d}\right.}{4e}$
derivativedivides	$\frac{\frac{a\,c^2\ln\left(c^2e\,x^2+c^2d\right)}{2e}+b\,c^2\left(-\frac{i\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}-\frac{\mathrm{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{i\,\mathrm{dilog}\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}-\frac{i\left(\_R1=\right.}{}\right.}{}$
default	$\frac{\frac{a\,c^2\ln\left(c^2e\,x^2+c^2d\right)}{2e}+b\,c^2\left(-\frac{i\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}-\frac{\mathrm{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{i\,\mathrm{dilog}\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}-\frac{i\left(\_R1=\right.}{}\right.}{}$

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/e*ln(e*x^2+d)-1/4*I*b/e*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*
d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/
c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^
2*d))-b/e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I*b/e*dilog(1+I/c/x+(
1-1/c^2/x^2)^(1/2))-I*b/e*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I*b*c^2*d/e*
sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2
/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^
2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{ex^2+d}\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*x*arccsc(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx=\int \frac{x(a+b\,\mathrm{acsc}\,(cx))}{d+ex^2}\,dx$$

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d),x)
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{ex^2+d}\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] b*integrate(x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x) + 1/2
*a*log(e*x^2 + d)/e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

3.100 $\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 529

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx = &-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

```
[Out] -1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/
e^(1/2)-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccsc(c*x))*ln(1
+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)
^(1/2)/e^(1/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,I*c*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(
```

```
1/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5339, 4757, 4825, 4615, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx = &-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2),x]
```

```
[Out] -1/2*((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCsc[c*x])*Log[1 + (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqr
t[e]) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt
[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCsc[c*x])*Log[1
+ (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d
]*Sqrt[e]) - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[
e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[
-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) - (
(I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*
d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCs
c[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5339

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{e}}$$

$$= -\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{2\sqrt{-d}\sqrt{e}}$$

$$= -\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{2\sqrt{-d}\sqrt{e}}$$

$$= -\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1068 vs. $2(529) = 1058$.

Time = 0.45 (sec) , antiderivative size = 1068, normalized size of antiderivative = 2.02

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx = -\frac{i\left(4ia\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+8ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2\csc^{-1}(cx)\right)\right)}{\sqrt{c^2d+e}}\right)-8ib\arcsin\left(\frac{\sqrt{1+}}{\sqrt{}}\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2),x]
```

```
[Out] ((-1/4*I)*((4*I)*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*ArcSin[Sqrt[1 - (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi
+ 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c
*x])/4])/Sqrt[c^2*d + e]] + b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - 2*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] - b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] + 2*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))] - 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*
```

```
Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*Pi*
Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*b*Ar
cCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))
] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + b*Pi*Log[1 + (Sqrt[e] +
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*ArcCsc[c*x]*Log[1 +
(Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*ArcSin[Sqr
t[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])
/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - b*Pi*
Log[Sqrt[e] + (I*Sqrt[d])/x] + (2*I)*b*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)*b*PolyLog[2, (-Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)*b*PolyLog[2, -((Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (2*I)*b*PolyLog[2, (Sqrt[
e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]))/(Sqrt[d]*Sqrt[e])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.99 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.51

method	result
parts	$\frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - \frac{bc\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)} \frac{i\operatorname{arccsc}(cx)\ln\left(\frac{\_R1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R\cdots}{\cdots}\right.}{\_R1\left(\_R1^2c^2d-c^2d-2e\right)}\right.}{2}$
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)} \frac{i\operatorname{arccsc}(cx)\ln\left(\frac{\_R1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\cdots\right.}{\_R1\left(\_R1^2c^2d-c^2d-2e\right.}\right.}{2}\right.}{\cdots}$
default	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)} \frac{i\operatorname{arccsc}(cx)\ln\left(\frac{\_R1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\cdots\right.}{\_R1\left(\_R1^2c^2d-c^2d-2e\right.}\right.}{2}\right.}{\cdots}$

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*b*c*sum(1/_R1/(_R1^2*c^2*d-c^2*d-
2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/
x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*
d))-1/2*b*c*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Roo
tOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx=\int \frac{b\operatorname{arccsc}(cx)+a}{ex^2+d}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx=\int \frac{a+b\operatorname{acsc}(cx)}{d+ex^2}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{e\,x^2+d}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2), x)
```

3.101 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 479

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)}\,dx = \frac{i(a+b\csc^{-1}(cx))^2}{2bd} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$

```
[Out] 1/2*I*(a+b*arccsc(c*x))^2/b/d-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsc(c*x))
*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/d-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,-I*c*(
I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d+1/2*I*b*
polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/
2)))/d+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5349, 4817, 4825, 4615, 2221, 2317, 2438}

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)}\,dx = -\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d} - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d} + \frac{i(a+b\csc^{-1}(cx))^2}{2bd} + \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcC
sc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e
])])/(2*d) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) + ((I/2)*b*Pol
yLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d
 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/d + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/d + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)\\ &= -\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\frac{1}{x}\right)\end{aligned}$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}} + \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{-d}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{-d}}$$
$$+ \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{-d}}$$
$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{-d}}$$
$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{-d}}$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2bd} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int \log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \csc^{-1}(cx)\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int \log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \csc^{-1}(cx)\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int \log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \csc^{-1}(cx)\right)}{2d}$$
$$+ \frac{b\text{Subst}\left(\int \log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \csc^{-1}(cx)\right)}{2d}$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2bd} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{2d}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{2d}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{2d}$$
$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{2d}$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2bd} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$
$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$
$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1089 vs. 2(479) = 958.

Time = 0.38 (sec) , antiderivative size = 1089, normalized size of antiderivative = 2.27

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)}\,dx =$$

$$-\frac{ib\pi^2-4ib\pi\csc^{-1}(cx)+4ib\csc^{-1}(cx)^2-16ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2\csc^{-1}(cx)\right)\right)}{\sqrt{c^2d+e}}\right)}{}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] -1/8*(I*b*Pi^2 - (4*I)*b*Pi*ArcCsc[c*x] + (4*I)*b*ArcCsc[c*x]^2 - (16*I)*b*
ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] +
 Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*b*ArcSin[S
qrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*C
ot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*b*Pi*Log[1 + (Sqrt[e] - Sq
rt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (Sq
rt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1
 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c
^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] - 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(
I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*S
qrt[d]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/S
qrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 - 2*b*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 + 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] + 8*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 +
(Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*b*Pi*Log[Sqr
t[e] - (I*Sqrt[d])/x] + 2*b*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] - 8*a*Log[x] +
4*a*Log[d + e*x^2] + (4*I)*b*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (4*I)*b*PolyLog[2, (Sqrt[e] + Sqrt[
c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/d
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.11 (sec) , antiderivative size = 1934, normalized size of antiderivative = 4.04

method	result	size
parts	Expression too large to display	1934
derivativedivides	Expression too large to display	1961
default	Expression too large to display	1961

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*ln(x)-1/2*a/d*ln(e*x^2+d)+b*(I*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arccsc
(c*x)^2*e/d^3/c^4+1/4*I*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*arccsc(c*x)^2*c^2-1
/8*I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*po
lylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*
e))/d/e/(c^2*d+e)+1/4*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^
(1/2)*e+2*e^2)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+
e))^(1/2)+2*e))*arccsc(c*x)/d/e/(c^2*d+e)+1/4*I*(c^2*d+2*(e*(c^2*d+e))^(1/2
)+2*e)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))
^(1/2)+2*e))/c^2/d^2+((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(
1/2)*e+2*e^2)*e*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d
+e))^(1/2)+2*e))*arccsc(c*x)/c^4/(c^2*d+e)/d^3+1/2*I/d*arccsc(c*x)^2-(c^2*d
+2*(e*(c^2*d+e))^(1/2)+2*e)*e*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2
*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^3/c^4+1/8*I*(e*(c^2*d+e))^(1/2
)/e/(c^2*d+e)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^
2*d+e))^(1/2)+2*e))*c^2+((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e)
)^(1/2)*e+2*e^2)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e))*arccsc(c*x)/c^2/(c^2*d+e)/d^2-1/4*(e*(c^2*d+e))^(1/2)/e/(
c^2*d+e)*c^2*arccsc(c*x)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*
(e*(c^2*d+e))^(1/2)+2*e))+1/4*I*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*polylog(2,d
*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))+1/2*I
*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*arccsc(c*x)^2+1/2*I*(c^2*d+2*(e*(c^2*d+e))
^(1/2)+2*e)*arccsc(c*x)^2/c^2/d^2-I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*
(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arccsc(c*x)^2/c^4/(c^2*d+e)/d^3-1/4*I*((e*(c
^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arccsc(c*x)^2
/d/e/(c^2*d+e)-1/2*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*arccsc(c*x)*ln(1-d*c^2*(
I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))-1/2*(c^2*d+
2*(e*(c^2*d+e))^(1/2)+2*e)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-
2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/c^2/d^2-1/2*I*((e*(c^2*d+e))^(1/2)*
c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*polylog(2,d*c^2*(I/c/x+(1-1/
c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/c^2/(c^2*d+e)/d^2-I*((
e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arccsc(c*
x)^2/c^2/(c^2*d+e)/d^2+1/2*I*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*polylog(2,d*
c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e/d^3/
```

```
c^4+1/2*I/d*sum((_R1^2*c^2*d-2*c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc
(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2
)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/2*I*((e*
(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*polylog(2
,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/c^4
/(c^2*d+e)/d^3)
```

Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x\,(d+ex^2)}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)x}\,dx$$

[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="fricas")

[Out] integral((b*arccsc(c*x) + a)/(e*x^3 + d*x), x)

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x\,(d+ex^2)}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{x\,(d+ex^2)}\,dx$$

[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d),x)

[Out] Integral((a + b*acsc(c*x))/(x*(d + e*x**2)), x)

Maxima [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x\,(d+ex^2)}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)x}\,dx$$

[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="maxima")

[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^3 + d*x), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{x\,(d + ex^2)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,(e\,x^2 + d)}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)), x)
```

3.102 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 572

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx = & -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}-\frac{a}{dx}-\frac{b\csc^{-1}(cx)}{dx} \\ & -\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\ & +\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\ & -\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\ & +\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\ & -\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\ & +\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\ & -\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} \\ & +\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\end{aligned}$$

```
[Out] -a/d/x-b*arccsc(c*x)/d/x-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+b*
arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*
d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/
2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2
*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*I*b*polyl
og(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*
e^(1/2)/(-d)^(3/2)-b*c*(1-1/c^2/x^2)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5349, 4817, 4715, 267, 4757, 4825, 4615, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx = &-\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\&+\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\&-\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}\\&+\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}\\&-\frac{a}{dx}-\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}\\&+\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}\\&-\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}\\&+\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}-\frac{b\csc^{-1}(cx)}{dx}\end{aligned}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -((b*c*Sqrt[1 - 1/(c^2*x^2)])/d) - a/(d*x) - (b*ArcCsc[c*x])/(d*x) - (Sqrt[
e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] -
Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1 + (I
*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2))
 - (Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(S
qrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcCsc[c*x])*L
og[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-
d)^(3/2)) - ((I/2)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))
/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*PolyLog[2, (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(3/2) - ((
I/2)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*PolyLog[2, (I*c*Sqrt[-d]*E^(
I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)^(3/2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
```

```
 && PosQ[a^2 - b^2]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{d}-\frac{e\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \left(a+b\arcsin\left(\frac{x}{c}\right)\right)\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}
\end{aligned}$$

$$= -\frac{a}{dx} - \frac{b\text{Subst}\left(\int \arcsin\left(\frac{x}{c}\right) \, dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right) \, dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} + \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x^2}{c^2}}} \, dx, x, \frac{1}{x}\right)}{cd} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x} \, dx, x, \frac{1}{x}\right)}{2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x} \, dx, x, \frac{1}{x}\right)}{2d}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)} \, dx, x, \csc^{-1}(cx)\right)}{2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)} \, dx, x, \csc^{-1}(cx)\right)}{2d}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}} \, dx, x, \csc^{-1}(cx)\right)}{2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}} \, dx, x, \csc^{-1}(cx)\right)}{2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}} \, dx, x, \csc^{-1}(cx)\right)}{2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}} \, dx, x, \csc^{-1}(cx)\right)}{2d}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} - \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$+ \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$- \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$+ \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$+ \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}}$$
$$- \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}}$$
$$+ \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}}$$
$$- \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} - \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$+ \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$- \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$+ \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

$$- \frac{\left(ib\sqrt{e}\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{2(-d)^{3/2}}$$

$$+ \frac{\left(ib\sqrt{e}\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{2(-d)^{3/2}}$$

$$- \frac{\left(ib\sqrt{e}\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{2(-d)^{3/2}}$$

$$+ \frac{\left(ib\sqrt{e}\right)\text{Subst}\left(\int \frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{2(-d)^{3/2}}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} - \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$+ \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$- \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$+ \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$- \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$
$$- \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1241 vs. 2(572) = 1144.

Time = 1.81 (sec) , antiderivative size = 1241, normalized size of antiderivative = 2.17

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx = -\frac{a}{dx} - \frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + b\left(-\frac{c\sqrt{1-\frac{1}{c^2x^2}}x+\csc^{-1}(cx)}{dx}\right.$$
$$+ \frac{\sqrt{e}\left(\pi^2-4\pi\csc^{-1}(cx)+8\csc^{-1}(cx)^2-32\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}(\pi+2\csc^{-1}(cx))\right)}{\sqrt{c^2d+e}}\right)\right.}{}$$
$$- \frac{\sqrt{e}\left(\pi^2-4\pi\csc^{-1}(cx)+8\csc^{-1}(cx)^2-32\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}(\pi+2\csc^{-1}(cx))\right)}{\sqrt{c^2d+e}}\right)+\right.}{}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -(a/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + b*(-((c*Sqrt
[1 - 1/(c^2*x^2)]*x + ArcCsc[c*x])/(d*x)) + (Sqrt[e]*(Pi^2 - 4*Pi*ArcCsc[c*
```

```
x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d
 + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqr
t[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
(4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log
[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*Arc
Csc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d]
)/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*
x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(16*d^(3/2)) - (Sqrt[e]*(Pi^
2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])
/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sq
rt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c
*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sq
rt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(16*d^(3/2)))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 43.16 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.58

method	result
parts	$-\frac{a}{dx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{bc\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arccsc}(cx)}{dx}+\frac{bce\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\,\mathrm{ar}}{}\right.}{}$
derivativedivides	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}-\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arccsc}(cx)}{dcx}+\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\right.}{}\right.$
default	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}-\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arccsc}(cx)}{dcx}+\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\right.}{}\right.$

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/x-a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-b*c/d*((c^2*x^2-1)/c^2/x^2
)^(1/2)-b*arccsc(c*x)/d/x+1/2*b*c*e/d*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*
arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c
^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*
b*c*e/d*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c
^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(
c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{(ex^2+d)x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{x^2(d+ex^2)}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**2*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

3.103 $\int \frac{x^5 (a+b \csc^{-1}(cx))}{(d+ex^2)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 628

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2}+\frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2(a+b\csc^{-1}(cx))}{2e^2}-\frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{5/2}\sqrt{c^2d+e}}-\frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}-\frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}+\frac{2d(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3}+\frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}+\frac{ibd\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}+\frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}+\frac{ibd\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}-\frac{ibd\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{e^3}$$

```
[Out] 1/2*d*(a+b*arccsc(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arccsc(c*x))/e^2+2*d*(a+
b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e^3-d*(a+b*arccsc(c*x))*
ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/
e^3-d*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-d*(a+b*arccsc(c*x))*ln(
1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3
-I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e^3+I*b*d*polylog(2,-I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+I*b*d*p
olylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
```

```
)))/e^3+I*b*d*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))/e^3+I*b*d*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*b*d*arctan((c^2*d+e)^(1/2)/c/x/e
^(1/2)/(1-1/c^2/x^2)^(1/2))/e^(5/2)/(c^2*d+e)^(1/2)+1/2*b*x*(1-1/c^2/x^2)^(
1/2)/c/e^2
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5349, 4817, 4723, 270, 4721, 3798, 2221, 2317, 2438, 4813, 385, 211, 4825, 4615}

$$\begin{aligned}\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = &-\frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}\\
&-\frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}\\
&-\frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}\\
&-\frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}\\
&+\frac{2d\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{e^3}\\
&+\frac{d(a+b\csc^{-1}(cx))}{2e^2\left(\frac{d}{x^2}+e\right)}+\frac{x^2(a+b\csc^{-1}(cx))}{2e^2}\\
&-\frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{5/2}\sqrt{c^2d+e}}+\frac{ibd\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}\\
&+\frac{ibd\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}\\
&+\frac{ibd\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}\\
&+\frac{ibd\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}\\
&+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2ce^2}-\frac{ibd\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{e^3}\end{aligned}$$

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*ArcCsc[c*x]))/(2*e^2*(e +
 d/x^2)) + (x^2*(a + b*ArcCsc[c*x]))/(2*e^2) - (b*d*ArcTan[Sqrt[c^2*d + e]/
(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]) - (d*(a +
 b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^
2*d + e])])/e^3 - (d*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[
c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcCsc[c*x])*Log[1 - (
I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (d*(a +
 b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^
2*d + e])])/e^3 + (2*d*(a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/
e^3 + (I*b*d*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt
[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sq
rt[e] - Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*Ar
cCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt
[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (I*b*d*PolyLog[
2, E^((2*I)*ArcCsc[c*x])])/e^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4813

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
 - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x^3\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{e^2x^3}-\frac{2d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e^3x}+\frac{d^2x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2}\right.\right.\\
&\qquad\left.\left.+\frac{2d^2x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e^3\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)\\
&= \frac{(2d)\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3}-\frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3}\\
&\quad-\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x^3}\,dx, x, \frac{1}{x}\right)}{e^2}-\frac{d^2\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e^2}\\
&= \frac{d\left(a+b\csc^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2\left(a+b\csc^{-1}(cx)\right)}{2e^2}\\
&\quad+\frac{(2d)\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)}{e^3}\\
&\quad-\frac{\left(2d^2\right)\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{e^3}\\
&\quad-\frac{b\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce^2}-\frac{(bd)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{2ce^2}
\end{aligned}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2} - \frac{id(a+b\csc^{-1}(cx))^2}{be^3} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{e^3} - \frac{(4id)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)}{e^3} - \frac{(bd)\text{Subst}\left(\int \frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2ce^2}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2} - \frac{id(a+b\csc^{-1}(cx))^2}{be^3} - \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} + \frac{2d(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{e^3} + \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{e^3} - \frac{(2bd)\text{Subst}\left(\int \log\left(1-e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{e^3}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2} - \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} + \frac{2d(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3} + \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{e^3} + \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{e^3} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{e^3} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{e^3} + \frac{(ibd)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)}{e^3}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2}$$
$$- \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$
$$+ \frac{2d(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3} - \frac{ibd\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{e^3}$$
$$+ \frac{(bd)\mathrm{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{e^3}$$
$$+ \frac{(bd)\mathrm{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{e^3}$$
$$+ \frac{(bd)\mathrm{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{e^3}$$
$$+ \frac{(bd)\mathrm{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)\,dx,x,\csc^{-1}(cx)\right)}{e^3}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2}$$

$$- \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

$$- \frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$

$$+ \frac{2d(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3} - \frac{ibd\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{e^3}$$

$$- \frac{(ibd)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{e^3}$$

$$- \frac{(ibd)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{e^3}$$

$$- \frac{(ibd)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{e^3}$$

$$- \frac{(ibd)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{e^3}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2}$$
$$- \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$
$$+ \frac{2d(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$
$$+ \frac{ibd\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$
$$+ \frac{ibd\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} - \frac{ibd\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{e^3}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1480 vs. 2(628) = 1256.

Time = 4.22 (sec) , antiderivative size = 1480, normalized size of antiderivative = 2.36

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx =$$

$$-\frac{-2aex^2+\frac{2ad^2}{d+ex^2}+4ad\log(d+ex^2)+b\left(id\pi^2-\frac{2e\sqrt{1-\frac{1}{c^2x^2}}x}{c}-4id\pi\csc^{-1}(cx)-2ex^2\csc^{-1}(cx)+\frac{d^{3/2}\mathrm{cs}}{\sqrt{d}-}\right.}{}$$

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*(I*d*Pi
^2 - (2*e*Sqrt[1 - 1/(c^2*x^2)]*x)/c - (4*I)*d*Pi*ArcCsc[c*x] - 2*e*x^2*Arc
Csc[c*x] + (d^(3/2)*ArcCsc[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)*ArcCsc[
c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (8*I)*d*ArcCsc[c*x]^2 - 2*d*ArcSin[1/(c*x)]
 - (16*I)*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)
```

```
*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I
)*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d]
+ Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*d*Pi*Log[1 + (
Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]
*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*d*A
rcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*d*Pi*Log[1 + (-Sqrt[e] + Sqrt[
c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 + (-Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*d*ArcSin[Sqrt[1 +
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*d*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Arc
Csc[c*x]))] - 2*d*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))] + 8*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[
2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*
d*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + 2*d*Pi*Log[Sqrt[e] - (I*Sqrt
[d])/x] + 2*d*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + (d*Sqrt[e]*Log[(2*Sqrt[d]*S
qrt[e]*(Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^
2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]
+ (d*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(
c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqr
t[e]*x))])/Sqrt[-(c^2*d) - e] + (4*I)*d*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2, (-Sqrt[e] + Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2, -((Sqrt[e] +
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (4*I)*d*PolyLog[2, (Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2,
E^((2*I)*ArcCsc[c*x])]))/e^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.69 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.02

method	result
parts	$\frac{a\,x^2}{2e^2}-\frac{a\,d^2}{2e^3(e\,x^2+d)}-\frac{ad\ln(e\,x^2+d)}{e^3}+\frac{b\left(\frac{c^4\left(2c^4d\ \mathrm{arccsc}(cx)x^2+\mathrm{arccsc}(cx)e\,c^4x^4+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c^3x^3-\right.}{2\left(c^2e\,x^2+c^2d\right)e^2}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^6x^2}{2e^2}-\frac{a\,c^8d^2}{2e^3\left(c^2e\,x^2+c^2d\right)}-\frac{a\,c^6d\ln\left(c^2e\,x^2+c^2d\right)}{e^3}+b\,c^4\left(\frac{2c^4d\ \mathrm{arccsc}(cx)x^2+\mathrm{arccsc}(cx)e\,c^4x^4+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c^3x}{2\left(c^2e\,x^2+c^2d\right)e^2}\right.}{}$
default	$\frac{\frac{a\,c^6x^2}{2e^2}-\frac{a\,c^8d^2}{2e^3\left(c^2e\,x^2+c^2d\right)}-\frac{a\,c^6d\ln\left(c^2e\,x^2+c^2d\right)}{e^3}+b\,c^4\left(\frac{2c^4d\ \mathrm{arccsc}(cx)x^2+\mathrm{arccsc}(cx)e\,c^4x^4+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3dx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,e\,c^3x}{2\left(c^2e\,x^2+c^2d\right)e^2}\right.}{}$

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2/e^2-1/2*a*d^2/e^3/(e*x^2+d)-a*d/e^3*ln(e*x^2+d)+b/c^6*(1/2*c^4*(2
*c^4*d*arccsc(c*x)*x^2+arccsc(c*x)*e*c^4*x^4+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^
3*d*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*e*c^3*x^3-I*c^2*d-I*e*c^2*x^2)/(c^2*e*x^2
+c^2*d)/e^2-2*I/e^3*d*c^6*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))+1/2*I/e^3*d^2*
c^8*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1
/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootO
f(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e
)/e^3*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*
d*e+e^2)^(1/2))*d*c^6+2*I/e^3*d*c^6*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))+1/2*I/
e^3*d*c^6*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x
)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1
/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+2/e^3*d*c^6*ar
ccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2)))
```

Fricas [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^5*arccsc(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^5(a+b\,\mathrm{acsc}\,(cx))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**5*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] -1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*integ
rate(x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2
), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.104 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 593

$$\begin{aligned}\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx &= \frac{-a-b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)}+\frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}}\\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}\\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}\\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}\\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^2}\\
&-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}\\
&-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}\\
&-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}\\
&-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}+\frac{ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^2}\end{aligned}$$

```
[Out] 1/2*(-a-b*arccsc(c*x))/e/(e+d/x^2)-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x
^2)^(1/2))^2)/e^2+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2)
)*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsc(c*x))*ln(1+I*c*
(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(
a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(
c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*I*b*polylog(2,(I/c/x+(1-
1/c^2/x^2)^(1/2))^2)/e^2-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))
*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,I*c*(I/c/x+(1-
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog
(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
^2-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))/e^2+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^
(1/2))/e^(3/2)/(c^2*d+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 590, normalized size of antiderivative = 0.99, number of steps used = 29, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules

used = {5349, 4817, 4721, 3798, 2221, 2317, 2438, 4813, 385, 211, 4825, 4615}

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2} - \frac{a+b\csc^{-1}(cx)}{2e\left(\frac{d}{x^2}+e\right)} - \frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{e^2} + \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{3/2}\sqrt{c^2d+e}} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^2}$$

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcCsc[c*x])/(e*(e + d/x^2)) + (b*ArcTan[Sqrt[c^2*d + e]/(c*Sqr
t[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(3/2)*Sqrt[c^2*d + e]) + ((a + b*ArcCs
c[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]
)])/(2*e^2) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))
/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*
Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) + ((a + b
*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*
d + e])])/(2*e^2) - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/e^
2 - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt
[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(
Sqrt[e] - Sqrt[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(
I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^2 + ((I/2)*b*
PolyLog[2, E^((2*I)*ArcCsc[c*x])])/e^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4813

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
 - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^2}\,dx,x,\frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{e^2x}-\frac{dx\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2}-\frac{dx\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right)dx,x,\frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right)}{e^2}+\frac{d\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)}{e^2}\\
&\quad+\frac{d\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx,x,\frac{1}{x}\right)}{e}
\end{aligned}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)}{e^2} + \frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \frac{1}{x}\right)}{e^2} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}(e+dx^2)}\,dx, x, \frac{1}{x}\right)}{2ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be^2} + \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)}{e^2} - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^2} + \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^2} + \frac{b\text{Subst}\left(\int \frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be^2} + \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}} - \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^2} + \frac{b\text{Subst}\left(\int \log\left(1-e^{2ix}\right)dx, x, \csc^{-1}(cx)\right)}{e^2} - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2e^2} + \frac{\sqrt{-d}\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}}$$

$$-\frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^2} - \frac{(ib)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i\csc^{-1}(cx)}\right)}{2e^2}$$

$$+\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^2}$$

$$+\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^2}$$

$$-\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^2}$$

$$-\frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^2} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^2}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^2}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^2}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^2}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^2} + \frac{ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^2}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^2}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^2}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^2}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} - \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^2}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1442 vs. 2(593) = 1186.

Time = 1.83 (sec) , antiderivative size = 1442, normalized size of antiderivative = 2.43

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$$

$$= \frac{ib\pi^2 + \frac{4ad}{d+ex^2} - 4ib\pi\csc^{-1}(cx) + \frac{2b\sqrt{d}\csc^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x} + \frac{2b\sqrt{d}\csc^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x} + 8ib\csc^{-1}(cx)^2 - 4b\arcsin\left(\frac{1}{cx}\right) - 16ib\,\mathrm{arcsi}}{}$$

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (I*b*Pi^2 + (4*a*d)/(d + e*x^2) - (4*I)*b*Pi*ArcCsc[c*x] + (2*b*Sqrt[d]*Arc
Csc[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (2*b*Sqrt[d]*ArcCsc[c*x])/(Sqrt[d] + I*
Sqrt[e]*x) + (8*I)*b*ArcCsc[c*x]^2 - 4*b*ArcSin[1/(c*x)] - (16*I)*b*ArcSin[
Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e
])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*b*ArcSin[Sqrt[1 +
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi
```

```
+ 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] -
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc
[c*x]))] - 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*P
i*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*
ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]
))] + 8*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e
] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcCsc[c*x]*Log[1
 - E^((2*I)*ArcCsc[c*x])] + 2*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 2*b*Pi*Lo
g[Sqrt[e] + (I*Sqrt[d])/x] + (2*b*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] +
 c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(
c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + (2*b*Sqrt[e]*Lo
g[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqr
t[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt
[-(c^2*d) - e] + 4*a*Log[d + e*x^2] + (4*I)*b*PolyLog[2, (Sqrt[e] - Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2, (-Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2, -((Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (4*I)*b*PolyLog[2,
 (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyL
og[2, E^((2*I)*ArcCsc[c*x])])/(8*e^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.55 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.88

method	result
parts	$\frac{ad}{2e^2(e\,x^2+d)}+\frac{a\ln(e\,x^2+d)}{2e^2}-\frac{b\,c^2x^2\,\text{arccsc}(cx)}{2(c^2e\,x^2+c^2d)e}-\frac{ib\,c^2d\left(\sum_{\_R1=\text{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{\left(\_R1^2\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^6d}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^4\ln\left(c^2e\,x^2+c^2d\right)}{2e^2}+b\,c^4\left(-\frac{c^2x^2\,\text{arccsc}(cx)}{2\left(c^2e\,x^2+c^2d\right)e}-\frac{i\,\text{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e^2}-\frac{ic^2d\left(\sum_{\_R1=\text{RootOf}\left(c^2d\_Z^4+\left(\right.\right.}\right.}{}\right.}{}$
default	$\frac{\frac{a\,c^6d}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^4\ln\left(c^2e\,x^2+c^2d\right)}{2e^2}+b\,c^4\left(-\frac{c^2x^2\,\text{arccsc}(cx)}{2\left(c^2e\,x^2+c^2d\right)e}-\frac{i\,\text{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e^2}-\frac{ic^2d\left(\sum_{\_R1=\text{RootOf}\left(c^2d\_Z^4+\left(\right.\right.}\right.}{}\right.}{}$

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*a*d/e^2/(e*x^2+d)+1/2*a/e^2*ln(e*x^2+d)-1/2*b*c^2*x^2*arccsc(c*x)/(c^2*
e*x^2+c^2*d)/e-1/4*I*b*c^2/e^2*d*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*a
rccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^
2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/2*I
*b*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/e^2*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x
^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2))-I*b/e^2*dilog(I/c/x+(1-1/c^2
/x^2)^(1/2))-1/4*I*b/e^2*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e
)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(
1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
+I*b/e^2*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-b/e^2*arccsc(c*x)*ln(1+I/c/x+(1
-1/c^2/x^2)^(1/2))
```

Fricas [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^3}{(ex^2 + d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^3*arccsc(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^3}{(ex^2 + d)^2}\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*arctan2(
1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{x^3\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^2}\,dx$$

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.105 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{-a-b\csc^{-1}(cx)}{2e(d+ex^2)} - \frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}} + \frac{bcx\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}}$$

```
[Out] 1/2*(-a-b*arccsc(c*x))/e/(e*x^2+d)-1/2*b*c*x*arctan((c^2*x^2-1)^(1/2))/d/e/
(c^2*x^2)^(1/2)+1/2*b*c*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2))
/d/e^(1/2)/(c^2*d+e)^(1/2)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5345, 457, 88, 65, 211}

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{a+b\csc^{-1}(cx)}{2e(d+ex^2)} - \frac{bcx\arctan\left(\sqrt{c^2x^2-1}\right)}{2de\sqrt{c^2x^2}} + \frac{bcx\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2x^2}\sqrt{c^2d+e}}$$

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcCsc[c*x])/(e*(d + e*x^2)) - (b*c*x*ArcTan[Sqrt[-1 + c^2*x^2]
])/(2*d*e*Sqrt[c^2*x^2]) + (b*c*x*ArcTan[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sqrt[
c^2*d + e]])/(2*d*Sqrt[e]*Sqrt[c^2*d + e]*Sqrt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\csc^{-1}(cx)}{2e(d+ex^2)}-\frac{(bcx)\int\frac{1}{x\sqrt{-1+c^2x^2}(d+ex^2)}\,dx}{2e\sqrt{c^2x^2}}\\
&= -\frac{a+b\csc^{-1}(cx)}{2e(d+ex^2)}-\frac{(bcx)\mathrm{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}(d+ex)}\,dx,x,x^2\right)}{4e\sqrt{c^2x^2}}\\
&= -\frac{a+b\csc^{-1}(cx)}{2e(d+ex^2)}+\frac{(bcx)\mathrm{Subst}\left(\int\frac{1}{\sqrt{-1+c^2x}(d+ex)}\,dx,x,x^2\right)}{4d\sqrt{c^2x^2}}\\
&\quad-\frac{(bcx)\mathrm{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{4de\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)} + \frac{(bx)\text{Subst}\left(\int \frac{1}{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{2cd\sqrt{c^2x^2}} - \frac{(bx)\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{2cde\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}} + \frac{bcx\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.13

$$\int \frac{x(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^2}\,dx =$$

$$-\frac{\frac{2a}{d+ex^2} + \frac{2b\csc^{-1}(cx)}{d+ex^2} - \frac{2b\arcsin\left(\frac{1}{cx}\right)}{d} + \frac{b\sqrt{e}\log\left(\frac{4ide-4cd\sqrt{e}\left(c\sqrt{d}+i\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x}{b\sqrt{-c^2d-e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{d\sqrt{-c^2d-e}} + \frac{b\sqrt{e}\log\left(\frac{4i\left(-de+cd\sqrt{e}\left(ic\sqrt{d}+\sqrt{-c^2d-e}\right.\right.}{b\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}\right.}\right.}{d\sqrt{-c^2d-e}}}{4e}$$

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*((2*a)/(d + e*x^2) + (2*b*ArcCsc[c*x])/(d + e*x^2) - (2*b*ArcSin[1/(c*
x)])/d + (b*Sqrt[e]*Log[((4*I)*d*e - 4*c*d*Sqrt[e]*(c*Sqrt[d] + I*Sqrt[-(c^
2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)/(b*Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt
[e]*x))])/(d*Sqrt[-(c^2*d) - e]) + (b*Sqrt[e]*Log[((4*I)*(-(d*e) + c*d*Sqrt
[e]*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*Sqrt[-(
c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/(d*Sqrt[-(c^2*d) - e]))/e
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(112) = 224.

Time = 8.61 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.98

method	result
parts	$-\frac{a}{2e(e\,x^2+d)}+\frac{b\left(-\frac{c^4\,\mathrm{arccsc}(cx)}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{c\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(\frac{2\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e-2\sqrt{-c^2de}}{cex+\sqrt{-c^2de}}\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd\sqrt{-\frac{c^2d+}{e}}}}{c^2}$
derivativedivides	$\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\mathrm{arccsc}(cx)}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(\frac{2\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e-2\sqrt{-c^2}}{cex+\sqrt{-c^2de}}\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3xd\sqrt{-}}\right.}{c^2}$
default	$\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\mathrm{arccsc}(cx)}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(\frac{2\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e-2\sqrt{-c^2}}{cex+\sqrt{-c^2de}}\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3xd\sqrt{-}}\right.}{c^2}$

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/e/(e*x^2+d)+b/c^2*(-1/2*c^4/e/(c^2*e*x^2+c^2*d)*arccsc(c*x)+1/4*c/e*
(c^2*x^2-1)^(1/2)*(2*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)-ln(2*
((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-
c^2*d*e)^(1/2)))-ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)
^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2))))/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(
-(c^2*d+e)/e)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.87

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$$
$$=\left[-\frac{2\,ac^2d^2+2\,ade+\sqrt{-c^2de-e^2}(bex^2+bd)\log\left(\frac{c^2ex^2-c^2d-2\sqrt{-c^2de-e^2}\sqrt{c^2x^2-1}-2\,e}{ex^2+d}\right)+2\,(bc^2d^2+bde)\,\mathrm{arcc}}{4\,(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3)x}\right.$$
$$-\frac{ac^2d^2+ade-\sqrt{c^2de+e^2}(bex^2+bd)\arctan\left(\frac{\sqrt{c^2de+e^2}\sqrt{c^2x^2-1}}{c^2d+e}\right)+(bc^2d^2+bde)\,\mathrm{arccsc}\,(cx)+2\,(bc^2d^2+}{2\,(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3)x^2)}$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 + 2*a*d*e + sqrt(-c^2*d*e - e^2)*(b*e*x^2 + b*d)*log((c^
2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2 +
d)) + 2*(b*c^2*d^2 + b*d*e)*arccsc(c*x) + 4*(b*c^2*d^2 + b*d*e + (b*c^2*d*e
 + b*e^2)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^
2*d^2*e^2 + d*e^3)*x^2), -1/2*(a*c^2*d^2 + a*d*e - sqrt(c^2*d*e + e^2)*(b*e
*x^2 + b*d)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2 - 1)/(c^2*d + e)) + (b*
c^2*d^2 + b*d*e)*arccsc(c*x) + 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x
^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 +
 d*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x(a+b\,\mathrm{acsc}\,(cx))}{(d+ex^2)^2}\,dx$$

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(c^2*e^2*x^2 + c^2*d*e)*integrate(1/2*x*e^(1/2*log(c*x + 1) + 1/2*l
og(c*x - 1))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*e^2*x^4 + (c^2
*d*e - e^2)*x^2 - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) + arctan2(1, sq
rt(c*x + 1)*sqrt(c*x - 1)))*b/(e^2*x^2 + d*e) - 1/2*a/(e^2*x^2 + d*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.106 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 566

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx &= -\frac{e(a+b\csc^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{i(a+b\csc^{-1}(cx))^2}{2bd^2}\\
&+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\end{aligned}$$

```
[Out] -1/2*e*(a+b*arccsc(c*x))/d^2/(e+d/x^2)+1/2*I*(a+b*arccsc(c*x))^2/b/d^2-1/2*
(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsc(c*x))*ln(1-
```

```
I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2-1
/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))
*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,I*c*(I/c/x+(1-
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog
(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
^2+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))/d^2+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^
(1/2))*e^(1/2)/d^2/(c^2*d+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5349, 4817, 4813, 385, 211, 4825, 4615, 2221, 2317, 2438}

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = -\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} -\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} -\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2} -\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2} -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(\frac{d}{x^2}+e\right)} +\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^2} +\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\sqrt{c^2d+e}} +\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2} +\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2} +\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2} +\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] -1/2*(e*(a + b*ArcCsc[c*x]))/(d^2*(e + d/x^2)) + ((I/2)*(a + b*ArcCsc[c*x])
^2)/(b*d^2) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*
x^2)]*x)])/(2*d^2*Sqrt[c^2*d + e]) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt
[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*Arc
Csc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
e])])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]
```

```
))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 + (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) + ((I/2
)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
e])])/d^2 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] -
 Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[
c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d
]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4813

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
 - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^3\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(-\frac{ex\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2}+\frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}\\
&= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}-\frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{d}\\
&\quad+\frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{2cd^2}
\end{aligned}
$$

$$= -\frac{e(a+b\csc^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}} + \frac{(be)\text{Subst}\left(\int \frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2cd^2}$$

$$= -\frac{e(a+b\csc^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$= -\frac{e(a+b\csc^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\csc^{-1}(cx))^2}{2bd^2} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}} + \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}} + \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}}$$

$$= -\frac{e(a+b\csc^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\csc^{-1}(cx))^2}{2bd^2} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2d^2}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2d^2}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2d^2}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2d^2}$$

$$= -\frac{e(a+b\csc^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\csc^{-1}(cx))^2}{2bd^2} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}}$$

$$-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$-\frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2d^2}$$

$$-\frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2d^2}$$

$$-\frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2d^2}$$

$$-\frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2d^2}$$

$$= -\frac{e(a+b\csc^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\csc^{-1}(cx))^2}{2bd^2} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$
$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$
$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1408 vs. 2(566) = 1132.

Time = 1.27 (sec) , antiderivative size = 1408, normalized size of antiderivative = 2.49

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

$$= \frac{-ib\pi^2+\frac{4ad}{d+ex^2}+4ib\pi\csc^{-1}(cx)+\frac{2b\sqrt{d}\csc^{-1}(cx)}{\sqrt{d}-i\sqrt{e}x}+\frac{2b\sqrt{d}\csc^{-1}(cx)}{\sqrt{d}+i\sqrt{e}x}-4ib\csc^{-1}(cx)^2-4b\arcsin\left(\frac{1}{cx}\right)+16ib\,\text{arc}}{}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] ((-I)*b*Pi^2 + (4*a*d)/(d + e*x^2) + (4*I)*b*Pi*ArcCsc[c*x] + (2*b*Sqrt[d]*
ArcCsc[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (2*b*Sqrt[d]*ArcCsc[c*x])/(Sqrt[d] +
 I*Sqrt[e]*x) - (4*I)*b*ArcCsc[c*x]^2 - 4*b*ArcSin[1/(c*x)] + (16*I)*b*ArcS
in[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqr
t[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (16*I)*b*ArcSin[Sqrt[
1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(
Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + 2*b*Pi*Log[1 + (Sqrt[e] - Sqrt[c
^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e
] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 - (
```

```
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] + 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*S
qrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d
 + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Arc
Csc[c*x]))] + 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] - 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[
2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*
b*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4
*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c
*x]))] - 8*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqr
t[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[Sqrt[e]
 - (I*Sqrt[d])/x] - 2*b*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 8*a*Log[x] + (2*b
*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d
) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*
x))])/Sqrt[-(c^2*d) - e] + (2*b*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] +
c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c
^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] - 4*a*Log[d + e*x^
2] - (4*I)*b*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] - (4*I)*b*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*A
rcCsc[c*x]))] - (4*I)*b*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x])))] - (4*I)*b*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*S
qrt[d]*E^(I*ArcCsc[c*x]))])/(8*d^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.16 (sec) , antiderivative size = 2071, normalized size of antiderivative = 3.66

method	result	size
parts	Expression too large to display	2071
derivativedivides	Expression too large to display	2120
default	Expression too large to display	2120

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a/d^2*ln(x)+1/2*a/d/(e*x^2+d)-1/2*a/d^2*ln(e*x^2+d)+b*(1/2*I/d^2*sum((_R1^2
*c^2*d-2*c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Roo
tOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*I*arccsc(c*x)^2/d^2-1/2*(c^2
*d+2*(e*(c^2*d+e))^(1/2)+2*e)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2
*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/c^2/d^3-1/2*(e*(c^2*d+e))^(1/2)/
(c^2*d+e)/d^2*arccsc(c*x)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2
```

```
*(e*(c^2*d+e))^(1/2)+2*e))+1/4*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/d^2*polylog(
2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))+1/
2*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/d^2*arccsc(c*x)^2+1/4*I*(c^2*d+2*(e*(c^2*
d+e))^(1/2)+2*e)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*
(c^2*d+e))^(1/2)+2*e))/c^2/d^3-1/2*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/d^2*arct
anh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^2)^(
1/2))+1/2*I*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arccsc(c*x)^2/c^2/d^3-1/4*I*(
(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arccsc(c
*x)^2/(c^2*d+e)/d^2/e+I*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arccsc(c*x)^2*e/d
^4/c^4-I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2
)*arccsc(c*x)^2/c^2/d^3/(c^2*d+e)-1/8*I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*
e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2)
)^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/(c^2*d+e)/d^2/e-1/2*I*((e*(c^2*d+e))
^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*polylog(2,d*c^2*(I/c/
x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/c^2/d^3/(c^2*d+
e)+1/4*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*
ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))
*arccsc(c*x)/(c^2*d+e)/d^2/e+((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2
*d+e))^(1/2)*e+2*e^2)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*
(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/c^2/d^3/(c^2*d+e)-(c^2*d+2*(e*(c^2*d+e))
^(1/2)+2*e)*e*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e
))^(1/2)+2*e))*arccsc(c*x)/d^4/c^4-1/2*x^2*c^2*e*arccsc(c*x)/(c^2*e*x^2+c^2
*d)/d^2+1/2*I*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*polylog(2,d*c^2*(I/c/x+(1-1
/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e/d^4/c^4-1/4*(e*(c^2
*d+e))^(1/2)/d/e/(c^2*d+e)*c^2*arccsc(c*x)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^
(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))+1/8*I*(e*(c^2*d+e))^(1/2)/d/e/(
c^2*d+e)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e))*c^2-I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(
1/2)*e+2*e^2)*e*arccsc(c*x)^2/c^4/d^4/(c^2*d+e)+((e*(c^2*d+e))^(1/2)*c^2*d+
2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^
(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/c^4/d^4/(c^2*d+e)+1
/4*I*(e*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*arccsc(c*x)^2*c^2-1/2*I*((e*(c^2*d+e
))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*polylog(2,d*c^2*(
I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/c^4/d^4/(c^
2*d+e))
```

Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\left(ex^2+d\right)^2 x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrate
(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

3.107 $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 803

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{x(a+b\csc^{-1}(cx))}{e^2}$$
$$+\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2}+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}$$
$$+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}$$
$$-\frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+\frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$-\frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+\frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$-\frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+\frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$-\frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+\frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

```
[Out] x*(a+b*arccsc(c*x))/e^2+b*arctanh((1-1/c^2/x^2)^(1/2))/c/e^2-3/4*(a+b*arccs
c(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^
(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+
b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^
2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-
3/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*I*b*po
```

```
lylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))*(-d)^(1/2)/e^(5/2)+3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-1/4*d*(a+b*arccsc(c*
x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*d*(a+b*arccsc(c*x))/e^2/(d/x+(-d)^(1/
2)*e^(1/2))+1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^
(1/2)/(1-1/c^2/x^2)^(1/2))*d^(1/2)/e^2/(c^2*d+e)^(1/2)+1/4*b*arctanh((c^2*d
+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))*d^(1/
2)/e^2/(c^2*d+e)^(1/2)
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules

used = {5349, 4817, 4723, 272, 65, 214, 4757, 4827, 739, 212, 4825, 4615, 2221, 2317, 2438}

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{x(a+b\csc^{-1}(cx))}{e^2} - \frac{3\sqrt{-d}\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)(a+b\csc^{-1}(cx))}{4e^{5/2}} + \frac{3\sqrt{-d}\log\left(\frac{i\sqrt{-d}e^{i\csc^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)(a+b\csc^{-1}(cx))}{4e^{5/2}} - \frac{3\sqrt{-d}\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)(a+b\csc^{-1}(cx))}{4e^{5/2}} + \frac{3\sqrt{-d}\log\left(\frac{i\sqrt{-d}e^{i\csc^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)(a+b\csc^{-1}(cx))}{4e^{5/2}} - \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} + \frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{dc^2+e}} + \frac{b\sqrt{d}\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{dc^2+e}} - \frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}} + \frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}} - \frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}} + \frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}$$

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(d*(a + b*ArcCsc[c*x]))/(e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*Arc
Csc[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcCsc[c*x]))/e^2 +
 (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d - (
Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4
*e^2*Sqrt[c^2*d + e]) + (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(
c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*e^2*Sqrt[c^2*d + e])
- (3*Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCsc[c*x])
```

```
*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*
e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCs
c[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*Arc
Csc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d +
e])])/(4*e^(5/2)) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*
ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4)*b*Sqrt[-d]
*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/
e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x
]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4)*b*Sqrt[-d]*PolyLog[2
, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2)
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

<u>Rule 272</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

<u>Rule 739</u>

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

<u>Rule 2221</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{e^2x^2} - \frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2}\\
&\quad + \frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e}\\
&= \frac{x\left(a+b\csc^{-1}(cx)\right)}{e^2} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce^2}\\
&\quad + \frac{d\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{e^2}\\
&\quad + \frac{d\text{Subst}\left(\int \left(-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)}{e}
\end{aligned}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}}$$

$$+ \frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ce^2}$$

$$- \frac{d^2\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e^2}$$

$$- \frac{d^2\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e^2} - \frac{d^2\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2e^2}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\csc^{-1}(cx))}{e^2}$$

$$+ \frac{d\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{d\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{(bc)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx, x, \sqrt{1-\frac{1}{c^2x^2}}\right)}{e^2}$$

$$+ \frac{(bd)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce^2}$$

$$- \frac{(bd)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce^2}$$

$$- \frac{d^2\text{Subst}\left(\int \left(-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{2e^2}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\csc^{-1}(cx))}{e^2}$$

$$+ \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} + \frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4e^{5/2}}$$

$$+ \frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4e^{5/2}} + \frac{d\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{d\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{d\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2e^{5/2}}$$

$$+ \frac{d\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2e^{5/2}}$$

$$- \frac{(bd)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4ce^2}$$

$$+ \frac{(bd)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4ce^2}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\csc^{-1}(cx))}{e^2} + \frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} + \frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} + \frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} + \frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^{5/2}} - \frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^{5/2}} + \frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^{5/2}} - \frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^{5/2}} + \frac{d\text{Subst}\left(\int\frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}dx,x,\csc^{-1}(cx)\right)}{4e^{5/2}} + \frac{d\text{Subst}\left(\int\frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}dx,x,\csc^{-1}(cx)\right)}{4e^{5/2}}$$

$$
\begin{aligned}
&= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+ \frac{x(a+b\csc^{-1}(cx))}{e^2}+\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} \\
&+ \frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}+\frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} \\
&- \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&+ \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&- \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&+ \frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&- \frac{(ib\sqrt{-d})\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^{5/2}} \\
&+ \frac{(ib\sqrt{-d})\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^{5/2}} \\
&- \frac{(ib\sqrt{-d})\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^{5/2}} \\
&+ \frac{(ib\sqrt{-d})\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^{5/2}} \\
&+ \frac{d\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}dx,x,\csc^{-1}(cx)\right)}{4e^{5/2}} \\
&+ \frac{d\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}dx,x,\csc^{-1}(cx)\right)}{4e^{5/2}} \\
&+ \frac{d\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}dx,x,\csc^{-1}(cx)\right)}{4e^{5/2}} \\
&+ \frac{d\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}dx,x,\csc^{-1}(cx)\right)}{4e^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+\frac{x(a+b\csc^{-1}(cx))}{e^2}+\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} \\
&+\frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}+\frac{b\sqrt{d}\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} \\
&-\frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+\frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&-\frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+\frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}}+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&+\frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4e^{5/2}} \\
&-\frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4e^{5/2}} \\
&+\frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4e^{5/2}} \\
&-\frac{(b\sqrt{-d})\,\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4e^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+ \frac{x(a+b\csc^{-1}(cx))}{e^2} + \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} \\
&+ \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} + \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} \\
&- \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+ \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&- \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+ \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&- \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} + \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&- \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} + \frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
&- \frac{\left(ib\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{4e^{5/2}} \\
&+ \frac{\left(ib\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{4e^{5/2}} \\
&- \frac{\left(ib\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{4e^{5/2}} \\
&+ \frac{\left(ib\sqrt{-d}\right)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{4e^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+ \frac{x(a+b\csc^{-1}(cx))}{e^2} + \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} \\
&+ \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} + \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}} \\
&- \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+ \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&- \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+ \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&- \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} + \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&- \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} + \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1634 vs. $2(803) = 1606$.

Time = 6.05 (sec) , antiderivative size = 1634, normalized size of antiderivative = 2.03

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{ax}{e^2}+\frac{adx}{2e^2(d+ex^2)}-\frac{3a\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{5/2}}$$

$$+ b\left(-\frac{d\left(-\frac{\csc^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex}+\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(-ic\sqrt{d}-\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)}{\sqrt{d}}\right)}{4e^2}\right.$$

$$-\frac{d\left(-\frac{\csc^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex}-\frac{i\left(\frac{\arcsin\left(\frac{1}{cx}\right)}{\sqrt{e}}-\frac{\log\left(\frac{2\sqrt{d}\sqrt{e}\left(-\sqrt{e}+c\left(-ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d-e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{\sqrt{-c^2d-e}}\right)}{\sqrt{d}}\right)}{4e^2}$$

$$+\frac{3\sqrt{d}\left(\pi^2-4\pi\csc^{-1}(cx)+8\csc^{-1}(cx)^2-32\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2\csc^{-1}(cx)\right)\right)}{\sqrt{c^2d+e}}\right)\cdots}{\cdots}$$

$$-\frac{3\sqrt{d}\left(\pi^2-4\pi\csc^{-1}(cx)+8\csc^{-1}(cx)^2-32\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2\csc^{-1}(cx)\right)\right)}{\sqrt{c^2d+e}}\right)\cdots}{\cdots}$$

$$+\frac{\frac{1}{2}\csc^{-1}(cx)\cot\left(\frac{1}{2}\csc^{-1}(cx)\right)+\log\left(\cos\left(\frac{1}{2}\csc^{-1}(cx)\right)\right)-\log\left(\sin\left(\frac{1}{2}\csc^{-1}(cx)\right)\right)+\frac{1}{2}\csc^{-1}(cx)\tan\left(\frac{1}{2}\csc\cdots\right.}{ce^2}$$

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*x)/e^2 + (a*d*x)/(2*e^2*(d + e*x^2)) - (3*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(2*e^(5/2)) + b*(-1/4*(d*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*
x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*((-
I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d)
 - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^2 - (d*(-(
ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[
(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[
1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(
c^2*d) - e]))/Sqrt[d]))/(4*e^2) + (3*Sqrt[d]*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*A
rcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[
(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] +
 (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))
] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*A
rcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Lo
g[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi
*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)
*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sq
rt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]
*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 8
*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8
*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] +
 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(32*e^(5/2)) - (3*Sqrt[d]*(Pi^2 - 4*
Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/S
qrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e]
- (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(32*e^(5/2)) + ((Arc
Csc[c*x]*Cot[ArcCsc[c*x]/2])/2 + Log[Cos[ArcCsc[c*x]/2]] - Log[Sin[ArcCsc[c
*x]/2]] + (ArcCsc[c*x]*Tan[ArcCsc[c*x]/2])/2)/(c*e^2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 44.66 (sec) , antiderivative size = 965, normalized size of antiderivative = 1.20

method	result	size
parts	Expression too large to display	965
derivativedivides	Expression too large to display	987
default	Expression too large to display	987

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/e^2*x-1/e^2*d*(-1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)
)))+b/c^5*(1/2*x*c^5*arccsc(c*x)*(2*c^2*e*x^2+3*c^2*d)/e^2/(c^2*e*x^2+c^2*d
)-1/e^2*c^4*ln(-1+I/c/x+(1-1/c^2/x^2)^(1/2))+1/e^2*c^4*ln(1+I/c/x+(1-1/c^2/
x^2)^(1/2))-3/16/e^3*c^6*d*sum((_R1^2*c^2*d-c^2*d-4*e)/_R1/(_R1^2*c^2*d-c^2
*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I
/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c
^2*d))+3/16/e^3*c^6*d*sum((_R1^2*c^2*d+4*_R1^2*e-c^2*d)/_R1/(_R1^2*c^2*d-c^
2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-
I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+
c^2*d))-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/
2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctan(c*d*(I/c/x+(1-1/
c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)
/d^2-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*
c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctanh(c*d*(I/c/x+(1-1/c^
2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+e)/d^
2+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(
1/2)+2*e)*c*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))
^(1/2)-2*e)*d)^(1/2))/d^2/e^2+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/
2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^2/e^2)
```

Fricas [F]

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\operatorname{arccsc}(cx)+a)x^4}{(ex^2+d)^2}\,dx$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a + b \csc^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{x^4(a + b \operatorname{acsc}(cx))}{(d + ex^2)^2} \, dx$$

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**4*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b \csc^{-1}(cx))}{(d + ex^2)^2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \csc^{-1}(cx))}{(d + ex^2)^2} \, dx = \text{Timed out}$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \csc^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{x^4 \left(a + b \operatorname{asin}\left(\frac{1}{c x}\right)\right)}{(e x^2 + d)^2} \, dx$$

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.108 $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 765

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

```
[Out] -1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccsc(c*x))*ln(1+I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d
)^(1/2)-1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccsc(c*x))*ln(1
+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3
/2)/(-d)^(1/2)-1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*I*b*polylog(2,I*c*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(
1/2)-1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(
```

```
a+b*arccsc(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a-b*arccsc(c*x))/e/(d/x+
(-d)^(1/2)*e^(1/2))-1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c
^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/d^(1/2)/(c^2*d+e)^(1/2)-1/4*b*arctanh(
(c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))
/e/d^(1/2)/(c^2*d+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5349, 4757, 4827, 739, 212, 4825, 4615, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = &-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}\\&+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}\\&+\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\&-\frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}}-\frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}}\\&-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}\\&+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}\end{aligned}$$

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcCsc[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCsc[c*x])/(4
*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c
```

```
*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*Sqrt[c^2*d +
 e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]
*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*Sqrt[c^2*d + e]) - ((a + b*ArcCsc[c*
x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/
(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcC
sc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcC
sc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e
])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I
*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((I/4)*
b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]
)])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]
))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, (
(-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*
e^(3/2)) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] +
Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= \frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} + \frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} + \frac{d\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2e}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce} + \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4ce} + \frac{d\text{Subst}\left(\int \left(-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{2e}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4e^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4e^{3/2}} + \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4ce} - \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4ce}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{4e^{3/2}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{4e^{3/2}}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{4e^{3/2}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{4e^{3/2}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{4e^{3/2}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{4e^{3/2}}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\text{Subst}\left(\int \log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}} - \frac{b\text{Subst}\left(\int \log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\text{Subst}\left(\int \log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}} - \frac{b\text{Subst}\left(\int \log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4\sqrt{-d}e^{3/2}}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}}$$

$$- \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

$$- \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}}$$

$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}}$$

$$- \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}}$$

$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{4\sqrt{-d}e^{3/2}}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}}$$
$$- \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$
$$- \frac{ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 1.70 (sec) , antiderivative size = 1482, normalized size of antiderivative = 1.94

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$$

$$= \frac{-\frac{4a\sqrt{e}x}{d+ex^2} + \frac{4a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + b\left(\frac{2\csc^{-1}(cx)}{i\sqrt{d}-\sqrt{e}x} - \frac{2\csc^{-1}(cx)}{i\sqrt{d}+\sqrt{e}x} + \frac{8\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2\csc^{-1}(cx)\right)\right)}{\sqrt{c^2d+e}}\right)}{\sqrt{d}} - \right.}{}$$

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 b*((2*ArcCsc[c*x])/(I*Sqrt[d] - Sqrt[e]*x) - (2*ArcCsc[c*x])/(I*Sqrt[d] +
Sqrt[e]*x) + (8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((
(-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]])/Sq
rt[d] - (8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*S
qrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]])/Sqrt[d] -
(I*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/S
qrt[d] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d
```

```
])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))])/Sqrt[d] + (I*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))])/Sqrt[d] - ((2*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d
 + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + ((4*I)*ArcSin[Sqrt[1 + (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + (I*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((2*I)*ArcCsc[c*x]*Log[1 - (Sqr
t[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((4*I)*Ar
cSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (I*Pi*Log[1 + (Sqrt[e] +
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + ((2*I)*ArcCsc[c
*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqr
t[d] + ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sq
rt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (I*Pi*Lo
g[Sqrt[e] - (I*Sqrt[d])/x])/Sqrt[d] + (I*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x])/S
qrt[d] - ((2*I)*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*((-I)*c*Sqrt[d]
 - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[
d] + I*Sqrt[e]*x))])/(Sqrt[d]*Sqrt[-(c^2*d) - e]) + ((2*I)*Sqrt[e]*Log[(2*S
qrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 -
1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/(Sqrt[d]*S
qrt[-(c^2*d) - e]) + (2*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))])/Sqrt[d] - (2*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (2*PolyLog[2, -((Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))])/Sqrt[d] + (2*PolyLog[2, (Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d]))/(8*e^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 39.56 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.10

method	result
parts	$-\frac{ax}{2e(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+\frac{b\left(-\frac{c^5\,\mathrm{arccsc}(cx)x}{2e\left(c^2e\,x^2+c^2d\right)}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\arctan\left(\right.}{2c\,d^3e}\right.}{}$
derivativedivides	$\frac{-\frac{a\,c^5x}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b\,c^4\left(-\frac{cx\,\mathrm{arccsc}(cx)}{2\left(c^2e\,x^2+c^2d\right)e}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\mathrm{arcta}}{2e\,c^5d^3}\right.}{}$
default	$\frac{-\frac{a\,c^5x}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b\,c^4\left(-\frac{cx\,\mathrm{arccsc}(cx)}{2\left(c^2e\,x^2+c^2d\right)e}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\mathrm{arcta}}{2e\,c^5d^3}\right.}{}$

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/e*x/(e*x^2+d)+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c^3*(-1/
2*c^5*arccsc(c*x)/e*x/(c^2*e*x^2+c^2*d)-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+
2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(I/c/x+(1-1/c^2/
x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/c/d^3/e+1/2*(-(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*
e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-
c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e/d^3/c-1/2*((c^2*d+2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*arctanh
(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2
))/c/d^3/e+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^
(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctanh(c*d*(I/c/x+(1-
1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/e/
d^3/c-1/4/e*c^4*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/
c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_
R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/4/e*c^4*sum(_R1/(_R1^2*c
^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilo
```

```
g((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e
)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^2}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{\left(ex^2+d\right)^2}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^2}\,dx=\int \frac{x^2(a+b\,\mathrm{acsc}\,(cx))}{\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.109 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 762

$$
\begin{aligned}
\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx =& \frac{-a-b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\
&+\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}+\frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} \\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}
\end{aligned}
$$

```
[Out] 1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/
c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e
```

```
^(1/2)+1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccsc(c*x))*ln(1+
I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^
(3/2)/e^(1/2)+1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,I*c*(I/c/x
+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1
/2)+1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/
x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(-
a-b*arccsc(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(a+b*arccsc(c*x))/d/(d/x+(
-d)^(1/2)*e^(1/2))+1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^
2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)+1/4*b*arctanh((c^
2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^
(3/2)/(c^2*d+e)^(1/2)
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 759, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5339, 4817, 4757, 4827, 739, 212, 4825, 4615, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx &= \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\ &- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\ &+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\ &- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\ &- \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} \\ &+ \frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4d^{3/2}\sqrt{c^2d+e}} + \frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4d^{3/2}\sqrt{c^2d+e}} \\ &+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\ &+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\end{aligned}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(a + b*ArcCsc[c*x])/(d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcCsc[c*x])
/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)
/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d
+ e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e
]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d + e]) + ((a + b*ArcCsc[c*x
])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(
4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*Arc
Csc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*A
rcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d
+ e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]
*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) +
((I/4)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*Ar
cCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e]) + ((I/4)*b*P
olyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])
/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x])
)/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5339

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx,x,\frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2}+\frac{a+b\arcsin\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)}\right)\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)}{d}+\frac{e\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx,x,\frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx,x,\frac{1}{x}\right)}{d}$$
$$+\frac{e\text{Subst}\left(\int \left(-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx,x,\frac{1}{x}\right)}{d}$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx,x,\frac{1}{x}\right)\right)$$
$$-\frac{1}{4}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx,x,\frac{1}{x}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx,x,\frac{1}{x}\right)$$
$$-\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{2d\sqrt{e}}-\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\frac{1}{x}\right)}{2d\sqrt{e}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$-\frac{1}{2}\text{Subst}\left(\int \left(-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx,x,\frac{1}{x}\right)$$
$$+\frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{4cd}-\frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{4cd}$$
$$-\frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx,x,\csc^{-1}(cx)\right)}{2d\sqrt{e}}$$
$$-\frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx,x,\csc^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cd}$$

$$+ \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cd} + \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d\sqrt{e}}$$

$$+ \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2d\sqrt{e}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}$$

$$+ \frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\,\mathrm{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\,\mathrm{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\,\mathrm{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\,\mathrm{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{\mathrm{Subst}\left(\int\frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}dx, x, \csc^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$+ \frac{\mathrm{Subst}\left(\int\frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}dx, x, \csc^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}$$

$$+ \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(ib)\text{Subst}\left(\int \frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$+ \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$+ \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$+ \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4d\sqrt{e}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}$$

$$+ \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$- \frac{b\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}$$
$$+ \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}} - \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}\sqrt{e}}$$
$$- \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$- \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$
$$+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

$$
\begin{aligned}
&= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}} \\
&+\frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 2.06 (sec) , antiderivative size = 1477, normalized size of antiderivative = 1.94

$$
\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx = \frac{1}{2}\left(\frac{ax}{d^2+dex^2}+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}\right.
$$

$$
+\frac{b\left(\frac{2\sqrt{d}\csc^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex}+\frac{2\sqrt{d}\csc^{-1}(cx)}{i\sqrt{d}\sqrt{e}+ex}+\frac{8\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2\csc^{-1}(cx)\right)\right)}{\sqrt{c^2d+e}}\right)}{\sqrt{e}}-\frac{8\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\mathrm{arctan}\cdots\right.}{\cdots}
$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*((2*Sqrt[d]*ArcCsc[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) + (2*Sqrt[d]*Arc
Csc[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + (8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])
/4])/Sqrt[c^2*d + e]])/Sqrt[e] - (8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqr
t[c^2*d + e]])/Sqrt[e] - (I*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((4*I)*ArcSin[Sqrt[
1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + (I*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((2*I)*ArcCsc[c*x]*Log[1
 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + (
(4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] +
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + (I*Pi*Log[1 - (
Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((2*I)
*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))])/Sqrt[e] - ((4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Lo
g[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] -
 (I*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/
Sqrt[e] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))])/Sqrt[e] + ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[
d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*
x]))])/Sqrt[e] - (I*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x])/Sqrt[e] + (I*Pi*Log[Sq
rt[e] + (I*Sqrt[d])/x])/Sqrt[e] + ((2*I)*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] +
c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c
^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] - ((2*I)*Log[(2*Sq
rt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1
/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt[-(c^2*
d) - e] + (2*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))])/Sqrt[e] - (2*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^
(I*ArcCsc[c*x]))])/Sqrt[e] - (2*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x])))])/Sqrt[e] + (2*PolyLog[2, (Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e]))/(4*d^(3/2)))/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 33.95 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.09

method	result
parts	$\frac{ax}{2d(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+\frac{b\left(\frac{c^3\operatorname{arccsc}(cx)x}{2d\left(c^2e\,x^2+c^2d\right)}-\frac{c^2\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\operatorname{arccsc}(cx)\ln\left(\ldots\right.}{\ldots}\right.}{4d}\ldots\right.}{\ldots}$
derivativedivides	$\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\operatorname{arccsc}(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\operatorname{arccsc}(cx)\ln\left(\ldots\right.}{\ldots}}{4d\,c^2}\ldots\right.$
default	$\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\operatorname{arccsc}(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\operatorname{arccsc}(cx)\ln\left(\ldots\right.}{\ldots}}{4d\,c^2}\ldots\right.$

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c^
3*arccsc(c*x)*x/d/(c^2*e*x^2+c^2*d)-1/4/d*c^2*sum(1/_R1/(_R1^2*c^2*d-c^2*d-
2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/
x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*
d))-1/4/d*c^2*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-
(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=R
ootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(I/c/x+(1-1
/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/c^3-1/2*
(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c
^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2)
)/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)/c^3+1/2*((c^2
*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*ar
ctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)
^(1/2))/d^4/c^3-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d
+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctanh(c*d*(I/c/
```

```
x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^
2*d+e)/c^3)
```

Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx = \int \frac{b\operatorname{arccsc}(cx)+a}{(ex^2+d)^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx = \int \frac{a+b\operatorname{acsc}(cx)}{(d+ex^2)^2}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**2,x)
[Out] Integral((a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex^2)^2}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(e\,x^2 + d)^2}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^2, x)
```

3.110 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 806

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^2}\,dx = -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2}-\frac{a}{d^2x}-\frac{b\csc^{-1}(cx)}{d^2x}+\frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b e\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}-\frac{b e\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}+\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

```
[Out] -a/d^2/x-b*arccsc(c*x)/d^2/x+3/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*(
a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(
c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2
)-3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*I*b*polylog(2,-I*c*(I/c/x+(1
-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)
```

```
-3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*I*b*p
olylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))*e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arccsc(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2)
)-1/4*e*(a+b*arccsc(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))-1/4*b*e*arctanh((c^2
*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(
5/2)/(c^2*d+e)^(1/2)-1/4*b*e*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)
/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)-b*c*(1-1/c^2/
x^2)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5349, 4817, 4715, 267, 4757, 4827, 739, 212, 4825, 4615, 2221, 2317, 2438}

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx &= -\frac{a}{d^2x}-\frac{b\csc^{-1}(cx)}{d^2x}+\frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}\\
&-\frac{be\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}}-\frac{be\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}}\\
&+\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}\\
&-\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(\frac{i\sqrt{-d}e^{i\csc^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}}\\
&+\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}\\
&-\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(\frac{i\sqrt{-d}e^{i\csc^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}}\\
&+\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}\\
&-\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}\\
&+\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}\\
&-\frac{3ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2}\end{aligned}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -((b*c*Sqrt[1 - 1/(c^2*x^2)])/d^2) - a/(d^2*x) - (b*ArcCsc[c*x])/(d^2*x) +
(e*(a + b*ArcCsc[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b*ArcCsc
[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*
Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(5/2)*
Sqrt[c^2*d + e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*S
qrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c^2*d + e]) + (3*Sq
rt[e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1
 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(
5/2)) + (3*Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*
x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCs
c[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]
)])/(4*(-d)^(5/2)) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*
ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(5/2) - (((3*I)/4)*b*Sqrt[
e]*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]
)/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)^(5/2) - (((3*I)/4)*b*Sqrt[e]*Pol
yLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)
^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^4\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx,x,\frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{d^2}+\frac{e^2\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2}-\frac{2e\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right)\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \left(a+b\arcsin\left(\frac{x}{c}\right)\right)\,dx,x,\frac{1}{x}\right)}{d^2}\\
&\quad+\frac{(2e)\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)}{d^2}-\frac{e^2\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx,x,\frac{1}{x}\right)}{d^2}\\
&= -\frac{a}{d^2x}-\frac{b\text{Subst}\left(\int \arcsin\left(\frac{x}{c}\right)\,dx,x,\frac{1}{x}\right)}{d^2}\\
&\quad+\frac{(2e)\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx,x,\frac{1}{x}\right)}{d^2}\\
&\quad-\frac{e^2\text{Subst}\left(\int \left(-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx,x,\frac{1}{x}\right)}{d^2}
\end{aligned}
$$

$$= -\frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd^2}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{d^2}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{e\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4d}$$
$$+ \frac{e\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4d} + \frac{e\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2d}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{d^2}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{d^2}$$
$$- \frac{(be)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4cd^2}$$
$$+ \frac{(be)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4cd^2}$$
$$+ \frac{e\text{Subst}\left(\int \left(-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{2d}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$- \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d^2} - \frac{\sqrt{e}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{4d^2}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{d^2}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{d^2}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{d^2}$$
$$+ \frac{\sqrt{e}\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{d^2}$$
$$+ \frac{(be)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cd^2}$$
$$- \frac{(be)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cd^2}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$- \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b e \operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}$$

$$- \frac{b e \operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} + \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$- \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$+ \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$- \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$

$$- \frac{(b\sqrt{e})\operatorname{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{(-d)^{5/2}}$$

$$+ \frac{(b\sqrt{e})\operatorname{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{(-d)^{5/2}}$$

$$- \frac{(b\sqrt{e})\operatorname{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{(-d)^{5/2}}$$

$$+ \frac{(b\sqrt{e})\operatorname{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{(-d)^{5/2}}$$

$$- \frac{\sqrt{e}\operatorname{Subst}\left(\int\frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}dx, x, \csc^{-1}(cx)\right)}{4d^2}$$

$$- \frac{\sqrt{e}\operatorname{Subst}\left(\int\frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}dx, x, \csc^{-1}(cx)\right)}{4d^2}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}$$
$$- \frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} + \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$
$$- \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$
$$+ \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$
$$- \frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}}$$
$$+ \frac{(ib\sqrt{e})\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{(-d)^{5/2}}$$
$$- \frac{(ib\sqrt{e})\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{(-d)^{5/2}}$$
$$+ \frac{(ib\sqrt{e})\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{(-d)^{5/2}}$$
$$- \frac{(ib\sqrt{e})\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx, x, e^{i\csc^{-1}(cx)}\right)}{(-d)^{5/2}}$$
$$- \frac{\sqrt{e}\operatorname{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4d^2}$$
$$- \frac{\sqrt{e}\operatorname{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4d^2}$$
$$- \frac{\sqrt{e}\operatorname{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4d^2}$$
$$- \frac{\sqrt{e}\operatorname{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4d^2}$$

$$
\begin{aligned}
&= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&\quad - \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} \\
&\quad - \frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} + \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad - \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad - \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} \\
&\quad + \frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} - \frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} \\
&\quad + \frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} - \frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} \\
&\quad + \frac{(b\sqrt{e})\operatorname{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4(-d)^{5/2}} \\
&\quad - \frac{(b\sqrt{e})\operatorname{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4(-d)^{5/2}} \\
&\quad + \frac{(b\sqrt{e})\operatorname{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4(-d)^{5/2}} \\
&\quad - \frac{(b\sqrt{e})\operatorname{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{4(-d)^{5/2}}
\end{aligned}
$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} - \frac{be\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} + \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} - \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} + \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} - \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{(-d)^{5/2}} - \frac{(ib\sqrt{e})\,\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{4(-d)^{5/2}} + \frac{(ib\sqrt{e})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{4(-d)^{5/2}} - \frac{(ib\sqrt{e})\,\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{4(-d)^{5/2}} + \frac{(ib\sqrt{e})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}\,dx,x,e^{i\csc^{-1}(cx)}\right)}{4(-d)^{5/2}}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$- \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}$$
$$- \frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}} + \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$- \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+ \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$- \frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+ \frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+ \frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 1.86 (sec) , antiderivative size = 1525, normalized size of antiderivative = 1.89

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$$

$$= \frac{-\frac{8a\sqrt{d}}{x} - \frac{4a\sqrt{d}ex}{d+ex^2} - 12a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + b\left(-8c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}} - \frac{8\sqrt{d}\csc^{-1}(cx)}{x} - \frac{2\sqrt{d}e\csc^{-1}(cx)}{-i\sqrt{d}\sqrt{e}+ex} - \frac{2\sqrt{d}e\csc^{-1}(c}{i\sqrt{d}\sqrt{e}+ex}\right.}{}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] + b*(-8*c*Sqrt[d]*Sqrt[1 - 1/(c^2*x^2)] - (8*Sqrt[d]*ArcC
sc[c*x])/x - (2*Sqrt[d]*e*ArcCsc[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) - (2*Sq
rt[d]*e*ArcCsc[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - 24*Sqrt[e]*ArcSin[Sqrt[1 -
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(
Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + 24*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sq
```

```
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*Ar
cCsc[c*x])/4])/Sqrt[c^2*d + e]] + (3*I)*Sqrt[e]*Pi*Log[1 + (Sqrt[e] - Sqrt[
c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (6*I)*Sqrt[e]*ArcCsc[c*x]*Log[
1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (12*I)*Sqr
t[e]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (3*I)*Sqrt[e]*Pi*Log[1 + (
-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (6*I)*Sqrt[e]*
ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] - (12*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log
[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (3*I)*Sq
rt[e]*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 + (6*I)*Sqrt[e]*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + (12*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] + (3*I)*Sqrt[e]*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] - (6*I)*Sqrt[e]*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (12*I)*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] + (3*I)*Sqrt[e]*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (3*I)
*Sqrt[e]*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] - ((2*I)*e*Log[(2*Sqrt[d]*Sqrt[e]*
(Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)
)/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + ((2*I
)*e*Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]
)/Sqrt[-(c^2*d) - e] - 6*Sqrt[e]*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] + 6*Sqrt[e]*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 6*Sqrt[e]*PolyLog[2, -((Sqrt[e] + Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] - 6*Sqrt[e]*PolyLog[2, (Sqrt[
e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]))/(8*d^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 35.46 (sec) , antiderivative size = 925, normalized size of antiderivative = 1.15

method	result	size
parts	Expression too large to display	925
derivativedivides	Expression too large to display	952
default	Expression too large to display	952

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/d^2/x-e/d^2*(1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
)+b*c*(1/2*(I*((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x-1)*(arccsc(c*x)+I)/d^2/x/c-1/
```

```
2*(I*((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x+1)/x/c*(arccsc(c*x)-I)/d^2-1/2*arccsc(
c*x)/d^2*e*x*c/(c^2*e*x^2+c^2*d)-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)
^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*e*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^
(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/c^5+1/2*(-(c^2*d-2
*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(
e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2
*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/c^5/(c^2*d+e)-1/2*((c^2*d+2*(e*
(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*e*arctanh(
c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)
)/d^5/c^5+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(
1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctanh(c*d*(I/c/x+(1
-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5/c^5/(c^
2*d+e)+3/4*e/d^2*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I
/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),
_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+3/4*e/d^2*sum(_R1/(_R1^2*
c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dil
og((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*
e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{acsc}\left(cx\right)}{x^2\left(d+ex^2\right)^2}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**2,x)
[Out] Integral((a + b*acsc(c*x))/(x**2*(d + e*x**2)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{x^2\left(d + ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{x^2\left(d + ex^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{x^2\left(d + ex^2\right)^2}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\left(e\,x^2 + d\right)^2}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

3.111 $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 727

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} + \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}(c^2d+e)^{3/2}} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} - \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^3}$$

```
[Out] 1/4*(-a-b*arccsc(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arccsc(c*x))/e^2/(e+d/x^2)+1
/8*b*(c^2*d+2*e)*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e^
(5/2)/(c^2*d+e)^(3/2)-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
/e^3+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsc(c*
x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/e^3+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1
/2))^2)/e^3-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(
```

```
e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,-I*c*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*I*b*po
lylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)
))/e^3+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e^(5/2
)/(c^2*d+e)^(1/2)+1/8*b*c*d*(1-1/c^2/x^2)^(1/2)/e^2/(c^2*d+e)/(e+d/x^2)/x
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5349, 4817, 4721, 3798, 2221, 2317, 2438, 4813, 390, 385, 211, 4825, 4615}

$$
\begin{aligned}
\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx &= \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} \\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} \\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3} \\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}-\frac{a+b\csc^{-1}(cx)}{2e^2\left(\frac{d}{x^2}+e\right)} \\
&-\frac{a+b\csc^{-1}(cx)}{4e\left(\frac{d}{x^2}+e\right)^2}-\frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{e^3} \\
&+\frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{8e^{5/2}(c^2d+e)^{3/2}}+\frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{5/2}\sqrt{c^2d+e}} \\
&-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3} \\
&-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3} \\
&-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3} \\
&-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3} \\
&+\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2x(c^2d+e)\left(\frac{d}{x^2}+e\right)}+\frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^3}
\end{aligned}
$$

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*d*Sqrt[1 - 1/(c^2*x^2)])/(8*e^2*(c^2*d + e)*(e + d/x^2)*x) - (a + b*Ar
cCsc[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcCsc[c*x])/(2*e^2*(e + d/x^2)) +
(b*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/2)*
Sqrt[c^2*d + e]) + (b*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[
1 - 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)) + ((a + b*ArcCsc[c*x])*
Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e
^3) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e
] - Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]
*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcCsc[
c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]
)/(2*e^3) - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/e^3 - ((I/
2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
 e])])/e^3 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
- Sqrt[c^2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-
d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + ((I/2)*b*PolyLog[
2, E^((2*I)*ArcCsc[c*x])])/e^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4813

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
 - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{e^3x} - \frac{dx\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^3} - \frac{dx\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2} - \frac{dx\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e^3\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3}\\
&\quad + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{e}\\
&= -\frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)}{e^3}\\
&\quad + \frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{e^3}\\
&\quad + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)}{2ce^2} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{4ce}
\end{aligned}
$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be^3}$$
$$+ \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)}{e^3} - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^3}$$
$$+ \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2e^3} + \frac{b\text{Subst}\left(\int \frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2ce^2}$$
$$+ \frac{\left(b(c^2d+2e)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}(e+dx^2)}\,dx, x, \frac{1}{x}\right)}{8ce^2\left(c^2d+e\right)}$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be^3}$$
$$+ \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} - \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3}$$
$$+ \frac{b\text{Subst}\left(\int \log\left(1-e^{2ix}\right)\,dx, x, \csc^{-1}(cx)\right)}{e^3}$$
$$- \frac{\sqrt{-d}\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2e^3}$$
$$+ \frac{\sqrt{-d}\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2e^3}$$
$$+ \frac{\left(b(c^2d+2e)\right)\text{Subst}\left(\int \frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2}\,dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8ce^2\left(c^2d+e\right)}$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$

$$+ \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} + \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}\left(c^2d+e\right)^{3/2}}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3} - \frac{(ib)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i\csc^{-1}(cx)}\right)}{2e^3}$$

$$+ \frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^3}$$

$$+ \frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^3}$$

$$- \frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^3}$$

$$- \frac{\sqrt{-d}\text{Subst}\left(\int\frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2e^3}$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$
$$+ \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} + \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}(c^2d+e)^{3/2}}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^3}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^3}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^3}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^3}$$
$$- \frac{b\mathrm{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx,x,\csc^{-1}(cx)\right)}{2e^3}$$

$$
\begin{aligned}
&= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} \\
&+ \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} + \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}\left(c^2d+e\right)^{3/2}} \\
&+ \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} \\
&+ \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} \\
&+ \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} \\
&+ \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} \\
&- \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3} + \frac{ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^3} \\
&+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^3} \\
&+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^3} \\
&+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^3} \\
&+ \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx,x,e^{i\csc^{-1}(cx)}\right)}{2e^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} \\
&+ \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{c^2d+e}} + \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}\left(c^2d+e\right)^{3/2}} \\
&+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} \\
&+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} \\
&+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} \\
&+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} \\
&- \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3} - \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} \\
&- \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} \\
&- \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^3}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2053 vs. 2(727) = 1454.

Time = 7.39 (sec) , antiderivative size = 2053, normalized size of antiderivative = 2.82

$$
\int \frac{x^5(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^3}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x
^2])/(2*e^3) + b*((((7*I)/16)*Sqrt[d]*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*Sqrt[e]
+ e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c
*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^
2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2)
- (((7*I)/16)*Sqrt[d]*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSi
n[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] +
 Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d]
```

```
 - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2) - (d*((I*c*Sqrt[e]
*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))
- ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*S
qrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*
(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I
)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)) - (d*(((-I)*
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e
]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(
d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[
e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d +
e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)) + ((I/16
)*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e
])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*Arc
Csc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sq
rt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*
Log[Sqrt[e] + (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/e^3
+ ((I/16)*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi +
2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[
e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1
 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (
4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/
e^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.67 (sec) , antiderivative size = 1269, normalized size of antiderivative = 1.75

method	result	size
parts	Expression too large to display	1269
derivativedivides	Expression too large to display	1285
default	Expression too large to display	1285

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/4*d^2/e^3/(e*x^2+d)^2+1/2/e^3*ln(e*x^2+d)+d/e^3/(e*x^2+d))+b/c^6*(-1/
8*c^6*(4*c^6*d^2*arccsc(c*x)*x^2+6*c^6*d*e*arccsc(c*x)*x^4-((c^2*x^2-1)/c^2
/x^2)^(1/2)*c^5*d^2*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^5*d*e*x^3+I*c^4*d^2+2*I
*c^4*d*e*x^2+I*e^2*c^4*x^4+4*c^4*d*e*arccsc(c*x)*x^2+6*arccsc(c*x)*e^2*c^4*
x^4)/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)-1/4*I/(c^2*d+e)/e^2*c^8*d*sum((_R1^2
-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2
))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(
-2*c^2*d-4*e)*_Z^2+c^2*d))+I/(c^2*d+e)/e^2*c^6*dilog(1+I/c/x+(1-1/c^2/x^2)^
(1/2))-I/(c^2*d+e)/e^2*c^6*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I/(c^2*d+e)
/e^2*c^6*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)
*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/
2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/(c^2*d+e)/e^2
*c^6*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/(c^2*d+e)/e^3*c^8*d*arcc
sc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I/(c^2*d+e)/e^3*c^8*d*sum((_R1^
2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-
1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Root
Of(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/4*I/(c^2*d+e)/e^3*c^10*d^2*sum(
(_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2
)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*
_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-5/8*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/e^3*
arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^
2)^(1/2))*c^8*d+I/(c^2*d+e)/e^3*c^8*d*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-I/
(c^2*d+e)/e^3*c^8*d*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))-3/4*I*(e*(c^2*d+e))^(1
/2)/(c^2*d+e)^2/e^2*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^
2*d-4*e)/(c^2*d*e+e^2)^(1/2))*c^6)
```

Fricas [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*x^5*arccsc(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
 d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
+ d)/e^3) + b*integrate(x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^3*x^
6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

3.112 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bcx\sqrt{-1+c^2x^2}}{8e(c^2d+e)\sqrt{c^2x^2}(d+ex^2)}+\frac{x^4(a+b\csc^{-1}(cx))}{4d(d+ex^2)^2} +\frac{bc(c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}(c^2d+e)^{3/2}\sqrt{c^2x^2}}$$

```
[Out] 1/4*x^4*(a+b*arccsc(c*x))/d/(e*x^2+d)^2+1/8*b*c*(c^2*d+2*e)*x*arctan(e^(1/2
)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2))/d/e^(3/2)/(c^2*d+e)^(3/2)/(c^2*x^2)^(1
/2)-1/8*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5347, 12, 457, 79, 65, 211}

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{x^4(a+b\csc^{-1}(cx))}{4d(d+ex^2)^2}+\frac{bcx(c^2d+2e)\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\sqrt{c^2x^2}(c^2d+e)^{3/2}} -\frac{bcx\sqrt{c^2x^2-1}}{8e\sqrt{c^2x^2}(c^2d+e)(d+ex^2)}$$

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*c*x*Sqrt[-1 + c^2*x^2])/(e*(c^2*d + e)*Sqrt[c^2*x^2]*(d + e*x^2)) +
 (x^4*(a + b*ArcCsc[c*x]))/(4*d*(d + e*x^2)^2) + (b*c*(c^2*d + 2*e)*x*ArcTa
```

```
n[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*d + e]])/(8*d*e^(3/2)*(c^2*d + e)^(
3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4(a+b\csc^{-1}(cx))}{4d\left(d+ex^2\right)^2}+\frac{(bcx)\int\frac{x^3}{4d\sqrt{-1+c^2x^2}(d+ex^2)^2}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{x^4(a+b\csc^{-1}(cx))}{4d\left(d+ex^2\right)^2}+\frac{(bcx)\int\frac{x^3}{\sqrt{-1+c^2x^2}(d+ex^2)^2}\,dx}{4d\sqrt{c^2x^2}}\\
&= \frac{x^4(a+b\csc^{-1}(cx))}{4d\left(d+ex^2\right)^2}+\frac{(bcx)\text{Subst}\left(\int\frac{x}{\sqrt{-1+c^2x}(d+ex)^2}\,dx,x,x^2\right)}{8d\sqrt{c^2x^2}}\\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{8e\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}+\frac{x^4(a+b\csc^{-1}(cx))}{4d\left(d+ex^2\right)^2}\\
&\quad+\frac{\left(bc(c^2d+2e)\,x\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+c^2x}(d+ex)}\,dx,x,x^2\right)}{16de\left(c^2d+e\right)\sqrt{c^2x^2}}\\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{8e\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}+\frac{x^4(a+b\csc^{-1}(cx))}{4d\left(d+ex^2\right)^2}\\
&\quad+\frac{\left(b(c^2d+2e)\,x\right)\text{Subst}\left(\int\frac{1}{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{8cde\left(c^2d+e\right)\sqrt{c^2x^2}}\\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{8e\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}+\frac{x^4(a+b\csc^{-1}(cx))}{4d\left(d+ex^2\right)^2}+\frac{bc(c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\left(c^2d+e\right)^{3/2}\sqrt{c^2x^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.48

$$\int\frac{x^3(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^3}\,dx$$

$$= \frac{\frac{4ad}{(d+ex^2)^2}-\frac{8a}{d+ex^2}-\frac{2bce\sqrt{1-\frac{1}{c^2x^2}}\,x}{(c^2d+e)(d+ex^2)}-\frac{4b(d+2ex^2)\csc^{-1}(cx)}{(d+ex^2)^2}+\frac{4b\arcsin\left(\frac{1}{cx}\right)}{d}+\frac{b\sqrt{e}(c^2d+2e)\log\left(\frac{16d\sqrt{-c^2d-e}\,e^{3/2}\left(i\sqrt{e}+c\left(c\sqrt{d}-i\sqrt{-\cdots}\right.\right.}{b\left(c^2d+2e\right)\left(\sqrt{d}+i\sqrt{\cdots}\right.}\right.}{d(-c^2d-e)^{3/2}}}{16e^2}$$

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((4*a*d)/(d + e*x^2)^2 - (8*a)/(d + e*x^2) - (2*b*c*e*Sqrt[1 - 1/(c^2*x^2)]
*x)/((c^2*d + e)*(d + e*x^2)) - (4*b*(d + 2*e*x^2)*ArcCsc[c*x])/(d + e*x^2)
^2 + (4*b*ArcSin[1/(c*x)])/d + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(16*d*Sqrt[-(c^
2*d) - e]*e^(3/2)*(I*Sqrt[e] + c*(c*Sqrt[d] - I*Sqrt[-(c^2*d) - e]*Sqrt[1 -
 1/(c^2*x^2)])*x))/(b*(c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(-(c^2*d)
 - e)^(3/2)) + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(-16*d*Sqrt[-(c^2*d) - e]*e^(3/
2)*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)]
)*x))/(b*(c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(-(c^2*d) - e)^(3/2)))
/(16*e^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(135) = 270$.

Time = 6.54 (sec) , antiderivative size = 945, normalized size of antiderivative = 6.02

method	result
parts	$a\left(\frac{d}{4e^2(ex^2+d)^2}-\frac{1}{2e^2(ex^2+d)}\right)+\frac{b\left(-\frac{c^6\operatorname{arccsc}(cx)}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{c^8\operatorname{arccsc}(cx)d}{4e^2\left(c^2e\,x^2+c^2d\right)^2}-\frac{c^3\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-}\right.}{}\right.}{}$
derivativedivides	$\frac{a\,c^6\left(-\frac{1}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}\right)+b\,c^6\left(-\frac{\operatorname{arccsc}(cx)}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{\operatorname{arccsc}(cx)d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}+\frac{\sqrt{c^2x^2-1}\left(-4\arctan\left(\frac{1}{\sqrt{c^2x}}\right.\right.}{}\right.}{}$
default	$\frac{a\,c^6\left(-\frac{1}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}\right)+b\,c^6\left(-\frac{\operatorname{arccsc}(cx)}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{\operatorname{arccsc}(cx)d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}+\frac{\sqrt{c^2x^2-1}\left(-4\arctan\left(\frac{1}{\sqrt{c^2x}}\right.\right.}{}\right.}{}$

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/4*d/e^2/(e*x^2+d)^2-1/2/e^2/(e*x^2+d))+b/c^4*(-1/2*c^6*arccsc(c*x)/e^2
/(c^2*e*x^2+c^2*d)+1/4*c^8*arccsc(c*x)*d/e^2/(c^2*e*x^2+c^2*d)^2-1/16*c^3*(
c^2*x^2-1)^(1/2)/e*(4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4*
d*e*x^2+4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4*d^2-ln(2*((c
^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2
*d*e)^(1/2)))*c^4*d*e*x^2-ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-
```

```
c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-ln(-2*((c^2*x^2-1)^
(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/
2)))*c^4*d*e*x^2-ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)
^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2+4*arctan(1/(c^2*x^2-1)^(1/
2))*(-(c^2*d+e)/e)^(1/2)*e^2*c^2*x^2+4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d
+e)/e)^(1/2)*c^2*d*e-2*(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*c^2*d*e-2*ln(
2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+
(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/
2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e-2*ln(-2*((c^
2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2
*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e
+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e)/((c^2*x^2-1)/c
^2/x^2)^(1/2)/x/d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-c*e*x+(-c^2*d*e)^(1/2))/
(c*e*x+(-c^2*d*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(135) = 270$.

Time = 0.50 (sec) , antiderivative size = 1015, normalized size of antiderivative = 6.46

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$$

$$= \Bigg[-\frac{4\,ac^4d^4 + 8\,ac^2d^3e + 4\,ad^2e^2 + 8\,(ac^4d^3e + 2\,ac^2d^2e^2 + ade^3)x^2 + (bc^2d^3 + (bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e + \cdots}{\cdots}$$

$$-\frac{2\,ac^4d^4 + 4\,ac^2d^3e + 2\,ad^2e^2 + 4\,(ac^4d^3e + 2\,ac^2d^2e^2 + ade^3)x^2 - (bc^2d^3 + (bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e + \cdots}{\cdots}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e + 2*a*c^
2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d
^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*log((c^2*e*x^2
 - c^2*d - 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2 + d)) + 4
*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2
+ b*d*e^3)*x^2)*arccsc(c*x) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b
*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^
2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^2*d^3*e + b*d^2
*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^5*e^2 + 2*c
^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d
^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e +
 2*a*d^2*e^2 + 4*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 - (b*c^2*d^3
 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^
```

```
2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2 - 1)/(c^2*d
+ e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2
*d^2*e^2 + b*d*e^3)*x^2)*arccsc(c*x) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2
*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c
^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*d^3*e
+ b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^5*e^
2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2
*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] -1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/4*(2*e*x^2*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1)) + 4*(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)*integrate(1/4*(2*c^2*e*x^3 + c^2*
d*x)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^4*x^6 + (2*c^2*d*e^3 -
e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*d*e^3)*x^2 + (c^2*e^4*x^6 + (2*c^2*d*
e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*d*e^3)*x^2)*e^(log(c*x + 1) + l
og(c*x - 1))), x))*b/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

3.113 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 193

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{bcx\sqrt{-1+c^2x^2}}{8d\,(c^2d+e)\,\sqrt{c^2x^2}\,(d+ex^2)} - \frac{a+b\csc^{-1}(cx)}{4e\,(d+ex^2)^2} - \frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}} + \frac{bc(3c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\,(c^2d+e)^{3/2}\sqrt{c^2x^2}}$$

```
[Out] 1/4*(-a-b*arccsc(c*x))/e/(e*x^2+d)^2-1/4*b*c*x*arctan((c^2*x^2-1)^(1/2))/d^
2/e/(c^2*x^2)^(1/2)+1/8*b*c*(3*c^2*d+2*e)*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2
)/(c^2*d+e)^(1/2))/d^2/(c^2*d+e)^(3/2)/e^(1/2)/(c^2*x^2)^(1/2)+1/8*b*c*x*(c
^2*x^2-1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5345, 457, 105, 162, 65, 211}

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{a+b\csc^{-1}(cx)}{4e\,(d+ex^2)^2} - \frac{bcx\arctan\left(\sqrt{c^2x^2-1}\right)}{4d^2e\sqrt{c^2x^2}} + \frac{bcx(3c^2d+2e)\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\sqrt{c^2x^2}\,(c^2d+e)^{3/2}} + \frac{bcx\sqrt{c^2x^2-1}}{8d\sqrt{c^2x^2}\,(c^2d+e)\,(d+ex^2)}$$

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*x*Sqrt[-1 + c^2*x^2])/(8*d*(c^2*d + e)*Sqrt[c^2*x^2]*(d + e*x^2)) - (a
 + b*ArcCsc[c*x])/(4*e*(d + e*x^2)^2) - (b*c*x*ArcTan[Sqrt[-1 + c^2*x^2]])/
(4*d^2*e*Sqrt[c^2*x^2]) + (b*c*(3*c^2*d + 2*e)*x*ArcTan[(Sqrt[e]*Sqrt[-1 +
c^2*x^2])/Sqrt[c^2*d + e]])/(8*d^2*Sqrt[e]*(c^2*d + e)^(3/2)*Sqrt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
```

```
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{(bcx)\int\frac{1}{x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^2}\,dx}{4e\sqrt{c^2x^2}}\\
&= -\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{(bcx)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}(d+ex)^2}\,dx,x,x^2\right)}{8e\sqrt{c^2x^2}}\\
&= \frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bcx)\text{Subst}\left(\int\frac{-c^2d-e+\frac{1}{2}c^2ex}{x\sqrt{-1+c^2x}(d+ex)}\,dx,x,x^2\right)}{8de\left(c^2d+e\right)\sqrt{c^2x^2}}\\
&= \frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{(bcx)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{8d^2e\sqrt{c^2x^2}}\\
&\quad+\frac{\left(bc\left(3c^2d+2e\right)x\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+c^2x}(d+ex)}\,dx,x,x^2\right)}{16d^2\left(c^2d+e\right)\sqrt{c^2x^2}}\\
&= \frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}\\
&\quad-\frac{(bx)\text{Subst}\left(\int\frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{4cd^2e\sqrt{c^2x^2}}\\
&\quad+\frac{\left(b\left(3c^2d+2e\right)x\right)\text{Subst}\left(\int\frac{1}{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{8cd^2\left(c^2d+e\right)\sqrt{c^2x^2}}\\
&= \frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}\\
&\quad-\frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}}+\frac{bc\left(3c^2d+2e\right)x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\left(c^2d+e\right)^{3/2}\sqrt{c^2x^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.99

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$$
$$=\frac{1}{16}\left(-\frac{4a}{e(d+ex^2)^2}+\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}x}{d(c^2d+e)(d+ex^2)}-\frac{4b\csc^{-1}(cx)}{e(d+ex^2)^2}+\frac{4b\arcsin\left(\frac{1}{cx}\right)}{d^2e}\right.$$
$$+\frac{b(3c^2d+2e)\log\left(\frac{16d^2\sqrt{-c^2d-e}\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}-i\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d^2(-c^2d-e)^{3/2}\sqrt{e}}$$
$$\left.+\frac{b(3c^2d+2e)\log\left(-\frac{16d^2\sqrt{-c^2d-e}\sqrt{e}\left(-\sqrt{e}+c\left(-ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2(-c^2d-e)^{3/2}\sqrt{e}}\right)$$

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((-4*a)/(e*(d + e*x^2)^2) + (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d + e)*
(d + e*x^2)) - (4*b*ArcCsc[c*x])/(e*(d + e*x^2)^2) + (4*b*ArcSin[1/(c*x)])/
(d^2*e) + (b*(3*c^2*d + 2*e)*Log[(16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(I*Sqrt
[e] + c*(c*Sqrt[d] - I*Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(3*
c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e])
+ (b*(3*c^2*d + 2*e)*Log[(-16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(-Sqrt[e] + c*
((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(3*c^2*d
 + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e]))/16
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. 2(168) = 336.

Time = 7.18 (sec) , antiderivative size = 894, normalized size of antiderivative = 4.63

method	result
parts	$-\frac{a}{4e(ex^2+d)^2}+\frac{b\left(-\frac{c^6\operatorname{arccsc}(cx)}{4e\left(c^2ex^2+c^2d\right)^2}-\frac{c\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4dex^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d}{e}}\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{ac^6}{4e\left(c^2ex^2+c^2d\right)^2}+bc^6\left(-\frac{\operatorname{arccsc}(cx)}{4e\left(c^2ex^2+c^2d\right)^2}-\frac{\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4dex^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\right.}{}\right.}{}$
default	$\frac{-\frac{ac^6}{4e\left(c^2ex^2+c^2d\right)^2}+bc^6\left(-\frac{\operatorname{arccsc}(cx)}{4e\left(c^2ex^2+c^2d\right)^2}-\frac{\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\,c^4dex^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\right.}{}\right.}{}$

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arccsc(c*x)-1/16
*c*(c^2*x^2-1)^(1/2)*(4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^
4*d*e*x^2+4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4*d^2-3*ln(-
2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x
+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-3*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(
1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-3*ln(2*((
c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^
2*d*e)^(1/2)))*c^4*d*e*x^2-3*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e
-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2+4*arctan(1/(c^2*
x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*e^2*c^2*x^2+4*arctan(1/(c^2*x^2-1)^(1/2)
)*(-(c^2*d+e)/e)^(1/2)*c^2*d*e+2*(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*c^2
*d*e-2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x
-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^
2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*
e-2*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/
(c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)
/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^2*d*e)/((c^
2*x^2-1)/c^2/x^2)^(1/2)/x/d^2/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(c*e*x+(-c^2*d
*e)^(1/2))/(-c*e*x+(-c^2*d*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(165) = 330.

Time = 0.50 (sec) , antiderivative size = 890, normalized size of antiderivative = 4.61

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex^2)^3}\,dx$$

$$= \left[-\frac{4\,ac^4d^4 + 8\,ac^2d^3e + 4\,ad^2e^2 + (3\,bc^2d^3 + (3\,bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e + 2\,(3\,bc^2d^2e + 2\,bde^2)x^2)\sqrt{-c^2d}}{}\right.$$

$$-\frac{2\,ac^4d^4 + 4\,ac^2d^3e + 2\,ad^2e^2 - (3\,bc^2d^3 + (3\,bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e + 2\,(3\,bc^2d^2e + 2\,bde^2)x^2)\sqrt{c^2de +}}{}$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^2
*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt
(-c^2*d*e - e^2)*log((c^2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x
^2 - 1) - 2*e)/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*arc
csc(c*x) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*
c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*a
rctan(-c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e
^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3
+ (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*
e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e + 2*a*d^2*e^2 - (3*b
*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2
*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2
- 1)/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*arccsc(c*x) +
 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3
+ b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x
 + sqrt(c^2*x^2 - 1)) - (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3
)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e
^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^
4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex^2)^3} dx = \text{Timed out}$$

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex^2)^3} dx = \int \frac{(b\,\text{arccsc}\,(cx) + a)x}{(ex^2 + d)^3} dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(c^2*e^3*x^4 + 2*c^2*d*e^2*x^2 + c^2*d^2*e)*integrate(1/4*x*e^(1/2*
log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 - d
^2*e + (c^2*d^2*e - 2*d*e^2)*x^2 + (c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 -
 d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) +
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
- 1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex^2)^3} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \csc^{-1}(cx))}{(d + ex^2)^3} \, dx = \int \frac{x \left(a + b \operatorname{asin}\left(\frac{1}{c x}\right)\right)}{\left(e x^2 + d\right)^3} \, dx$$

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

3.114 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 704

$$\begin{aligned}
\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = &-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x}+\frac{e^2\left(a+b\csc^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)}\\
&+\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^3}+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{d^3\sqrt{c^2d+e}}\\
&-\frac{b\sqrt{e}\left(c^2d+2e\right)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{8d^3\left(c^2d+e\right)^{3/2}}\\
&-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}\\
&-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}\\
&-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}\\
&-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}
\end{aligned}$$

```
[Out] 1/4*e^2*(a+b*arccsc(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arccsc(c*x))/d^3/(e+d/x^2)
+1/2*I*(a+b*arccsc(c*x))^2/b/d^3-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsc(
c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
/2)))/d^3-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polyl
og(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/d^3+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/8*b*(c^2*d+2*e)*
arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))*e^(1/2)/d^3/(c^2*d+
e)^(3/2)+b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))*e^(1/2)/
d^3/(c^2*d+e)^(1/2)-1/8*b*c*e*(1-1/c^2/x^2)^(1/2)/d^2/(c^2*d+e)/(e+d/x^2)/x
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules

used = {5349, 4817, 4813, 390, 385, 211, 4825, 4615, 2221, 2317, 2438}

$$\int \frac{a+b\csc^{-1}(cx)}{x\,(d+ex^2)^3}\,dx = -\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3} - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(\frac{d}{x^2}+e\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(\frac{d}{x^2}+e\right)} + \frac{i(a+b\csc^{-1}(cx))^2}{2bd^3} - \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{8d^3\left(c^2d+e\right)^{3/2}} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d^3\sqrt{c^2d+e}} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3} - \frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2x\left(c^2d+e\right)\left(\frac{d}{x^2}+e\right)}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] -1/8*(b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(d^2*(c^2*d + e)*(e + d/x^2)*x) + (e^2*(
a + b*ArcCsc[c*x]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcCsc[c*x]))/(d^3*(e
 + d/x^2)) + ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d^3) + (b*Sqrt[e]*ArcTan[Sqrt
[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(d^3*Sqrt[c^2*d + e]) - (
b*Sqrt[e]*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x
^2)]*x)])/(8*d^3*(c^2*d + e)^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*Ar
cCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
 e])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x
]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 + (I
*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) + ((I/
2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
 e])])/d^3 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
- Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
```

```
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-
d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
```

```
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4813

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
 - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^5\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx,x,\frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{e^2x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3}-\frac{2ex\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2}+\frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right)dx,x,\frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)}{d^2}+\frac{(2e)\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx,x,\frac{1}{x}\right)}{d^2}\\
&\quad-\frac{e^2\text{Subst}\left(\int \frac{x\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx,x,\frac{1}{x}\right)}{d^2}
\end{aligned}$$

$$= \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}(a+b\arcsin(\frac{x}{c}))}{2d(\sqrt{e}-\sqrt{-d}x)} + \frac{\sqrt{-d}(a+b\arcsin(\frac{x}{c}))}{2d(\sqrt{e}+\sqrt{-d}x)}\right) dx, x, \frac{1}{x}\right)}{d^2} + \frac{(be)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}(e+dx^2)} dx, x, \frac{1}{x}\right)}{cd^3} - \frac{(be^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}(e+dx^2)^2} dx, x, \frac{1}{x}\right)}{4cd^3}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\arcsin(\frac{x}{c})}{\sqrt{e}-\sqrt{-d}x} dx, x, \frac{1}{x}\right)}{2(-d)^{5/2}} + \frac{\text{Subst}\left(\int \frac{a+b\arcsin(\frac{x}{c})}{\sqrt{e}+\sqrt{-d}x} dx, x, \frac{1}{x}\right)}{2(-d)^{5/2}} + \frac{(be)\text{Subst}\left(\int \frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2} dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{cd^3} - \frac{(be(c^2d+2e))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}(e+dx^2)} dx, x, \frac{1}{x}\right)}{8cd^3(c^2d+e)}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)} dx, x, \csc^{-1}(cx)\right)}{2(-d)^{5/2}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)} dx, x, \csc^{-1}(cx)\right)}{2(-d)^{5/2}} - \frac{(be(c^2d+2e))\,\text{Subst}\left(\int \frac{1}{e-\left(-d-\frac{e}{c^2}\right)x^2} dx, x, \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8cd^3(c^2d+e)}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)}$$

$$+ \frac{i(a+b\csc^{-1}(cx))^2}{2bd^3} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}}$$

$$- \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d+e\right)^{3/2}}$$

$$+ \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{5/2}}$$

$$+ \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{5/2}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{5/2}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2(-d)^{5/2}}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)}$$

$$+ \frac{i(a+b\csc^{-1}(cx))^2}{2bd^3} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}}$$

$$- \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d+e\right)^{3/2}}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2d^3}$$

$$+ \frac{b\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{2d^3}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)}$$

$$+ \frac{i(a+b\csc^{-1}(cx))^2}{2bd^3} + \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}}$$

$$- \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d+e\right)^{3/2}}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$- \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{2d^3}$$

$$- \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{2d^3}$$

$$- \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{2d^3}$$

$$- \frac{(ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{2d^3}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2}$$
$$- \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\csc^{-1}(cx))^2}{2bd^3}$$
$$+ \frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}} - \frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8d^3(c^2d+e)^{3/2}}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$
$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$
$$+ \frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2114 vs. 2(704) = 1408.

Time = 6.06 (sec) , antiderivative size = 2114, normalized size of antiderivative = 3.00

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] a/(4*d*(d + e*x^2)^2) + a/(2*d^2*(d + e*x^2)) + (a*Log[x])/d^3 - (a*Log[d +
 e*x^2])/(2*d^3) + b*((((5*I)/16)*Sqrt[e]*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*Sqrt
[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e]
 + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[
-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/d^(5
/2) - (((5*I)/16)*Sqrt[e]*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(A
rcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[
d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqr
```

t[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/d^(5/2) + (Sqrt[e]*((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*d^2) + (Sqrt[e]*(((-I)*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*d^2) - ((I/16)*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/d^3 - ((I/16)*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/d^3 + (-(ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])]) + (I/2)*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/d^3)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.31 (sec) , antiderivative size = 3479, normalized size of antiderivative = 4.94

method	result	size
parts	Expression too large to display	3479
derivativedivides	Expression too large to display	3553
default	Expression too large to display	3553

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a/d^3*ln(x)+1/2*a/d^2/(e*x^2+d)-1/2*a/d^3*ln(e*x^2+d)+1/4*a/d/(e*x^2+d)^2+b
*(-5/4*I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2
)*arccsc(c*x)^2/d^3/(c^4*d^2+2*c^2*d*e+e^2)+I/(c^2*d+e)/d^3*e*arccsc(c*x)^2
+1/2*I/(c^2*d+e)/d^2*c^2*sum((_R1^2*c^2*d-2*c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2
*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x
-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d
))+5/4*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*
ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))
*arccsc(c*x)/d^3/(c^4*d^2+2*c^2*d*e+e^2)+1/2*I*arccsc(c*x)^2*(c^2*d+2*(e*(c
^2*d+e))^(1/2)+2*e)/(c^2*d+e)/d^3-1/8*e*(8*c^6*d^2*arccsc(c*x)*x^2+6*c^6*d*
e*arccsc(c*x)*x^4+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^5*d^2*x+((c^2*x^2-1)/c^2/x^
2)^(1/2)*c^5*d*e*x^3-I*c^4*d^2-2*I*c^4*d*e*x^2-I*e^2*c^4*x^4+8*c^4*d*e*arcc
sc(c*x)*x^2+6*arccsc(c*x)*e^2*c^4*x^4)/d^3/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-5/
8*I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*pol
ylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e
))/d^3/(c^4*d^2+2*c^2*d*e+e^2)+1/4*I*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(
1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e
)/(c^2*d+e)/d^3-1/2*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*ln(1-d*c^2*(I/c/x+(1-
1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/(c^2*d+e
)/d^3+1/2*I/(c^2*d+e)/d^3*e*sum((_R1^2*c^2*d-2*c^2*d-4*e)/(_R1^2*c^2*d-c^2*
d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/
c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^
2*d))+I/(c^2*d+e)/d^2*c^2*arccsc(c*x)^2-1/2*I*arccsc(c*x)^2/d^3-1/2*(e*(c^2
*d+e))^(1/2)/(c^2*d+e)^2/d^3*e*arccsc(c*x)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^
(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))-3/4*(e*(c^2*d+e))^(1/2)/(c^2*d+
e)^2/d^2*c^2*arccsc(c*x)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*
(e*(c^2*d+e))^(1/2)+2*e))-7/8*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*arctanh
(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2
))*c^2+1/4*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*e*polylog(2,d*c^2*(I/c/x+(
1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))-1/4*I*((e*(c^2*d+e
))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c^2*arccsc(c*x)^2/d
^2/e/(c^4*d^2+2*c^2*d*e+e^2)-2*I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*
(c^2*d+e))^(1/2)*e+2*e^2)*e*arccsc(c*x)^2/d^4/(c^4*d^2+2*c^2*d*e+e^2)/c^2+2
```

```
*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*ln(1
-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arc
csc(c*x)/d^4/(c^4*d^2+2*c^2*d*e+e^2)/c^2-(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*
e^2*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2
*e))*arccsc(c*x)/c^4/d^5/(c^2*d+e)-1/2*I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d
*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e^2*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^
(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/(c^4*d^2+2*c^2*d*e+e^2)/d^5/c^4
+I*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arccsc(c*x)^2*e^2/c^4/d^5/(c^2*d+e)+((
e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e^2*ln(1-
d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arcc
sc(c*x)/(c^4*d^2+2*c^2*d*e+e^2)/d^5/c^4+1/4*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^
2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c^2*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1
/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^2/e/(c^4*d^2+2*c^2*
d*e+e^2)+1/8*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/e/d*polylog(2,d*c^2*(I/c/x+(
1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))*c^4-I*((e*(c^2*d+e
))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*polylog(2,d*c^2*(
I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^4/(c^4*d^
2+2*c^2*d*e+e^2)/c^2-1/4*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/e/d*c^4*arccsc(c*x
)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e
))+1/4*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/e/d*arccsc(c*x)^2*c^4+3/2*I*(c^2*d
+2*(e*(c^2*d+e))^(1/2)+2*e)*arccsc(c*x)^2*e/c^2/d^4/(c^2*d+e)-1/8*I*((e*(c^
2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c^2*polylog(2,
d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^2/
e/(c^4*d^2+2*c^2*d*e+e^2)+3/4*I*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*polylog(2
,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e/c
^2/d^4/(c^2*d+e)+1/2*I*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*polylog(2,d*c^2*(I
/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e^2/c^4/d^5/
(c^2*d+e)-I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*
e^2)*e^2*arccsc(c*x)^2/(c^4*d^2+2*c^2*d*e+e^2)/d^5/c^4-3/2*(c^2*d+2*(e*(c^2
*d+e))^(1/2)+2*e)*e*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c
^2*d+e))^(1/2)+2*e))*arccsc(c*x)/c^2/d^4/(c^2*d+e)+3/4*I*(e*(c^2*d+e))^(1/2
)/(c^2*d+e)^2/d^2*arccsc(c*x)^2*c^2-3/4*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d
^3*e*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d
*e+e^2)^(1/2))+3/8*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*polylog(2,d*c^2*(I
/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))*c^2+1/2*I*(e
*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*e*arccsc(c*x)^2)
```

Fricas [F]

$$\int \frac{a + b \csc^{-1}(cx)}{x\,(d + ex^2)^3}\,dx = \int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^3 x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \csc^{-1}(cx)}{x\,(d + ex^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a + b \csc^{-1}(cx)}{x\,(d + ex^2)^3}\,dx = \int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^3 x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)
/d^3 + 4*log(x)/d^3) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/
(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \csc^{-1}(cx)}{x\,(d + ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x\,(d+ex^2)^3}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,(e\,x^2+d)^3}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^3),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

3.115 $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 1144

$$
\begin{aligned}
\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = &-\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} \\
&-\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} \\
&+\frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} \\
&-\frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} \\
&-\frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} \\
&-\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} \\
&-\frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} \\
&-\frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+\frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&-\frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+\frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&-\frac{3ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+\frac{3ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&-\frac{3ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} \\
&+\frac{3ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}
\end{aligned}
$$

```
[Out] -3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arccsc(c*x))*ln(1+I*c*
(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(
-d)^(1/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arccsc(c*x))*
ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/
e^(5/2)/(-d)^(1/2)+3/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*I*b*polylog(2,I*c*
(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(
-d)^(1/2)-3/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*I*b*polylog(2,-I*c*(I/c/x+
(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/
2)-1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1
-1/c^2/x^2)^(1/2))/e/(c^2*d+e)^(3/2)/d^(1/2)-1/16*b*arctanh((c^2*d+(-d)^(1/
2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/(c^2*d+e)^(3
/2)/d^(1/2)+1/16*(a+b*arccsc(c*x))*(-d)^(1/2)/e^(3/2)/(-d/x+(-d)^(1/2)*e^(1
/2))^2+3/16*(a+b*arccsc(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arccs
c(c*x))*(-d)^(1/2)/e^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2-3/16*(a+b*arccsc(c*x)
)/e^2/(d/x+(-d)^(1/2)*e^(1/2))-3/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/
c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2*d+e)^(1/2)-
3/16*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/
c^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2*d+e)^(1/2)-1/16*b*c*(-d)^(1/2)*(1-1/c^2/x^
2)^(1/2)/e^(3/2)/(c^2*d+e)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(-d)^(1/2)*(1
-1/c^2/x^2)^(1/2)/e^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 1144, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules

used = {5349, 4757, 4827, 745, 739, 212, 4825, 4615, 2221, 2317, 2438}

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2} - \frac{3b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{dc^2+e}} - \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e(dc^2+e)^{3/2}} - \frac{3b\,\mathrm{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{dc^2+e}} - \frac{b\,\mathrm{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e(dc^2+e)^{3/2}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3(a+b\csc^{-1}(cx))\log\left(\frac{i\sqrt{-d}e^{i\csc^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3(a+b\csc^{-1}(cx))\log\left(\frac{i\sqrt{-d}e^{i\csc^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(e^(3/2)*(c^2*d + e)*(Sqrt[-d]*S
qrt[e] - d/x)) - (b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d +
e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcCsc[c*x]))/(16*e^(3/2)*(
Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcCsc[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt
[e] - d/x)) - (Sqrt[-d]*(a + b*ArcCsc[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e]
+ d/x)^2) - (3*(a + b*ArcCsc[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*
ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 -
1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) - (3*b*ArcTanh[(c^2*d - (S
qrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16
*Sqrt[d]*e^2*Sqrt[c^2*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(
c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e
)^(3/2)) - (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*
d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) - (3*(a +
b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt
[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2))
 - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
 + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1
+ (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-
d]*e^(5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2,
 (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e
^(5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

```
&& NeQ[m, -1]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{a + b\arcsin\left(\frac{x}{c}\right)}{\left(e + dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{d^3\left(a + b\arcsin\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e} - dx\right)^3} - \frac{3d\left(a + b\arcsin\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e} - dx\right)^2} - \frac{d^3\left(a + b\arcsin\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e} + dx\right)^3} - \frac{3d\left(a + b\arcsin\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e} + dx\right)^2} - \frac{3d\left(a + b\arcsin\left(\frac{x}{c}\right)\right)}{8e^2\left(-de - d^2x^2\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= \frac{(3d)\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2} + \frac{(3d)\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2} + \frac{(3d)\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{8e^2} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8e^{3/2}} - \frac{(-d)^{3/2}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8e^{3/2}}$$

$$= \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{(3b)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^2} + \frac{(3b)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^2} + \frac{(3d)\text{Subst}\left(\int \left(-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)\,dx, x, \frac{1}{x}\right)}{8e^2} - \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^{3/2}} + \frac{\left(b\sqrt{-d}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce^{3/2}}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16e^{5/2}} - \frac{3\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16e^{5/2}} + \frac{(3b)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16ce^2} - \frac{(3b)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16ce^2} - \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce\left(c^2d+e\right)} + \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16ce\left(c^2d+e\right)}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{3\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{16e^{5/2}} - \frac{3\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{16e^{5/2}} + \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16ce\left(c^2d+e\right)} - \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16ce\left(c^2d+e\right)}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{3\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{16e^{5/2}} - \frac{3\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{16e^{5/2}} - \frac{3\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{16e^{5/2}} - \frac{3\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{16e^{5/2}}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$- \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}}$$

$$- \frac{3b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}}$$

$$- \frac{3b\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{(3b)\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{(3b)\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

$$+ \frac{(3b)\text{Subst}\left(\int\log\left(1-\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

$$- \frac{(3b)\text{Subst}\left(\int\log\left(1+\frac{i\sqrt{-d}e^{ix}}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)dx, x, \csc^{-1}(cx)\right)}{16\sqrt{-d}e^{5/2}}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$
$$+ \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$
$$- \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}}$$
$$- \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}}$$
$$- \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$
$$+ \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$
$$- \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$
$$+ \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$
$$- \frac{(3ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}}$$
$$+ \frac{(3ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}}$$
$$- \frac{(3ib)\text{Subst}\left(\int\frac{\log\left(1-\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}}$$
$$+ \frac{(3ib)\text{Subst}\left(\int\frac{\log\left(1+\frac{i\sqrt{-d}x}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}}\right)}{x}dx, x, e^{i\csc^{-1}(cx)}\right)}{16\sqrt{-d}e^{5/2}}$$

$$= -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(c^2d+e\right)^{3/2}} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 6.07 (sec) , antiderivative size = 2067, normalized size of antiderivative = 1.81

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((5*(-(ArcCsc[c*x]/((-I)*Sqrt[
d]*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(
Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))
/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]
```

))/(16*e^2) + (5*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(
c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt
[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*
Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*e^2) + ((I/16)*Sqrt[d]*((I*
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqr
t[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c
*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqr
t[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d
+ e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^2 - ((I/16)*Sq
rt[d]*(((-I)*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqr
t[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcS
in[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e
]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))
/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^2 - (3
*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcC
sc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Arc
Csc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*L
og[Sqrt[e] + (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(128*
Sqrt[d]*e^(5/2)) + (3*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSi
n[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e]
)*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e
] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log
[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*A
rcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c
^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (S
qrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sq
rt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCs
c[c*x])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] -
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcC
sc[c*x])]))/(128*Sqrt[d]*e^(5/2)))

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 47.85 (sec) , antiderivative size = 1804, normalized size of antiderivative = 1.58

method	result	size
parts	Expression too large to display	1804
derivativedivides	Expression too large to display	1827
default	Expression too large to display	1827

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*((-5/8/e*x^3-3/8*d/e^2*x)/(e*x^2+d)^2+3/8/e^2/(d*e)^(1/2)*arctan(e*x/(d*e
)^(1/2)))+b/c^5*(-1/8*x*c^7*(3*d^2*c^4*arccsc(c*x)+5*c^4*d*e*arccsc(c*x)*x^
2+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*c^3
*x^3+3*c^2*d*e*arccsc(c*x)+5*e^2*arccsc(c*x)*c^2*x^2)/e^2/(c^2*d+e)/(c^2*e*
x^2+c^2*d)^2+3/8*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e)
)^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c^3*arctan(c*d*(I/c/
x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d
+e)^2/e^2/d^2-3/16/(c^2*d+e)/e^2*c^8*d*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*a
rccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^
2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*(
-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^
2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2
))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3-3/16/(c^
2*d+e)/e^2*c^8*d*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I
/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),
_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/2*((c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(I/c
/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d
+e)/e/d^3+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(
1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctanh(c*d*(I/c/x+(1
-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2
/e/d^3-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+
e))^(1/2)+2*e)*c*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*
d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e/d^3-3/16/(c^2*d+e)/e*c^6*sum(1/_R1/(
_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R
1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^
2*d-4*e)*_Z^2+c^2*d))-3/8*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2
*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c^3*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((
-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e^2/d^2-3/8*((c^2*d+2
*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c^3*ar
ctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)
^(1/2))/(c^2*d+e)/e^2/d^2-3/16/(c^2*d+e)/e*c^6*sum(_R1/(_R1^2*c^2*d-c^2*d-2
*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x
```

```
-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d
))+3/8*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^
2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c^3*arctanh(c*d*(I/c/x+(1-1/c^
2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/e^2/
d^2)
```

Fricas [F]

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
 d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\csc^{-1}(cx))}{(d + ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\csc^{-1}(cx))}{(d + ex^2)^3}\,dx = \int \frac{x^4\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^3}\,dx$$

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

3.116 $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 1144

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}(c^2d+e)^{3/2}} - \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}(c^2d+e)^{3/2}} - \frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

```
[Out] 1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/
c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(3/2)+1/16*b*arctanh((c^2*d+(-d)^(1/2)*e^
(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(
3/2)+1/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arccsc(c*x))*ln(1+
I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^
(3/2)/e^(3/2)+1/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arccsc(c*
x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/(-d)^(3/2)/e^(3/2)+1/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*
(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*I*b*polylog(2
,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d
)^(3/2)/e^(3/2)-1/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*polylog(2,I*c*(I/
c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e
^(3/2)+1/16*(a+b*arccsc(c*x))/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))^
2+1/16*(a+b*arccsc(c*x))/d/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*(-a-b*arccsc(c*
x))/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+1/16*(-a-b*arccsc(c*x))/d
/e/(d/x+(-d)^(1/2)*e^(1/2))-1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d
^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/e/(c^2*d+e)^(1/2)-1/16*
b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x
^2)^(1/2))/d^(3/2)/e/(c^2*d+e)^(1/2)-1/16*b*c*(1-1/c^2/x^2)^(1/2)/(c^2*d+e)
/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(1-1/c^2/x^2)^(1/2)/
(c^2*d+e)/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 1144, normalized size of antiderivative = 1.00, number of steps used = 63, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules

used = {5349, 4817, 4757, 4827, 745, 739, 212, 4825, 4615, 2221, 2317, 2438}

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}\,(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$-\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}\,(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

$$-\frac{a+b\csc^{-1}(cx)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}$$

$$-\frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}-\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{dc^2+e}}$$

$$+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(dc^2+e\right)^{3/2}}-\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{dc^2+e}}$$

$$+\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(dc^2+e\right)^{3/2}}$$

$$+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

$$-\frac{(a+b\csc^{-1}(cx))\log\left(\frac{i\sqrt{-d}e^{i\csc^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}$$

$$+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

$$-\frac{(a+b\csc^{-1}(cx))\log\left(\frac{i\sqrt{-d}e^{i\csc^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}$$

$$+\frac{ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

$$-\frac{ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

$$+\frac{ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

$$-\frac{ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[1 - 1/(c^2*x^2)])/(Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*S
qrt[e] - d/x)) - (b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d +
e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCsc[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sq
rt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCsc[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] -
d/x)) - (a + b*ArcCsc[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] + d/x)^2
) - (a + b*ArcCsc[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2
*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)]
)])/(16*d^(3/2)*(c^2*d + e)^(3/2)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])
/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*e*Sqrt[
c^2*d + e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2
*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d + e)^(3/2)) - (b*ArcTan
h[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2
*x^2)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) + ((a + b*ArcCsc[c*x])*Log[1 - (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*
e^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(S
qrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCsc[c*x])*
Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*
(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCs
c[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((I/16)*b*
PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]
)/((-d)^(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x
]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(3/2)*e^(3/2)) + ((I/16)*b*PolyLog[
2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^
(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/((-d)^(3/2)*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
```

```
FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^2\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^3} + \frac{a+b\arcsin\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)^2}\right)\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)\,dx, x, \frac{1}{x}\right)}{d}$$

$$+ \frac{e\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d^3\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}+dx\right)^3} - \frac{3d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\right.}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e} - \frac{3\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e}$$

$$+ \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} + \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e}$$

$$- \frac{3\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{8e} + \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{2e}$$

$$- \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{e}} - \frac{\sqrt{-d}\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{e}}$$

$$= \frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}+\frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$-\frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{3\text{Subst}\left(\int\left(-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\frac{1}{x}\right)}{8e}$$

$$+\frac{\text{Subst}\left(\int\left(-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)}-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\frac{1}{x}\right)}{2e}$$

$$+\frac{(3b)\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16cde}$$

$$-\frac{(3b)\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16cde}$$

$$-\frac{b\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{4cde}+\frac{b\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{4cde}$$

$$-\frac{b\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16c\sqrt{-d}\sqrt{e}}+\frac{b\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16c\sqrt{-d}\sqrt{e}}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+\frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}+\frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$-\frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{3\text{Subst}\left(\int\frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{16de^{3/2}}$$

$$+\frac{3\text{Subst}\left(\int\frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{16de^{3/2}}-\frac{\text{Subst}\left(\int\frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{4de^{3/2}}$$

$$-\frac{\text{Subst}\left(\int\frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}dx,x,\frac{1}{x}\right)}{4de^{3/2}}-\frac{(3b)\text{Subst}\left(\int\frac{1}{d^2+\frac{de}{c^2}-x^2}dx,x,\frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cde}$$

$$+\frac{(3b)\text{Subst}\left(\int\frac{1}{d^2+\frac{de}{c^2}-x^2}dx,x,\frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cde}$$

$$+\frac{b\text{Subst}\left(\int\frac{1}{d^2+\frac{de}{c^2}-x^2}dx,x,\frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cde}-\frac{b\text{Subst}\left(\int\frac{1}{d^2+\frac{de}{c^2}-x^2}dx,x,\frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4cde}$$

$$+\frac{b\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16cd\left(c^2d+e\right)}-\frac{b\text{Subst}\left(\int\frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{16cd\left(c^2d+e\right)}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$- \frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}}$$

$$- \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} + \frac{3\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{16de^{3/2}}$$

$$+ \frac{3\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{16de^{3/2}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{4de^{3/2}}$$

$$- \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{4de^{3/2}} - \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cd\,(c^2d+e)}$$

$$+ \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cd\,(c^2d+e)}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d+e\right)^{3/2}} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} + \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d+e\right)^{3/2}} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} + \frac{3\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{16de^{3/2}} + \frac{3\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{16de^{3/2}} + \frac{3\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{16de^{3/2}} + \frac{3\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{16de^{3/2}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4de^{3/2}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4de^{3/2}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4de^{3/2}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{4de^{3/2}}$$

$$= \text{Too large to display}$$

Mathematica [A] (warning: unable to verify)

Time = 6.06 (sec) , antiderivative size = 2075, normalized size of antiderivative = 1.81

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(-1/16*(-(ArcCsc[c*x]/((-I)*Sqrt[d]
*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sq
rt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(
Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/
(d*e) - (-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/Sqr
t[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x
))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(16*d*e) - ((I/16)*((I*c*Sqrt[e]*Sqrt[1 -
 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsc[
c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqrt[e]) +
 (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c*Sqrt[d
] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*Sqrt[d]
 + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + ((I/16)*(((-I)*c*Sqr
t[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))
 - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqr
t[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e] +
c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*(I
*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) - (Pi^2 - 4*Pi
*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/
Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])
/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
 (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] +
(I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(128*d^(3/2)*e^(3/2
)) + (Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*Arc
Csc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d +
```

```
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] +
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*
Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(128*d
^(3/2)*e^(3/2)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 64.23 (sec) , antiderivative size = 1278, normalized size of antiderivative = 1.12

method	result	size
parts	Expression too large to display	1278
derivativedivides	Expression too large to display	1301
default	Expression too large to display	1301

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a*((1/8/d*x^3-1/8/e*x)/(e*x^2+d)^2+1/8/e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/
2)))+b/c^3*(1/8*x*c^5*(c^4*d*e*arccsc(c*x)*x^2-d^2*c^4*arccsc(c*x)+((c^2*x^
2-1)/c^2/x^2)^(1/2)*e^2*c^3*x^3+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x+e^2*a
rccsc(c*x)*c^2*x^2-c^2*d*e*arccsc(c*x))/d/e/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-1
/16/d/(c^2*d+e)*c^4*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R
1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1
)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/16/d/(c^2*d+e)*c^4*s
um(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(
1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^
4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/8*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1
/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/
2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e/d^3+1/8*(-(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*
e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/(
(-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^3-1/8*((c^2*d+
2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*arc
tanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^
(1/2))/(c^2*d+e)/e/d^3+1/8*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e
*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctanh(
```

```
c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)
)/(c^2*d+e)^2/e/d^3-1/16/(c^2*d+e)/e*c^6*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*
(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-
1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1
/16/(c^2*d+e)/e*c^6*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-
I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1))
,_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d)))
```

Fricas [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\operatorname{arccsc}(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 +
 d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

3.117 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1134

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx = -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} + \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\,\mathrm{earctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(c^2d+e)^{3/2}} + \frac{5b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d+e}} - \frac{b\,\mathrm{earctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(c^2d+e)^{3/2}} + \frac{5b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d+e}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

```
[Out] -1/16*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1
-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(3/2)-1/16*b*e*arctanh((c^2*d+(-d)^(1/
2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d
+e)^(3/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arccsc(c*x))*
ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/
(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arcc
sc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)
^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b*poly
log(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
/(-d)^(5/2)/e^(1/2)+3/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*I*b*polylog(2,-I*
c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5
/2)/e^(1/2)+1/16*(a+b*arccsc(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x+(-d)^(1/2)*e^(1
/2))^2-5/16*(a+b*arccsc(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arccs
c(c*x))*e^(1/2)/(-d)^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+5/16*(a+b*arccsc(c*x)
)/d^2/(d/x+(-d)^(1/2)*e^(1/2))+5/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/
c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)+5/16
*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/
x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)-1/16*b*c*e^(1/2)*(1-1/c^2/x^2)^(1/2)/(-
d)^(3/2)/(c^2*d+e)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*e^(1/2)*(1-1/c^2/x^2)
^(1/2)/(-d)^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A] (verified)

Time = 2.77 (sec) , antiderivative size = 1134, normalized size of antiderivative = 1.00, number of steps used = 81, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules

used = {5339, 4817, 4757, 4827, 745, 739, 212, 4825, 4615, 2221, 2317, 2438}

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx = -\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}\,(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}\,(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} - \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2} + \frac{5b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}} - \frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(dc^2+e\right)^{3/2}} + \frac{5b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}} - \frac{be\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(dc^2+e\right)^{3/2}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3(a+b\csc^{-1}(cx))\log\left(\frac{i\sqrt{-d}e^{i\csc^{-1}(cx)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3(a+b\csc^{-1}(cx))\log\left(\frac{i\sqrt{-d}e^{i\csc^{-1}(cx)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/((-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]
*Sqrt[e] - d/x)) - (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*
d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcCsc[c*x]))/(16*(-d)^(
3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcCsc[c*x]))/(16*d^2*(Sqrt[-d
]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcCsc[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*
Sqrt[e] + d/x)^2) + (5*(a + b*ArcCsc[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x
)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]
*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) + (5*b*ArcTanh[(c^
2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)
])])/(16*d^(5/2)*Sqrt[c^2*d + e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e]
)/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d
 + e)^(3/2)) + (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[
c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) - (3*(a +
b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sq
rt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt
[e]) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCsc[c*x])*
Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*
(-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*
PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(
(-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*
PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(
(-d)^(5/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
```

```
FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5339

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x^4\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{e^2\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2e\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{a+b\arcsin\left(\frac{x}{c}\right)}{d^2\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d^2}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\arcsin\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \left(-\frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right)dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d^3\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}+dx\right)^3} - \frac{3d\left(a+b\arcsin\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}+dx\right.}\right.\right.}{d^2}$$

$$= \frac{3\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{8d} - \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{-de-d^2x^2}\,dx, x, \frac{1}{x}\right)}{d} - \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}} - \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{-d}} - \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^3}\,dx, x, \frac{1}{x}\right)}{8\sqrt{-d}}$$

$$= \frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$+ \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{(3b)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16cd^2}$$

$$+ \frac{(3b)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16cd^2}$$

$$+ \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2cd^2} - \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2cd^2}$$

$$+ \frac{3\text{Subst}\left(\int \left(-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{8d}$$

$$- \frac{\text{Subst}\left(\int \left(-\frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} - \frac{a+b\arcsin\left(\frac{x}{c}\right)}{2d\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx, x, \frac{1}{x}\right)}{d}$$

$$- \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2d^2\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2d^2\sqrt{e}}$$

$$- \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16c(-d)^{3/2}}$$

$$+ \frac{\left(b\sqrt{e}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16c(-d)^{3/2}}$$

$$= -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}$$

$$+ \frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2}$$

$$+ \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{(3b)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cd^2}$$

$$- \frac{(3b)\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16cd^2} - \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2cd^2}$$

$$+ \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{d+\frac{\sqrt{-d}\sqrt{e}}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2cd^2} - \frac{3\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16d^2\sqrt{e}}$$

$$- \frac{3\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{16d^2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2d^2\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2d^2\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2d^2\sqrt{e}}$$

$$- \frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+e}}{c}+i\sqrt{-d}e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2d^2\sqrt{e}} + \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}}$$

$$+ \frac{\text{Subst}\left(\int \frac{a+b\arcsin\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx, x, \frac{1}{x}\right)}{2d^2\sqrt{e}} - \frac{(be)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}-dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16cd^2\left(c^2d+e\right)}$$

$$+ \frac{(be)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\sqrt{e}+dx\right)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{16cd^2\left(c^2d+e\right)}$$

= Too large to display

Mathematica [A] (warning: unable to verify)

Time = 6.05 (sec) , antiderivative size = 2060, normalized size of antiderivative = 1.82

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*((-3*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*
Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqr
t[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(S
qrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/
(16*d^2) - (3*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x
)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(
c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqr
t[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*d^2) + ((I/16)*((I*c*Sqrt[e]*S
qrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) -
ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqr
t[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c
*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*
Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/d^(3/2) - ((I/16)*(((-I)*c*
Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*
x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*
Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e]
 + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)
*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/d^(3/2) - (3*(Pi^2 - 4*
Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4]
)/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*S
qrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e]
+ (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(
I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^
(I*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(128*d^(5/2)*Sqr
t[e]) + (3*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 +
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi +
 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^
```

```
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[
1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sq
rt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] -
(4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))
/(128*d^(5/2)*Sqrt[e]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 106.52 (sec) , antiderivative size = 1798, normalized size of antiderivative = 1.59

method	result	size
parts	Expression too large to display	1798
derivativedivides	Expression too large to display	1823
default	Expression too large to display	1823

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x/d/(e*x^2+d)^2+3/8*a/d^2*x/(e*x^2+d)+3/8*a/d^2/(d*e)^(1/2)*arctan(e*
x/(d*e)^(1/2))+b/c*(1/8*x*c^3*(5*d^2*c^4*arccsc(c*x)+3*c^4*d*e*arccsc(c*x)*
x^2-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*c
^3*x^3+5*c^2*d*e*arccsc(c*x)+3*e^2*arccsc(c*x)*c^2*x^2)/d^2/(c^2*d+e)/(c^2*
e*x^2+c^2*d)^2+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e
*(c^2*d+e))^(1/2)+2*e)*e*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*
(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/c^3/d^5/(c^2*d+e)+1/2*((c^2*d+2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*e*arctanh(c*d*(
I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/c^3
/d^5/(c^2*d+e)-3/16/d/(c^2*d+e)*c^4*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*ar
ccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2
/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/16/d
^2/(c^2*d+e)*c^2*e*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I
/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),
_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-5/8*(-(c^2*d-2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e)
)^(1/2)*e+2*e^2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*
d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)^2/c-1/2*((c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/
```

```
2)*e+2*e^2)*e*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2))/d^5/(c^2*d+e)^2/c^3+5/8*(-(c^2*d-2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(I/c/x+(1-1/
c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/c/d^4/(c^2*d+
e)+5/8*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(
1/2)+2*e)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2))/c/d^4/(c^2*d+e)-3/16/d^2/(c^2*d+e)*c^2*e*sum(1/_R1/(_R1
^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+
dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d
-4*e)*_Z^2+c^2*d))-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^
2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctan(c*d*(
I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^
5/(c^2*d+e)^2/c^3-3/16/d/(c^2*d+e)*c^4*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*a
rccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^
2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-5/8*(
(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^
2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2)
)/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)^2/c)
```

Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx = \int \frac{b\operatorname{arccsc}(cx)+a}{(ex^2+d)^3}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex^2)^3} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex^2)^3} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex^2)^3} \, dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(e\,x^2 + d)^3} \, dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^3, x)
```

3.118 $\int x^5\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 403

$$\int x^5\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx = -\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^3} - \frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{105e^3\sqrt{c^2x^2}} + \frac{b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{c^2x^2}}$$

```
[Out] 1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*ar
ccsc(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arccsc(c*x))/e^3-8/105*b*c*d^(7/2)*
x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)+1/1
680*b*(105*c^6*d^3-35*c^4*d^2*e+63*c^2*d*e^2+75*e^3)*x*arctanh(e^(1/2)*(c^2
*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(5/2)/(c^2*x^2)^(1/2)-1/840*b*(29*c^
2*d-25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)+1/42*
b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)-1/1680*b*(23*c^
4*d^2+12*c^2*d*e-75*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^5/e^2/(c^2*x
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {272, 45, 5347, 12, 1629, 159, 163, 65, 223, 212, 95, 210}

$$\begin{aligned}
&\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx\\
&=\frac{d^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}-\frac{2d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}\\
&\quad+\frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{105e^3\sqrt{c^2x^2}}\\
&\quad+\frac{bx(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{c^2x^2}}\\
&\quad+\frac{bx\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}(29c^2d-25e)\,(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}}\\
&\quad-\frac{bx\sqrt{c^2x^2-1}(23c^4d^2+12c^2de-75e^2)\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}}
\end{aligned}$$

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] -1/1680*(b*(23*c^4*d^2 + 12*c^2*d*e - 75*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d +
 e*x^2])/(c^5*e^2*Sqrt[c^2*x^2]) - (b*(29*c^2*d - 25*e)*x*Sqrt[-1 + c^2*x^2
]*(d + e*x^2)^(3/2))/(840*c^3*e^2*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*
(d + e*x^2)^(5/2))/(42*c*e^2*Sqrt[c^2*x^2]) + (d^2*(d + e*x^2)^(3/2)*(a + b
*ArcCsc[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3
) + ((d + e*x^2)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^3) - (8*b*c*d^(7/2)*x*ArcT
an[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(105*e^3*Sqrt[c^2*x^2]) +
 (b*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*x*ArcTanh[(Sqrt[e]
*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(1680*c^6*e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3} + \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(8d^2-12dex^2+15e^2x^4)}{105e^3x\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= \frac{d^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} \\
&\quad + \frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3} + \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(8d^2-12dex^2+15e^2x^4)}{x\sqrt{-1+c^2x^2}}\,dx}{105e^3\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{d^2(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^3} + \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^{3/2}(8d^2-12dex+15e^2x^2)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{210e^3\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^3} + \frac{(bx)\text{Subst}\left(\int \frac{(d+ex)^{3/2}\left(24c^2d^2e-\frac{3}{2}(29c^2d-25e)e^2x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{630ce^4\sqrt{c^2x^2}}$$

$$= -\frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^3} + \frac{(bx)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(48c^4d^3e-\frac{3}{4}e^2(23c^4d^2+12c^2de-75e^2)x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{1260c^3e^4\sqrt{c^2x^2}}$$

$$= -\frac{b(23c^4d^2+12c^2de-75e^2)x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b(29c^2d-25e)x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^3} + \frac{(bx)\text{Subst}\left(\int \frac{48c^6d^4e+\frac{3}{8}e^2(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{1260c^5e^4\sqrt{c^2x^2}}$$

$$= -\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^3} + \frac{(4bcd^4x)\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{105e^3\sqrt{c^2x^2}} + \frac{(b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x)\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3360c^5e^2\sqrt{c^2x^2}}$$

$$= -\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^3} + \frac{(8bcd^4x)\,\mathrm{Subst}\left(\int\frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{105e^3\sqrt{c^2x^2}} + \frac{(b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x)\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{1680c^7e^2\sqrt{c^2x^2}}$$

$$= -\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - \frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} - \frac{2d(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^3} - \frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{105e^3\sqrt{c^2x^2}} + \frac{(b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x)\,\mathrm{Subst}\left(\int\frac{1}{1-\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{1680c^7e^2\sqrt{c^2x^2}}$$

$$
\begin{aligned}
&= -\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} \\
&\quad -\frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} \\
&\quad +\frac{d^2(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3}-\frac{2d(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3} \\
&\quad +\frac{(d+ex^2)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^3}-\frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{105e^3\sqrt{c^2x^2}} \\
&\quad +\frac{b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{c^2x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.56 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.81

$$
\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx
$$

$$
=\frac{32a(d+ex^2)^2\left(8d^2-12dex^2+15e^2x^4\right)+\frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)\left(75e^2+2c^2e(19d+25ex^2)+c^4(-41d^2+22dex^2+40e^2x^4)\right)}{c^5}+\frac{b\Big(}{}
$$

```
[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (32*a*(d + e*x^2)^2*(8*d^2 - 12*d*e*x^2 + 15*e^2*x^4) + (2*b*e*Sqrt[1 - 1/(
c^2*x^2)]*x*(d + e*x^2)*(75*e^2 + 2*c^2*e*(19*d + 25*e*x^2) + c^4*(-41*d^2
+ 22*d*e*x^2 + 40*e^2*x^4)))/c^5 + (b*(-128*c^4*d^4*Sqrt[1 + d/(e*x^2)]*App
ellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] - (e*(105*c^6*d^3 - 35*c^4
*d^2*e + 63*c^2*d*e^2 + 75*e^3)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/
d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^
5*x) + 32*b*(d + e*x^2)^2*(8*d^2 - 12*d*e*x^2 + 15*e^2*x^4)*ArcCsc[c*x])/(3
360*e^3*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x^{5}(a+b\ \mathrm{arccsc}\left(cx\right))\sqrt{e\,x^{2}+d}dx$$

```
[In] int(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
[Out] int(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 2.44 (sec) , antiderivative size = 1699, normalized size of antiderivative = 4.22

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] [1/6720*(128*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*
e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*
e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x
^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c
^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4
*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arccsc(c*x) + (40*b*c^5*e^3*x^4 - 41*b*c^5*
d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2
)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^3), -1/6720*(256*b*c^7*d^(7/2)
*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt
(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (105*b*c^6*d^3 - 35*b*c^4*
d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*
c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^
2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(240*a*c^7*e^3*x^6 + 48*a*c^7
*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*
b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arccsc(c*x) + (40*b*c^5*
e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2
+ 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^3), 1/3360*
(64*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt
(-d) + 8*d^2)/x^4) - (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*
b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt
(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(24
0*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 +
 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3
)*arccsc(c*x) + (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*
```

```
c*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^
2 + d))/(c^7*e^3), -1/3360*(128*b*c^7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)
*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 -
d*e)*x^2 - d^2)) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*
e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e
*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(240*
a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 1
6*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*
arccsc(c*x) + (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*
e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
+ d))/(c^7*e^3)]
```

Sympy [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^5(a+b\operatorname{acsc}(cx))\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**5*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x**5*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\operatorname{arccsc}(cx)+a)x^5\,dx$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^5\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^5*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^5*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.119 $\int x^3\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 294

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}-\frac{d(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^2\sqrt{c^2x^2}}-\frac{b(15c^4d^2-10c^2de-9e^2)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{c^2x^2}}$$

```
[Out] -1/3*d*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arccs
c(c*x))/e^2+2/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^2/(c^2*x^2)^(1/2)-1/120*b*(15*c^4*d^2-10*c^2*d*e-9*e^2)*x*arctanh(e
^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(3/2)/(c^2*x^2)^(1/2)+1/2
0*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e/(c^2*x^2)^(1/2)+1/120*b*(c^2*d+
9*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {272, 45, 5347, 12, 587, 159, 163, 65, 223, 212, 95, 210}

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{15e^2\sqrt{c^2x^2}}-\frac{bx(15c^4d^2-10c^2de-9e^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{20ce\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}(c^2d+9e)\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}$$

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(c^2*d + 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e*Sqrt[c^2*
x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e*Sqrt[c^2*x^2]) -
 (d*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) + ((d + e*x^2)^(5/2)*(a
+ b*ArcCsc[c*x]))/(5*e^2) + (2*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d
]*Sqrt[-1 + c^2*x^2])])/(15*e^2*Sqrt[c^2*x^2]) - (b*(15*c^4*d^2 - 10*c^2*d*
e - 9*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(12
0*c^4*e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} \\
&\quad+\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}+\frac{(bcx)\int\frac{(d+ex^2)^{3/2}(-2d+3ex^2)}{15e^2x\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} \\
&\quad+\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}+\frac{(bcx)\int\frac{(d+ex^2)^{3/2}(-2d+3ex^2)}{x\sqrt{-1+c^2x^2}}\,dx}{15e^2\sqrt{c^2x^2}} \\
&= -\frac{d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2}+\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} \\
&\quad+\frac{(bcx)\text{Subst}\left(\int\frac{(d+ex)^{3/2}(-2d+3ex)}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{30e^2\sqrt{c^2x^2}} \\
&= \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}-\frac{d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} \\
&\quad+\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}+\frac{(bx)\text{Subst}\left(\int\frac{\sqrt{d+ex}\left(-4c^2d^2+\frac{1}{2}e(c^2d+9e)x\right)}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{60ce^2\sqrt{c^2x^2}}
\end{aligned}
$$

$$= \frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2} + \frac{(bx)\text{Subst}\left(\int \frac{-4c^4d^3-\frac{1}{4}e(15c^4d^2-10c^2de-9e^2)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{60c^3e^2\sqrt{c^2x^2}}$$

$$= \frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2} - \frac{(bcd^3x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{15e^2\sqrt{c^2x^2}} - \frac{(b(15c^4d^2-10c^2de-9e^2)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{240c^3e\sqrt{c^2x^2}}$$

$$= \frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2} - \frac{(2bcd^3x)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{15e^2\sqrt{c^2x^2}} - \frac{(b(15c^4d^2-10c^2de-9e^2)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{120c^5e\sqrt{c^2x^2}}$$

$$= \frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2} + \frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^2\sqrt{c^2x^2}} - \frac{(b(15c^4d^2-10c^2de-9e^2)\,x)\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{120c^5e\sqrt{c^2x^2}}$$

$$
= \frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2} + \frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^2\sqrt{c^2x^2}} - \frac{b(15c^4d^2-10c^2de-9e^2)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{c^2x^2}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.44 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.89

$$
\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx
$$

$$
= \frac{16a(d+ex^2)^2\,(-2d+3ex^2) + \frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(9e+c^2(7d+6ex^2))}{c^3} + \frac{b\left(16c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+\right.}{}}{240e^2\sqrt{d+e}}
$$

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (16*a*(d + e*x^2)^2*(-2*d + 3*e*x^2) + (2*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
e*x^2)*(9*e + c^2*(7*d + 6*e*x^2)))/c^3 + (b*(16*c^2*d^3*Sqrt[1 + d/(e*x^2)
]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] + (e*(15*c^4*d^2 - 10
*c^2*d*e - 9*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1,
 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^3*x) + 16*b*(d
 + e*x^2)^2*(-2*d + 3*e*x^2)*ArcCsc[c*x])/(240*e^2*Sqrt[d + e*x^2])
```

Maple [F]

$$
\int x^3(a+b\ \text{arccsc}\,(cx))\,\sqrt{e\,x^2+d}dx
$$

```
[In] int(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.05 (sec) , antiderivative size = 1379, normalized size of antiderivative = 4.69

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(16*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2
*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)
*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2
*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2)
+ 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4
 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 + 7*b*c^3*d*
e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2), 1/480*(32*b*c
^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (15*b*c^4*d^2 -
10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8
*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*
sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16
*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x)
+ (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
 + d))/(c^5*e^2), 1/240*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e
^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*
d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e -
9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sq
rt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(
24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b*
c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9
*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2), 1/240*(16*b*c^5*d^
(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)
*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 - 10*b*
c^2*d*e - 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x
^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x
^2)) + 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^
2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 + 7*b*c
^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2)]
```

Sympy [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^3(a+b\operatorname{acsc}(cx))\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**3*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\operatorname{arccsc}(cx)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^3\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.120 $\int x\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 195

$$\int x\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx = \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e} - \frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}} + \frac{b(3c^2d+e)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e-1/3*b*c*d^(3/2)*x*arctan((e*x^2+d)^
(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/6*b*(3*c^2*d+e)*x*arct
anh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(1/2)/(c^2*x^2)^(1/2
)+1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5345, 457, 104, 163, 65, 223, 212, 95, 210}

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e\sqrt{c^2x^2}}+\frac{bx(3c^2d+e)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}$$

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(6*c*Sqrt[c^2*x^2]) + ((d + e*x^2)
^(3/2)*(a + b*ArcCsc[c*x]))/(3*e) - (b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(
Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*e*Sqrt[c^2*x^2]) + (b*(3*c^2*d + e)*x*ArcT
anh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*Sqrt[e]*Sqrt[
c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
```

```
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{(bcx)\int\frac{(d+ex^2)^{3/2}}{x\sqrt{-1+c^2x^2}}\,dx}{3e\sqrt{c^2x^2}}$$

$$= \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{(bcx)\mathrm{Subst}\left(\int \frac{(d+ex)^{3/2}}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{6e\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{(bx)\mathrm{Subst}\left(\int \frac{c^2d^2+\frac{1}{2}e(3c^2d+e)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6ce\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{(bcd^2x)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6e\sqrt{c^2x^2}} + \frac{(b(3c^2d+e)\,x)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{12c\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{(bcd^2x)\,\mathrm{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}} + \frac{(b(3c^2d+e)\,x)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{6c^3\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} - \frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}} + \frac{(b(3c^2d+e)\,x)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^3\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e} - \frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}} + \frac{b(3c^2d+e)\,x\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.57 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{-\frac{2bd^2\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cex}+\frac{-\frac{b\left(3c^2d+e\right)\sqrt{1-\frac{1}{c^2x^2}}\,x^3\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)}{\sqrt{1-c^2x^2}}+\frac{2\left(d+ex^2\right)\left(be\sqrt{1-\frac{1}{c^2x^2}}\,x+2ac\right.}{}}{c}}{12\sqrt{d+ex^2}}$$

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((-2*b*d^2*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e
*x^2))])/(c*e*x) + (-((b*(3*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1 + (
e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2
]) + (2*(d + e*x^2)*(b*e*Sqrt[1 - 1/(c^2*x^2)]*x + 2*a*c*(d + e*x^2) + 2*b*
c*(d + e*x^2)*ArcCsc[c*x]))/e)/c)/(12*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x(a+b\ \text{arccsc}\left(cx\right))\sqrt{e\,x^2+d}dx$$

```
[In] int(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 1098, normalized size of antiderivative = 5.63

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(2*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*s
qrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d
^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*
sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 + 2*a*c
^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arccsc(c*x))*sqr
t(e*x^2 + d))/(c^3*e), -1/24*(4*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)
```

```
*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 -
d*e)*x^2 - d^2)) - (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 -
6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(
c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(2*a*c^3*e*x^2 + 2*a*c^3*d
+ sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arccsc(c*x))*sqrt(e*x
^2 + d))/(c^3*e), 1/12*(b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x
^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sq
rt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d + b*e)*sqrt(-e)*arctan(1/
2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3
*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*x^2 + 2*a*c^3*d +
 sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arccsc(c*x))*sqrt(e*x^
2 + d))/(c^3*e), -1/12*(2*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2
*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x
^2 - d^2)) + (3*b*c^2*d + b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e
)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*
e - c*e^2)*x^2)) - 2*(2*a*c^3*e*x^2 + 2*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e +
 2*(b*c^3*e*x^2 + b*c^3*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c^3*e)]
```

Sympy [F]

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x(a+b\,\mathrm{acsc}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/3*(e*x^2 + d)^(3/2)*a/e + 1/3*(3*e*integrate(1/3*(c^2*e*x^3 + c^2*d*x)*e^
(1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e*x^2 + (c^
2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e), x) + (e*x^2*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1)) + d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sq
rt(e*x^2 + d))*b/e
```

Giac [F]

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\operatorname{arccsc}(cx)+a)x\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.121 $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^2+d}}{x}dx$$

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 10.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,\sqrt{d+ex^2}}{x}\,dx$$

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x,x)
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x, x)
```

3.122 $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.76 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\sqrt{e\,x^2+d}}{x^3}dx$$

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 14.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\sqrt{d+ex^2}}{x^3}\,dx$$

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**3,x)
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^3, x)
```

3.123 $\int x^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Int}\left(x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\text{integral}=\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^2+d}dx$$

```
[In] int(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
[Out] int(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 118.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2(a+b\,\mathrm{acsc}\,(cx))\,\sqrt{d+ex^2}\,dx$$

```
[In] integrate(x**2*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x**2*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int x^2\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.124 $\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\operatorname{Int}\left(\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\text{integral}=\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 17.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\, dx = \int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\, dx$$

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\,\operatorname{arccsc}\left(c x\right)\right)\sqrt{e\,x^{2}+d}dx$$

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\, dx = \int \sqrt{ex^2+d}(b\operatorname{arccsc}\left(cx\right)+a)\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 47.88 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\, dx = \int \left(a+b\operatorname{acsc}\left(cx\right)\right)\sqrt{d+ex^2}\,dx$$

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int \sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.125 $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\operatorname{arccsc}(cx))\sqrt{e\,x^2+d}}{x^2}dx$$

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\operatorname{arccsc}(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 7.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\operatorname{acsc}(cx))\sqrt{d+ex^2}}{x^2}\,dx$$

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**2,x)
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^2,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^2, x)
```

3.126 $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 328

$$\begin{aligned}
&\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx \\
&= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} \\
&\quad -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}}-\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3dx^3} \\
&\quad +\frac{2bc^2(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad -\frac{b(c^2d+e)\,(2c^2d+3e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

```
[Out] -1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/d/x^3-2/9*b*c*(c^2*d+2*e)*(c^2*x^2-1
)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)-1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+
d)^(1/2)/x^2/(c^2*x^2)^(1/2)+2/9*b*c^2*(c^2*d+2*e)*x*EllipticE(c*x,(-e/c^2/
d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^
(1/2)/(1+e*x^2/d)^(1/2)-1/9*b*(c^2*d+e)*(2*c^2*d+3*e)*x*EllipticF(c*x,(-e/c
^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^
2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {270, 5347, 12, 485, 597, 538, 438, 437, 435, 432, 430}

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx$$
$$= -\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3dx^3} - \frac{bx\sqrt{1-c^2x^2}(c^2d+e)\,(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{2bc^2x\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} - \frac{2bc\sqrt{c^2x^2-1}(c^2d+2e)\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] (-2*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*Sqrt[c^2*x^2
]) - (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*x^2*Sqrt[c^2*x^2]) - ((d +
 e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*d*x^3) + (2*b*c^2*(c^2*d + 2*e)*x*Sqr
t[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d*S
qrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(2*c^
2*d + 3*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -
(e/(c^2*d))])/(9*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{(bcx)\int -\frac{(d+ex^2)^{3/2}}{3dx^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}}{x^4\sqrt{-1+c^2x^2}}\,dx}{3d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{(bcx)\int \frac{-2d(c^2d+2e)-e(c^2d+3e)x^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d\sqrt{c^2x^2}}$$

$$= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{(bcx)\int \frac{-de(c^2d+3e)+2c^2de(c^2d+2e)x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{c^2x^2}}$$

$$= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{(2bc^3(c^2d+2e)\,x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{9d\sqrt{c^2x^2}} - \frac{(bc(c^2d+e)(2c^2d+3e)\,x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d\sqrt{c^2x^2}}$$

$$= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{\left(2bc^3(c^2d+2e)x\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}dx}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} - \frac{\left(bc(c^2d+e)(2c^2d+3e)x\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}dx}{9d\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{\left(2bc^3(c^2d+2e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}dx}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bc(c^2d+e)(2c^2d+3e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}dx}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{2bc^2(c^2d+2e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+e)(2c^2d+3e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.75

$$\int\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}dx$$

$$= -\frac{\sqrt{d+ex^2}\left(3a(d+ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+4ex^2)+3b(d+ex^2)\csc^{-1}(cx)\right)}{9dx^3} + \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d+2e)E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\left|-\frac{e}{c^2d}\right.\right)-(2c^4d^2+5c^2de+3e^2)\,\text{EllipticF}\left(i\right.\right.}{9\sqrt{-c^2}d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] -1/9*(Sqrt[d + e*x^2]*(3*a*(d + e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2
*c^2*d*x^2 + 4*e*x^2) + 3*b*(d + e*x^2)*ArcCsc[c*x]))/(d*x^3) + ((I/9)*b*c*
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d + 2*e)*Elliptic
E[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (2*c^4*d^2 + 5*c^2*d*e + 3*e^2)*
EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d*Sqrt[1 - c
^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^2+d}}{x^4}dx$$

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{\left(3\,acdex^2+3\,acd^2+3\,(bcdex^2+bcd^2)\,\mathrm{arccsc}\,(cx)+(bcd^2+2\,(bc^3d^2+2\,bcde)x^2)\sqrt{c^2x^2-1}\right)\sqrt{ex^2+d}}{}$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*((3*a*c*d*e*x^2 + 3*a*c*d^2 + 3*(b*c*d*e*x^2 + b*c*d^2)*arccsc(c*x) +
(b*c*d^2 + 2*(b*c^3*d^2 + 2*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d
) + (2*(b*c^6*d^2 + 2*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d)) -
(2*b*c^6*d^2 + (4*b*c^4 + b*c^2)*d*e + 3*b*e^2)*x^3*elliptic_f(arcsin(c*x),
 -e/(c^2*d)))*sqrt(-d))/(c*d^2*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,\sqrt{d+ex^2}}{x^4}\,dx$$

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^4}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^4, x)
```

3.127 $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 453

$$\begin{aligned}
&\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx \\
&= -\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} \\
&\quad -\frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} \\
&\quad -\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}+\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3} \\
&\quad +\frac{bc^2(24c^4d^2+19c^2de-31e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad -\frac{b(c^2d+e)(24c^4d^2+7c^2de-30e^2)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

```
[Out] -1/5*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*ar
ccsc(c*x))/d^2/x^3+2/15*b*c*e^2*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*
x^2)^(1/2)-1/45*b*c*e*(2*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^
2*x^2)^(1/2)-1/75*b*c*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+
d)^(1/2)/d^2/(c^2*x^2)^(1/2)-1/25*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^4
/(c^2*x^2)^(1/2)-1/45*b*c*e*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^
2)^(1/2)-1/75*b*c*(4*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*
x^2)^(1/2)-2/15*b*c^2*e^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)
+1/45*b*c^2*e*(2*c^2*d+e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)
```

```
+1/75*b*c^2*(8*c^4*d^2+3*c^2*d*e-2*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(
-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+
e*x^2/d)^(1/2)-1/75*b*c^2*(8*c^2*d-e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^
(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(
1/2)/(e*x^2+d)^(1/2)-2/45*b*c^2*e*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2
))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
/(e*x^2+d)^(1/2)+2/15*b*e^2*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c
^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*
x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {277, 270, 5347, 12, 594, 597, 538, 438, 437, 435, 432, 430}

$$\begin{aligned}
&\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx \\
&= \frac{2e(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{5dx^5} \\
&\quad -\frac{bx\sqrt{1-c^2x^2}(c^2d+e)\,(24c^4d^2+7c^2de-30e^2)\,\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} \\
&\quad +\frac{bc^2x\sqrt{1-c^2x^2}(24c^4d^2+19c^2de-31e^2)\,\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{225d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} \\
&\quad -\frac{bc\sqrt{c^2x^2-1}(12c^2d-e)\,\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} \\
&\quad -\frac{bc\sqrt{c^2x^2-1}(24c^4d^2+19c^2de-31e^2)\,\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}}
\end{aligned}$$

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(b*c*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d +
e*x^2])/(d^2*Sqrt[c^2*x^2]) - (b*c*(12*c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d
 + e*x^2])/(225*d*x^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^
(3/2))/(25*d*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(
5*d*x^5) + (2*e*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(15*d^2*x^3) + (b*c^
2*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*El
lipticE[ArcSin[c*x], -(e/(c^2*d))])/(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^
2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(24*c^4*d^2 + 7*c^2*d*e - 30*e^2)*
x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))
])/(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3} + \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(-3d+2ex^2)}{15d^2x^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3} + \frac{(bcx)\int \frac{(d+ex^2)^{3/2}(-3d+2ex^2)}{x^6\sqrt{-1+c^2x^2}}\,dx}{15d^2\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}(d(12c^2d-e)+(3c^2d-10e)ex^2)}{x^4\sqrt{-1+c^2x^2}}\,dx}{75d^2\sqrt{c^2x^2}}$$

$$= -\frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3} + \frac{(bcx)\int \frac{-d(24c^4d^2+19c^2de-31e^2)-2e(6c^4d^2+4c^2de-15e^2)x^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^2\sqrt{c^2x^2}}$$

$$= -\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3} + \frac{(bcx)\int \frac{-2de(6c^4d^2+4c^2de-15e^2)+c^2de(24c^4d^2+19c^2de-31e^2)x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^3\sqrt{c^2x^2}}$$

$$= -\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3} + \frac{(bc^3(24c^4d^2+19c^2de-31e^2)x)\int\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{225d^2\sqrt{c^2x^2}} - \frac{(bc(c^2d+e)(24c^4d^2+7c^2de-30e^2)x)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{225d^2\sqrt{c^2x^2}}$$

$$= -\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3} + \frac{\left(bc^3(24c^4d^2+19c^2de-31e^2)x\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} - \frac{\left(bc(c^2d+e)(24c^4d^2+7c^2de-30e^2)x\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{225d^2\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3} + \frac{\left(bc^3(24c^4d^2+19c^2de-31e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bc(c^2d+e)(24c^4d^2+7c^2de-30e^2)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3} + \frac{bc^2(24c^4d^2+19c^2de-31e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+e)(24c^4d^2+7c^2de-30e^2)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.69 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{\sqrt{d+ex^2}\left(15a(3d^2+dex^2-2e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(-31e^2x^4+dex^2(8+19c^2x^2)+3d^2(3+4c^2x^2+8\cdots\right.}{225d^2x^5} + \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(24c^4d^2+19c^2de-31e^2)E\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right)\left|-\frac{e}{c^2d}\right.\right)+(-24c^6d^3-31c^4d^2\cdots\right.}{225\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(Sqrt[d + e*x^2]*(15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*Sqrt[1 -
1/(c^2*x^2)]*x*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 + 4*c^2*x
^2 + 8*c^4*x^4)) + 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*ArcCsc[c*x]))/(d^2*x^
5) + ((I/225)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(24*c^
4*d^2 + 19*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d)
)] + (-24*c^6*d^3 - 31*c^4*d^2*e + 23*c^2*d*e^2 + 30*e^3)*EllipticF[I*ArcSi
nh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d
+ e*x^2])
```

Maple [F]

$$\int \frac{(a + b \operatorname{arccsc}(cx)) \sqrt{e x^2 + d}}{x^6} dx$$

[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x)

[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x)

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{d + ex^2}(a + b \csc^{-1}(cx))}{x^6} dx = \frac{(30\, acde^2x^4 - 15\, acd^2ex^2 - 45\, acd^3 + 15\, (2\, bcde^2x^4 - bcd^2ex^2 - 3\, bcd^3) \operatorname{arccsc}(cx) - (9\, bcd^3 + (24\, bc^5d^3 +}{}$$

[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")

[Out] 1/225*((30*a*c*d*e^2*x^4 - 15*a*c*d^2*e*x^2 - 45*a*c*d^3 + 15*(2*b*c*d*e^2*x^4 - b*c*d^2*e*x^2 - 3*b*c*d^3)*arccsc(c*x) - (9*b*c*d^3 + (24*b*c^5*d^3 + 19*b*c^3*d^2*e - 31*b*c*d*e^2)*x^4 + 4*(3*b*c^3*d^3 + 2*b*c*d^2*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((24*b*c^8*d^3 + 19*b*c^6*d^2*e - 31*b*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (24*b*c^8*d^3 + (19*b*c^6 + 12*b*c^4)*d^2*e - (31*b*c^4 - 8*b*c^2)*d*e^2 - 30*b*e^3)*x^5*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^3*x^5)

Sympy [F]

$$\int \frac{\sqrt{d + ex^2}(a + b \csc^{-1}(cx))}{x^6} dx = \int \frac{(a + b \operatorname{acsc}(cx)) \sqrt{d + ex^2}}{x^6} dx$$

[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**6,x)

[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**6, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{ex^2+d}(b\operatorname{arccsc}(cx)+a)}{x^6}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^6, x)
```

3.128 $\int x^3(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 374

$$\int x^3(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))\,dx = -\frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^2} + \frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{35e^2\sqrt{c^2x^2}} - \frac{b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\,x\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{c^2x^2}}$$

```
[Out] -1/5*d*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arccs
c(c*x))/e^2+2/35*b*c*d^(7/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^2/(c^2*x^2)^(1/2)-1/560*b*(35*c^6*d^3-35*c^4*d^2*e-63*c^2*d*e^2-25*
e^3)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(3/2)/(c^
2*x^2)^(1/2)+1/840*b*(13*c^2*d+25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c^
3/e/(c^2*x^2)^(1/2)+1/42*b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e/(c^2*x^2
)^(1/2)-1/560*b*(3*c^4*d^2-38*c^2*d*e-25*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)
^(1/2)/c^5/e/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {272, 45, 5347, 12, 587, 159, 163, 65, 223, 212, 95, 210}

$$\int x^3(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))\,dx = -\frac{d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^2} + \frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{35e^2\sqrt{c^2x^2}} - \frac{bx(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2-1}(13c^2d+25e)(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2-1}(3c^4d^2-38c^2de-25e^2)\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}}$$

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] -1/560*(b*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e
*x^2])/(c^5*e*Sqrt[c^2*x^2]) + (b*(13*c^2*d + 25*e)*x*Sqrt[-1 + c^2*x^2]*(d
 + e*x^2)^(3/2))/(840*c^3*e*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e
*x^2)^(5/2))/(42*c*e*Sqrt[c^2*x^2]) - (d*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*
x]))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^2) + (2*b*c*d^(
7/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(35*e^2*Sqrt[c
^2*x^2]) - (b*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2 - 25*e^3)*x*ArcTanh
[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^6*e^(3/2)*Sqrt[c
^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} \\
&\quad+\frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}+\frac{(bcx)\int \frac{(d+ex^2)^{5/2}(-2d+5ex^2)}{35e^2x\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} \\
&\quad+\frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}+\frac{(bcx)\int \frac{(d+ex^2)^{5/2}(-2d+5ex^2)}{x\sqrt{-1+c^2x^2}}\,dx}{35e^2\sqrt{c^2x^2}} \\
&= -\frac{d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2} \\
&\quad+\frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^{5/2}(-2d+5ex)}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{70e^2\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2} + \frac{(bx)\text{Subst}\left(\int \frac{(d+ex)^{3/2}\left(-6c^2d^2+\frac{1}{2}e(13c^2d+25e)x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{210ce^2\sqrt{c^2x^2}}$$

$$= \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2} + \frac{(bx)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(-12c^4d^3-\frac{3}{4}e(3c^4d^2-38c^2de-25e^2)x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{420c^3e^2\sqrt{c^2x^2}}$$

$$= -\frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2} + \frac{(bx)\text{Subst}\left(\int \frac{-12c^6d^4-\frac{3}{8}e(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{420c^5e^2\sqrt{c^2x^2}}$$

$$= -\frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2} - \frac{(bcd^4x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{35e^2\sqrt{c^2x^2}} - \frac{\left(b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\,x\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{1120c^5e\sqrt{c^2x^2}}$$

$$= -\frac{b(3c^4d^2 - 38c^2de - 25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^2} - \frac{(2bcd^4x)\,\mathrm{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{35e^2\sqrt{c^2x^2}} - \frac{(b(35c^6d^3 - 35c^4d^2e - 63c^2de^2 - 25e^3)\,x)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{560c^7e\sqrt{c^2x^2}}$$

$$= -\frac{b(3c^4d^2 - 38c^2de - 25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^2} + \frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{35e^2\sqrt{c^2x^2}} - \frac{(b(35c^6d^3 - 35c^4d^2e - 63c^2de^2 - 25e^3)\,x)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{560c^7e\sqrt{c^2x^2}}$$

$$= -\frac{b(3c^4d^2 - 38c^2de - 25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^2} + \frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{35e^2\sqrt{c^2x^2}} - \frac{b(35c^6d^3 - 35c^4d^2e - 63c^2de^2 - 25e^3)\,x\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.59 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.81

$$\int x^3(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))\ dx = \frac{96a(d+ex^2)^3(-2d+5ex^2)+\frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(75e^2+2c^2e(82d+25ex^2)+c^4(57d^2+106dex^2+40e^2x^4))}{c^5}}{}$$

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (96*a*(d + e*x^2)^3*(-2*d + 5*e*x^2) + (2*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
e*x^2)*(75*e^2 + 2*c^2*e*(82*d + 25*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 + 40
*e^2*x^4)))/c^5 + (3*b*(32*c^4*d^4*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2
, 2, 1/(c^2*x^2), -(d/(e*x^2))] + (e*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*
e^2 - 25*e^3)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2
, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^5*x) + 96*b*(d + e
*x^2)^3*(-2*d + 5*e*x^2)*ArcCsc[c*x])/(3360*e^2*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x^3\left(e\,x^2+d\right)^{\frac{3}{2}}(a+b\,\mathrm{arccsc}\,(cx))\,dx$$

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

Fricas [A] (verification not implemented)

none

Time = 2.24 (sec) , antiderivative size = 1697, normalized size of antiderivative = 4.54

$$\int x^3(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))\ dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] [1/6720*(96*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^
2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
 d)*sqrt(-d) + 8*d^2)/x^4) - 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*
e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*
```

```
e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x
^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*
c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*
c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arccsc(c*x) + (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2
*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*
sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^2), 1/6720*(192*b*c^7*d^(7/2)*ar
ctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)
/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 3*(35*b*c^6*d^3 - 35*b*c^4*d^
2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^
2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*
x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*
d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^7*e^3*x^6 + 8*b*c
^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arccsc(c*x) + (40*b*c^5*e^3*x
^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(53*b*c^5*d*e^2 + 25
*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^2), 1/3360*(48*
b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)
*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d)
 + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e
^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*
x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240*a
*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48
*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arcc
sc(c*x) + (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3
 + 2*(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d
))/(c^7*e^2), 1/3360*(96*b*c^7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*
d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^
2 - d^2)) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*s
qrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 +
 d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(240*a*c^7*
e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b
*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arccsc(c*
x) + (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*
(53*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c
^7*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^3(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^3(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int (ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)x^3\,dx$$

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int x^3\,\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^3*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

3.129 $\int x(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 262

$$\int x(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} - \frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{5e\sqrt{c^2x^2}} + \frac{b(15c^4d^2+10c^2de+3e^2)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}}$$

```
[Out] 1/5*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/e-1/5*b*c*d^(5/2)*x*arctan((e*x^2+d)^
(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/40*b*(15*c^4*d^2+10*c^
2*d*e+3*e^2)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(
1/2)/(c^2*x^2)^(1/2)+1/20*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)
^(1/2)+1/40*b*(7*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/(c^2*x^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules

used = {5345, 457, 104, 159, 163, 65, 223, 212, 95, 210}

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{5e\sqrt{c^2x^2}}+\frac{bx(15c^4d^2+10c^2de+3e^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}(7c^2d+3e)\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}}$$

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(7*c^2*d + 3*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(40*c^3*Sqrt[c^2*x
^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*Sqrt[c^2*x^2]) + ((
d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) - (b*c*d^(5/2)*x*ArcTan[Sqrt[d
+ e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(5*e*Sqrt[c^2*x^2]) + (b*(15*c^4*d^
2 + 10*c^2*d*e + 3*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d +
e*x^2])])/(40*c^4*Sqrt[e]*Sqrt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} + \frac{(bcx)\int \frac{(d+ex^2)^{5/2}}{x\sqrt{-1+c^2x^2}}\,dx}{5e\sqrt{c^2x^2}}$$

$$= \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} + \frac{(bcx)\text{Subst}\left(\int \frac{(d+ex)^{5/2}}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{10e\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} + \frac{(bx)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(2c^2d^2+\frac{1}{2}e(7c^2d+3e)x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{20ce\sqrt{c^2x^2}}$$

$$= \frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} + \frac{(bx)\text{Subst}\left(\int \frac{2c^4d^3+\frac{1}{4}e(15c^4d^2+10c^2de+3e^2)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{20c^3e\sqrt{c^2x^2}}$$

$$= \frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} + \frac{(bcd^3x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{10e\sqrt{c^2x^2}} + \frac{(b(15c^4d^2+10c^2de+3e^2)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{80c^3\sqrt{c^2x^2}}$$

$$= \frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e} + \frac{(bcd^3x)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{5e\sqrt{c^2x^2}} + \frac{(b(15c^4d^2+10c^2de+3e^2)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{40c^5\sqrt{c^2x^2}}$$

$$= \frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e} - \frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{5e\sqrt{c^2x^2}} + \frac{(b(15c^4d^2+10c^2de+3e^2)\,x)\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{40c^5\sqrt{c^2x^2}}$$

$$= \frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e} - \frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{5e\sqrt{c^2x^2}} + \frac{b(15c^4d^2+10c^2de+3e^2)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.32 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.95

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{\frac{16a(d+ex^2)^3}{e} + \frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2)(3e+c^2(9d+2ex^2))}{c^3} + \frac{b\left(-\frac{8c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{e} - \frac{(15c\cdots}{\cdots}\right)}{\cdots}}{80\sqrt{d+ex^2}}$$

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((16*a*(d + e*x^2)^3)/e + (2*b*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)*(3*e + c
^2*(9*d + 2*e*x^2)))/c^3 + (b*((-8*c^2*d^3*Sqrt[1 + d/(e*x^2)]*AppellF1[1,
1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/e - ((15*c^4*d^2 + 10*c^2*d*e + 3*
e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2,
 c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^3*x) + (16*b*(d + e*x^2)^3*
ArcCsc[c*x])/e)/(80*Sqrt[d + e*x^2])
```

Maple [F]

$$\int x(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arccsc}\,(cx))\,dx$$

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.98 (sec) , antiderivative size = 1375, normalized size of antiderivative = 5.25

$$\int x\big(d+ex^2\big)^{3/2}\,\big(a+b\csc^{-1}(cx)\big)\,dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] [1/160*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*
d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d
)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*
log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*
c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) +
 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2
*b*c^5*d*e*x^2 + b*c^5*d^2)*arccsc(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e +
3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e), -1/160*(16*b*c^5*d^
(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)
*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (15*b*c^4*d^2 + 10*b*
c^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4
*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(e) + e^2) - 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5
*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arccsc(c*x) + (2*b*c
^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(
c^5*e), 1/80*(4*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8
*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x
^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sq
rt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 +
d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(8*a*c^5*e^2
*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2
+ b*c^5*d^2)*arccsc(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt
(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e), -1/80*(8*b*c^5*d^(5/2)*arctan(-1/2
*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e
*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2
)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^
2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(8*a*c^5
```

```
*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*
x^2 + b*c^5*d^2)*arccsc(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*
sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e)]
```

Sympy [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x(a+b\operatorname{acsc}(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
[Out] Integral(x*(a + b*acsc(c*x))*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arccsc}(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] 1/5*(e*x^2 + d)^(5/2)*a/e + 1/5*(5*e*integrate(1/5*(c^2*e^2*x^5 + 2*c^2*d*e
*x^3 + c^2*d^2*x)*e^(1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x -
1))/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e), x) +
 (e^2*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*d*e*x^2*arctan2(1, sq
rt(c*x + 1)*sqrt(c*x - 1)) + d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*s
qrt(e*x^2 + d))*b/e
```

Giac [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arccsc}(cx)+a)x\,dx$$

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x\,\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

3.130 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.92 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arccsc}(cx))}{x}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arccsc}(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x, x
)
```

Sympy [N/A]

Not integrable

Time = 72.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx=\int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x,x)
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x}\,dx=\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x, x)
```

3.131 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx=\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arccsc}(cx))}{x^3}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arccsc}(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^3,
 x)
```

Sympy [N/A]

Not integrable

Time = 63.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\operatorname{acsc}(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**3,x)
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arccsc}(cx)+a)}{x^3}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^3, x)
```

3.132 $\int x^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Int}\left(x^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] Defer[Int][x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\text{integral}=\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccsc}\left(cx\right)\right)dx$$

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int x^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arccsc}\left(cx\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arccsc(c*x))*sqrt(e*x^2 +
 d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\,dx = \int (ex^2+d)^{\frac{3}{2}}(b\operatorname{arccsc}(cx)+a)x^2\,dx$$

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2 (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\,dx = \int x^2 \left(e\,x^2+d\right)^{3/2} \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int(x^2*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^2*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

3.133 $\int (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\, dx = \text{Int}\Big((d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\, ,x\Big)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\, dx = \int (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\, dx$$

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\text{integral} = \int (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\, dx$$

Mathematica [N/A]

Not integrable

Time = 18.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$$

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\operatorname{arccsc}\left(cx\right)\right)dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arccsc}\left(cx\right)+a\right)dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}} \left(b\,\mathrm{arccsc}\,(cx)+a\right)dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(e\,x^2+d\right)^{3/2} \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

3.134 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 36.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\ \mathrm{arccsc}\,(cx))}{x^2}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^2,
 x)
```

Sympy [N/A]

Not integrable

Time = 98.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**2,x)
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^2, x)
```

3.135 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \mathrm{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4, x]
```

Rubi steps

$$\text{integral} = \int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arccsc}(cx))}{x^4}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arccsc}(cx)+a)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^4,
 x)
```

Sympy [N/A]

Not integrable

Time = 65.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\operatorname{acsc}{(cx)})\,(d+ex^2)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**4,x)
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arccsc}(cx)+a)}{x^4}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^4, x)
```

3.136 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 416

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}}$$
$$-\frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}}$$
$$-\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5}$$
$$+\frac{bc^2(8c^4d^2+23c^2de+23e^2)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{b(c^2d+e)(8c^4d^2+19c^2de+15e^2)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\mathrm{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

```
[Out] -1/5*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/d/x^5-1/25*b*c*(e*x^2+d)^(3/2)*(c^2*
x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-1/75*b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*(c^
2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)-4/75*b*c*(c^2*d+2*e)*(c^2*
x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^2/(c^2*x^2)^(1/2)+1/75*b*c^2*(8*c^4*d^2+23*c
^2*d*e+23*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+
d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-1/75*b*(c^2*
d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*
x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d
)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {270, 5347, 12, 485, 594, 597, 538, 438, 437, 435, 432, 430}

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5} - \frac{bx\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} - \frac{4bc\sqrt{c^2x^2-1}(c^2d+2e)\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{bc\sqrt{c^2x^2-1}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/75*(b*c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*
x^2])/(d*Sqrt[c^2*x^2]) - (4*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d +
e*x^2])/(75*x^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))
/(25*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*d*x^5)
 + (b*c^2*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*
x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^
2*x^2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(8*c^4*d^2 + 19*c^2*d*e + 15*e
^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2
*d))])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5dx^5}+\frac{(bcx)\int -\frac{(d+ex^2)^{5/2}}{5dx^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5dx^5}-\frac{(bcx)\int \frac{(d+ex^2)^{5/2}}{x^6\sqrt{-1+c^2x^2}}\,dx}{5d\sqrt{c^2x^2}}\\
&= -\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}}-\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5dx^5}\\
&\quad+\frac{(bcx)\int \frac{\sqrt{d+ex^2}\,(-4d(c^2d+2e)-e(c^2d+5e)x^2)}{x^4\sqrt{-1+c^2x^2}}\,dx}{25d\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5} - \frac{(bcx)\int \frac{d(8c^4d^2+23c^2de+23e^2)+e(4c^4d^2+11c^2de+15e^2)x^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{75d\sqrt{c^2x^2}}$$

$$= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5} - \frac{(bcx)\int \frac{de(4c^4d^2+11c^2de+15e^2)-c^2de(8c^4d^2+23c^2de+23e^2)x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{75d^2\sqrt{c^2x^2}}$$

$$= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5} - \frac{(bc(c^2d+e)(8c^4d^2+19c^2de+15e^2)x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{75d\sqrt{c^2x^2}} + \frac{(bc^3(8c^4d^2+23c^2de+23e^2)x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{75d\sqrt{c^2x^2}}$$

$$= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{\left(bc^3(8c^4d^2+23c^2de+23e^2)x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} - \frac{\left(bc(c^2d+e)(8c^4d^2+19c^2de+15e^2)x\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{75d\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{\left(bc^3(8c^4d^2+23c^2de+23e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}dx}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bc(c^2d+e)(8c^4d^2+19c^2de+15e^2)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}dx}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{bc^2(8c^4d^2+23c^2de+23e^2)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,|-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+e)(8c^4d^2+19c^2de+15e^2)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.20 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.73

$$\int\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}dx = -\frac{\sqrt{d+ex^2}\left(15a(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}x(23e^2x^4+dex^2(11+23c^2x^2)+d^2(3+4c^2x^2+8c^4x^4))+15b(d+\right.}{75dx^5} + \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(8c^4d^2+23c^2de+23e^2)E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-(8c^6d^3+27c^4d^2e+34c^2\right.}{75\sqrt{-c^2}d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/75*(Sqrt[d + e*x^2]*(15*a*(d + e*x^2)^2 + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2
3*e^2*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4)) +
15*b*(d + e*x^2)^2*ArcCsc[c*x]))/(d*x^5) + ((I/75)*b*c*Sqrt[1 - 1/(c^2*x^2)
]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*EllipticE[
I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (8*c^6*d^3 + 27*c^4*d^2*e + 34*c^2
*d*e^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-
c^2]*d*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arccsc}\,(cx))}{x^6}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{\left(15\,acde^2x^4+30\,acd^2ex^2+15\,acd^3+15\,(bcde^2x^4+2\,bcd^2ex^2+bcd^3)\,\mathrm{arccsc}\,(cx)+(3\,bcd^3+(8\,bc^5d^3+23\right.}{}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/75*((15*a*c*d*e^2*x^4 + 30*a*c*d^2*e*x^2 + 15*a*c*d^3 + 15*(b*c*d*e^2*x^
4 + 2*b*c*d^2*e*x^2 + b*c*d^3)*arccsc(c*x) + (3*b*c*d^3 + (8*b*c^5*d^3 + 23
*b*c^3*d^2*e + 23*b*c*d*e^2)*x^4 + (4*b*c^3*d^3 + 11*b*c*d^2*e)*x^2)*sqrt(c
^2*x^2 - 1))*sqrt(e*x^2 + d) + ((8*b*c^8*d^3 + 23*b*c^6*d^2*e + 23*b*c^4*d*
e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (8*b*c^8*d^3 + (23*b*c^6 + 4
*b*c^4)*d^2*e + (23*b*c^4 + 11*b*c^2)*d*e^2 + 15*b*e^3)*x^5*elliptic_f(arcs
in(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^6}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**6,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x^6}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^6, x)
```

3.137 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 554

$$\begin{aligned}
&\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx = \\
&-\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} \\
&-\frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} \\
&-\frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} \\
&-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5} \\
&+\frac{bc^2(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&-\frac{2b(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

```
[Out] -1/7*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*ar
ccsc(c*x))/d^2/x^5-1/1225*b*c*(30*c^2*d+11*e)*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(
1/2)/d/x^4/(c^2*x^2)^(1/2)-1/49*b*c*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/d/x^6
/(c^2*x^2)^(1/2)-1/3675*b*c*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^
3)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)-1/3675*b*c*(120*c^
4*d^2+159*c^2*d*e-37*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^2)
^(1/2)+1/3675*b*c^2*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*x*Ell
ipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^
```

```
2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-2/3675*b*(c^2*d+e)*(120*c^6*d^
3+204*c^4*d^2*e+17*c^2*d*e^2-105*e^3)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c
^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*
x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {277, 270, 5347, 12, 594, 597, 538, 438, 437, 435, 432, 430}

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx = \frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5}$$
$$-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7}$$
$$-\frac{2bx\sqrt{1-c^2x^2}(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3675d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$
$$+\frac{bc^2x\sqrt{1-c^2x^2}(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3675d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}$$
$$-\frac{bc\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}(30c^2d+11e)(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}}$$
$$-\frac{bc\sqrt{c^2x^2-1}(120c^4d^2+159c^2de-37e^2)\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}}$$
$$-\frac{bc\sqrt{c^2x^2-1}(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}$$

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] -1/3675*(b*c*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[-
1 + c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[c^2*x^2]) - (b*c*(120*c^4*d^2 + 159
*c^2*d*e - 37*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^2*Sqrt[c^2
*x^2]) - (b*c*(30*c^2*d + 11*e)*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(1225
*d*x^4*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(5/2))/(49*d*x^
6*Sqrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(7*d*x^7) + (2*e
*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(35*d^2*x^5) + (b*c^2*(240*c^6*d^3
+ 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x
^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 +
 c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^
2*e + 17*c^2*d*e^2 - 105*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*Ellip
ticF[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]
*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{35d^2x^5} \\
&\quad+\frac{(bcx)\int\frac{(d+ex^2)^{5/2}(-5d+2ex^2)}{35d^2x^8\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{35d^2x^5} \\
&\quad+\frac{(bcx)\int\frac{(d+ex^2)^{5/2}(-5d+2ex^2)}{x^8\sqrt{-1+c^2x^2}}\,dx}{35d^2\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}}-\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{7dx^7} \\
&\quad+\frac{2e(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{35d^2x^5}-\frac{(bcx)\int\frac{(d+ex^2)^{3/2}(d(30c^2d+11e)+(5c^2d-14e)ex^2)}{x^6\sqrt{-1+c^2x^2}}\,dx}{245d^2\sqrt{c^2x^2}} \\
&= -\frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} \\
&\quad-\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{35d^2x^5} \\
&\quad+\frac{(bcx)\int\frac{\sqrt{d+ex^2}(-d(120c^4d^2+159c^2de-37e^2)-2e(15c^4d^2+18c^2de-35e^2)x^2)}{x^4\sqrt{-1+c^2x^2}}\,dx}{1225d^2\sqrt{c^2x^2}} \\
&= -\frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} \\
&\quad-\frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} \\
&\quad-\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{35d^2x^5} \\
&\quad-\frac{(bcx)\int\frac{d(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)+e(120c^6d^3+249c^4d^2e+71c^2de^2-210e^3)x^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^2\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} - \frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} - \frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5} - \frac{(bcx)\int \frac{de(120c^6d^3+249c^4d^2e+71c^2de^2-210e^3)-c^2de(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^3\sqrt{c^2x^2}}$$

$$= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} - \frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} - \frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5} + \frac{(bc^3(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{3675d^2\sqrt{c^2x^2}} - \frac{(2bc(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3675d^2\sqrt{c^2x^2}}$$

$$= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} - \frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} - \frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5} + \frac{\left(bc^3(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} - \frac{\left(2bc(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3675d^2\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}$$
$$-\frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}}$$
$$-\frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}}$$
$$-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5}$$
$$+\frac{\left(bc^3(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}dx}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{\left(2bc(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}$$
$$-\frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}}$$
$$-\frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}}$$
$$-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5}$$
$$+\frac{bc^2(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$-\frac{2b(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\text{EllipticF}\left(\arcsin(cx)\right.}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.82 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.69

$$\int\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}dx=$$
$$-\frac{\sqrt{d+ex^2}\left(105a(5d-2ex^2)(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}x(-247e^3x^6+de^2x^4(71+193c^2x^2)+3d^2ex^2(61+8\right.}{3675d^2x^7}$$
$$+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\left|-\frac{e}{c^2d}\right.\right)-2(120\right.}{3675\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d}}$$

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] -1/3675*(Sqrt[d + e*x^2]*(105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*c*Sqrt[1
- 1/(c^2*x^2)]*x*(-247*e^3*x^6 + d*e^2*x^4*(71 + 193*c^2*x^2) + 3*d^2*e*x^2
*(61 + 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c
^6*x^6)) + 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcCsc[c*x]))/(d^2*x^7) + ((
I/3675)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*c^6*d^3
 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*EllipticE[I*ArcSinh[Sqrt[-c^2]*
x], -(e/(c^2*d))] - 2*(120*c^8*d^4 + 324*c^6*d^3*e + 221*c^4*d^2*e^2 - 88*c
^2*d*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqr
t[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arccsc}(cx))}{x^8}dx$$

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.14 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^8}\,dx = \frac{(210\,acde^3x^6 - 105\,acd^2e^2x^4 - 840\,acd^3ex^2 - 525\,acd^4 + 105\,(2\,bcde^3x}{}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] 1/3675*((210*a*c*d*e^3*x^6 - 105*a*c*d^2*e^2*x^4 - 840*a*c*d^3*e*x^2 - 525*
a*c*d^4 + 105*(2*b*c*d*e^3*x^6 - b*c*d^2*e^2*x^4 - 8*b*c*d^3*e*x^2 - 5*b*c*
d^4)*arccsc(c*x) - ((240*b*c^7*d^4 + 528*b*c^5*d^3*e + 193*b*c^3*d^2*e^2 -
247*b*c*d*e^3)*x^6 + 75*b*c*d^4 + (120*b*c^5*d^4 + 249*b*c^3*d^3*e + 71*b*c
*d^2*e^2)*x^4 + 3*(30*b*c^3*d^4 + 61*b*c*d^3*e)*x^2)*sqrt(c^2*x^2 - 1))*sqr
t(e*x^2 + d) - ((240*b*c^10*d^4 + 528*b*c^8*d^3*e + 193*b*c^6*d^2*e^2 - 247
*b*c^4*d*e^3)*x^7*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (240*b*c^10*d^4 + 2
4*(22*b*c^8 + 5*b*c^6)*d^3*e + (193*b*c^6 + 249*b*c^4)*d^2*e^2 - (247*b*c^4
 - 71*b*c^2)*d*e^3 - 210*b*e^4)*x^7*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sq
rt(-d))/(c*d^3*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^8}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**8,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^8}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x^8}\,dx$$

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^8}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^8, x)
```

3.138 $\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 321

$$\begin{aligned}\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx =& -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} \\ &+ \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} \\ &- \frac{2d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} \\ &+ \frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3} - \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^3\sqrt{c^2x^2}} \\ &+ \frac{b(45c^4d^2-10c^2de+9e^2)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{c^2x^2}}\end{aligned}$$

```
[Out] -2/3*d*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arccs
c(c*x))/e^3-8/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^3/(c^2*x^2)^(1/2)+1/120*b*(45*c^4*d^2-10*c^2*d*e+9*e^2)*x*arctanh(e
^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(5/2)/(c^2*x^2)^(1/2)+1/2
0*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)+d^2*(a+b*arcc
sc(c*x))*(e*x^2+d)^(1/2)/e^3-1/120*b*(19*c^2*d-9*e)*x*(c^2*x^2-1)^(1/2)*(e*
x^2+d)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {272, 45, 5347, 12, 1629, 159, 163, 65, 223, 212, 95, 210}

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} - \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{15e^3\sqrt{c^2x^2}} + \frac{bx(45c^4d^2-10c^2de+9e^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} - \frac{bx\sqrt{c^2x^2-1}(19c^2d-9e)\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}}$$

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -1/120*(b*(19*c^2*d - 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(c^3*e^2*S
qrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e^2*Sqrt[c
^2*x^2]) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^3 - (2*d*(d + e*x^2)
^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]
))/(5*e^3) - (8*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2
*x^2])])/(15*e^3*Sqrt[c^2*x^2]) + (b*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*x*Ar
cTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(5/2)*S
qrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} \\
&\quad + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}(8d^2-4dex^2+3e^2x^4)}{15e^3x\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= \frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} \\
&\quad + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}(8d^2-4dex^2+3e^2x^4)}{x\sqrt{-1+c^2x^2}}\,dx}{15e^3\sqrt{c^2x^2}} \\
&= \frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} \\
&\quad + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(bcx)\mathrm{Subst}\left(\int \frac{\sqrt{d+ex}(8d^2-4dex+3e^2x^2)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{30e^3\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(bx)\text{Subst}\left(\int \frac{\sqrt{d+ex}\left(16c^2d^2e-\frac{1}{2}(19c^2d-9e)e^2x\right)}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{60ce^4\sqrt{c^2x^2}}$$

$$= -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(bx)\text{Subst}\left(\int \frac{16c^4d^3e+\frac{1}{4}e^2(45c^4d^2-10c^2de+9e^2)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{60c^3e^4\sqrt{c^2x^2}}$$

$$= -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(4bcd^3x)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{15e^3\sqrt{c^2x^2}} + \frac{(b(45c^4d^2-10c^2de+9e^2)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{240c^3e^2\sqrt{c^2x^2}}$$

$$= -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} + \frac{(8bcd^3x)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{15e^3\sqrt{c^2x^2}} + \frac{(b(45c^4d^2-10c^2de+9e^2)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{120c^5e^2\sqrt{c^2x^2}}$$

$$= -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3} - \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^3\sqrt{c^2x^2}} + \frac{(b(45c^4d^2-10c^2de+9e^2)\,x)\,\text{Subst}\left(\int\frac{1}{1-\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{120c^5e^2\sqrt{c^2x^2}}$$

$$= -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3} - \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^3\sqrt{c^2x^2}} + \frac{b(45c^4d^2-10c^2de+9e^2)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.01 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.88

$$\int\frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{16a(d+ex^2)\left(8d^2-4dex^2+3e^2x^4\right) + \frac{2be\sqrt{1-\frac{1}{c^2x^2}}(d+ex^2)(9ex+c^2(-13dx+6ex^3))}{c^3} + \frac{b\left(-64c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},\right.\right.}{}}{24}$$

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (16*a*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) + (2*b*e*Sqrt[1 - 1/(c^2*
x^2)]*(d + e*x^2)*(9*e*x + c^2*(-13*d*x + 6*e*x^3)))/c^3 + (b*(-64*c^2*d^3*
Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] + (
e*(-45*c^4*d^2 + 10*c^2*d*e - 9*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*
x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2])
)/(c^3*x) + 16*b*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4)*ArcCsc[c*x])/(
240*e^3*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arccsc}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 1.11 (sec) , antiderivative size = 1383, normalized size of antiderivative = 4.31

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(64*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2
*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
d)*sqrt(-d) + 8*d^2)/x^4) + (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)
*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2
*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2)
+ 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^
4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 - 13*b*c^
3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), -1/480*(1
28*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt
(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (45*b*c^4*
d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d
*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2
 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x
^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arc
csc(c*x) + (6*b*c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*
sqrt(e*x^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*
c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d -
e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (45*b*c^4*d^2 - 10*b
*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*
x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*
x^2)) + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*
e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 - 1
3*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), -1/
240*(64*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)
*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (45*b
*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d
```

```
 - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^
3*d*e - c*e^2)*x^2)) - 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^
2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arccsc(c*x) + (6*b*
c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))
/(c^5*e^3)]
```

Sympy [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^5(a+b\,\mathrm{acsc}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**5*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\, dx = \int \frac{x^5\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\, dx$$

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.139 $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 225

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2} + \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}} - \frac{b(3c^2d-e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^2+2/3*b*c*d^(3/2)*x*arctan((e*x^2+d
)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/(c^2*x^2)^(1/2)-1/6*b*(3*c^2*d-e)*x*
arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(3/2)/(c^2*x^2)^
(1/2)-d*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e^2+1/6*b*x*(c^2*x^2-1)^(1/2)*(e*
x^2+d)^(1/2)/c/e/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules

used = {272, 45, 5347, 12, 587, 159, 163, 65, 223, 212, 95, 210}

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = -\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^2\sqrt{c^2x^2}} - \frac{bx(3c^2d-e)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}}$$

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e*Sqrt[c^2*x^2]) - (d*Sqrt[d
+ e*x^2]*(a + b*ArcCsc[c*x]))/e^2 + ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))
/(3*e^2) + (2*b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x
^2])])/(3*e^2*Sqrt[c^2*x^2]) - (b*(3*c^2*d - e)*x*ArcTanh[(Sqrt[e]*Sqrt[-1
+ c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
```

```
]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{(bcx)\int\frac{(-2d+ex^2)\sqrt{d+ex^2}}{3e^2x\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{(bcx)\int\frac{(-2d+ex^2)\sqrt{d+ex^2}}{x\sqrt{-1+c^2x^2}}\,dx}{3e^2\sqrt{c^2x^2}}\\
&= -\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{(bcx)\mathrm{Subst}\left(\int\frac{(-2d+ex)\sqrt{d+ex}}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{6e^2\sqrt{c^2x^2}}\\
&= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{(bx)\mathrm{Subst}\left(\int\frac{-2c^2d^2-\frac{1}{2}(3c^2d-e)ex}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{6ce^2\sqrt{c^2x^2}}\\
&= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} - \frac{(bcd^2x)\,\mathrm{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3e^2\sqrt{c^2x^2}}\\
&\quad - \frac{(b(3c^2d-e)\,x)\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{12ce\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} - \frac{(2bcd^2x)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}} - \frac{(b(3c^2d-e)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{6c^3e\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}} - \frac{(b(3c^2d-e)\,x)\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^3e\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2} + \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}} - \frac{b(3c^2d-e)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.42 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{4bd^2\sqrt{1+\frac{d}{ex^2}}(-1+c^2x^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right) + be(-3c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x^4\sqrt{1-c^2x^2}\sqrt{1+}}{12c}$$

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (4*b*d^2*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c^2
*x^2), -(d/(e*x^2))] + b*e*(-3*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1
- c^2*x^2]*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/
d)] + 2*x*(-1 + c^2*x^2)*(d + e*x^2)*(-4*a*c*d + b*e*Sqrt[1 - 1/(c^2*x^2)]*
x + 2*a*c*e*x^2 + 2*b*c*(-2*d + e*x^2)*ArcCsc[c*x]))/(12*c*e^2*x*(-1 + c^2*
x^2)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^3(a + b\,\mathrm{arccsc}\,(cx))}{\sqrt{e\,x^2 + d}}dx$$

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
[Out] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 1107, normalized size of antiderivative = 4.92

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] [1/24*(4*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*s
qrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d
^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*
sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 - 4*a*c
^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arccsc(c*x))*s
qrt(e*x^2 + d))/(c^3*e^2), 1/24*(8*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^2 -
 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2
 - d*e)*x^2 - d^2)) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2
 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sq
rt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 - 4*a*c^3
*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arccsc(c*x))*sqr
t(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e
 + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2
- 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d - b*e)*sqrt(-e)*
arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt
(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*x^2 - 4*
a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arccsc(c*x)
)*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(4*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x^
2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*
d^2 - d*e)*x^2 - d^2)) + (3*b*c^2*d - b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2
 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d
*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*x^2 - 4*a*c^3*d + sqrt(c^2*x^2
- 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c^3
*e^2)]
```

Sympy [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\, dx = \int \frac{x^3(a + b\operatorname{acsc}(cx))}{\sqrt{d + ex^2}}\, dx$$

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**3*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\, dx = \int \frac{(b\operatorname{arccsc}(cx) + a)x^3}{\sqrt{ex^2 + d}}\, dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\, dx = \int \frac{x^3\left(a + b\operatorname{asin}\left(\frac{1}{cx}\right)\right)}{\sqrt{e\,x^2 + d}}\, dx$$

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.140 $\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e} - \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} + \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

```
[Out] -b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e/(c^2*x^2
)^(1/2)+b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(1/2)/(c
^2*x^2)^(1/2)+(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5345, 457, 132, 65, 223, 212, 12, 95, 210}

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e} - \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e\sqrt{c^2x^2}} + \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e - (b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*
x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(e*Sqrt[c^2*x^2]) + (b*x*ArcTanh[(Sqrt[
e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x\sqrt{-1+c^2x^2}}\,dx}{e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e} + \frac{(bcx)\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{2e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2\sqrt{c^2x^2}} + \frac{(bcx)\text{Subst}\left(\int \frac{d}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e} + \frac{(bx)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{c\sqrt{c^2x^2}} + \frac{(bcdx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e} + \frac{(bx)\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{c\sqrt{c^2x^2}} + \frac{(bcdx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e} - \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} + \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.81

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e-c^2ex^2}{c^2d+e},1-c^2x^2\right)}{\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}}+b\csc^{-1}(cx)\right)}{e}$$

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*AppellF1[1/2, -1/2, 1, 3
/2, (e - c^2*e*x^2)/(c^2*d + e), 1 - c^2*x^2])/Sqrt[(c^2*(d + e*x^2))/(c^2*
d + e)] + b*ArcCsc[c*x]))/e
```

Maple [F]

$$\int \frac{x(a+b\,\text{arccsc}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 870, normalized size of antiderivative = 6.59

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \Bigg[\frac{bc\sqrt{-d}\log\left(\frac{(c^4d^2-6c^2de+e^2)x^4-8(c^2d^2-de)x^2+4\sqrt{c^2x^2-1}((c^2d-e)x^2-2d)\sqrt{ex^2+d}\sqrt{-d}+8d^2}{x^4}\right)+b\sqrt{e}\log\left(8c^4e^2x^4+c^4d\cdots\right.}{\cdots}$$

$$-\frac{2bc\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}((c^2d-e)x^2-2d)\sqrt{ex^2+d}\sqrt{d}}{2(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)-b\sqrt{e}\log\left(8c^4e^2x^4+c^4d^2-6c^2de+8(c^4de-c^2e^2)x\cdots\right.}{4ce}$$

$$-\frac{bc\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}((c^2d-e)x^2-2d)\sqrt{ex^2+d}\sqrt{d}}{2(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)+b\sqrt{-e}\arctan\left(\frac{(2c^2ex^2+c^2d-e)\sqrt{c^2x^2-1}\sqrt{ex^2+d}\sqrt{-e}}{2(c^3e^2x^4-cde+(c^3de-ce^2)x^2)}\right)-2\sqrt{\cdots}}{2ce}$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)
*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d)
 + 8*d^2)/x^4) + b*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4
*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(e) + e^2) + 4*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e
), -1/4*(2*b*c*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d
)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - b*sq
rt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 +
 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) +
e^2) - 4*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e), 1/4*(b*c*sqrt(-d)*
log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x
^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - 2*
b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^
2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 4*sqrt(e*x
^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e), -1/2*(b*c*sqrt(d)*arctan(-1/2*sqrt(
c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 +
 (c^2*d^2 - d*e)*x^2 - d^2)) + b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d -
 e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*
d*e - c*e^2)*x^2)) - 2*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e)]
```

Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x(a+b\,\mathrm{acsc}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] (e*integrate((c^2*e*x^3 + c^2*d*x)*e^(-1/2*log(e*x^2 + d) + 1/2*log(c*x + 1
) + 1/2*log(c*x - 1))/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*
x - 1)) - e), x) + sqrt(e*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))
*b/e + sqrt(e*x^2 + d)*a/e
```

Giac [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.141 $\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x\sqrt{e\,x^{2}+d}}dx$$

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\sqrt{ex^2+d}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [N/A]

Not integrable

Time = 7.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{acsc}\left(cx\right)}{x\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsc(c*x))/(x*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{x\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a + b\csc^{-1}(cx)}{x\sqrt{d + ex^2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{\sqrt{ex^2 + d}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\csc^{-1}(cx)}{x\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

3.142 $\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\mathrm{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\ \mathrm{arccsc}\left(cx\right)}{x^{3}\sqrt{e\,x^{2}+d}}dx$$

```
[In] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\sqrt{ex^2+dx^3}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 27.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{acsc}\left(cx\right)}{x^3\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsc(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a + b\csc^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{\sqrt{ex^2 + dx^3}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\csc^{-1}(cx)}{x^3\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^3\,\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.143 $\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 36.98 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx$$

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a + b\ \mathrm{arccsc}\,(cx))}{\sqrt{e\,x^2 + d}}dx$$

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
[Out] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^2}{\sqrt{ex^2 + d}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 53.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^2(a + b\,\mathrm{acsc}\,(cx))}{\sqrt{d + ex^2}}\,dx$$

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x**2*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^2}{\sqrt{ex^2 + d}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^2\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\,dx$$

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.144 $\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\operatorname{arccsc}\left(cx\right)}{\sqrt{e\,x^{2}+d}}dx$$

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\operatorname{arccsc}\left(cx\right)+a}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 15.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\operatorname{acsc}\left(cx\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

3.145 $\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 247

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

```
[Out] -(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/d/x-b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2
)/d/(c^2*x^2)^(1/2)+b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-b
*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)
^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules

used = {270, 5347, 12, 486, 21, 434, 438, 437, 435, 432, 430}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx} - \frac{bx\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} - \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] -((b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d*Sqrt[c^2*x^2])) - (Sqrt[d + e
*x^2]*(a + b*ArcCsc[c*x]))/(d*x) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^
2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2
]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2
)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x
^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
```

```
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{dx^2\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x^2\sqrt{-1+c^2x^2}}\,dx}{d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx} + \frac{(bcx)\int \frac{-e+c^2ex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx} + \frac{(bcex)\int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx} + \frac{(bc^3x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{d\sqrt{c^2x^2}} - \frac{(bc(c^2d+e)\,x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} - \frac{\left(bc(c^2d+e)\,x\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{\left(bc(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.57

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}x+b\csc^{-1}(cx)\right)}{dx} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)\left|-\frac{c^2d}{e}\right.\right)}{d\sqrt{-\frac{e}{d}}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x^2]),x]
[Out] -((Sqrt[d + e*x^2]*(a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x + b*ArcCsc[c*x]))/(d*x)
) + (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqr
t[-(e/d)]*x], -((c^2*d)/e)])/(d*Sqrt[-(e/d)]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x
^2])
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arccsc}\,(cx)}{x^2\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.43

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\left(bcd\,\mathrm{arccsc}\,(cx)+\sqrt{c^2x^2-1}bcd+acd\right)\sqrt{ex^2+d}+\left(bc^4dxE(\arcsin\,(cx)\,|-\frac{e}{c^2d})-(bc^4d+be)xF(\arcsin\right.}{cd^2x}$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -((b*c*d*arccsc(c*x) + sqrt(c^2*x^2 - 1)*b*c*d + a*c*d)*sqrt(e*x^2 + d) + (
b*c^4*d*x*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (b*c^4*d + b*e)*x*elliptic_
f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x)
```

Sympy [F]

$$\int \frac{a + b\csc^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\mathrm{acsc}\,(cx)}{x^2\sqrt{d + ex^2}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\csc^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{\sqrt{ex^2 + d}x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{e\,x^2+d}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.146 $\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 362

$$
\begin{aligned}
\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx
&= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} \\
&\quad - \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x} \\
&\quad + \frac{bc^2(2c^2d-5e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{2b(c^2d-3e)\,(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

```
[Out] -1/3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arccsc(c*x))*(e*x^2
+d)^(1/2)/d^2/x-1/9*b*c*(2*c^2*d-5*e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2
/(c^2*x^2)^(1/2)-1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^2)^
(1/2)+1/9*b*c^2*(2*c^2*d-5*e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1
)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(
1/2)-2/9*b*(c^2*d-3*e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^
2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d
)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {277, 270, 5347, 12, 594, 597, 538, 438, 437, 435, 432, 430}

$$\begin{aligned}
&\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx\\
&=\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}\\
&-\frac{2bx\sqrt{1-c^2x^2}(c^2d-3e)\,(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\\
&+\frac{bc^2x\sqrt{1-c^2x^2}(2c^2d-5e)\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\\
&-\frac{bc\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}
\end{aligned}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/9*(b*c*(2*c^2*d - 5*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[c^2
*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*x^2*Sqrt[c^2*x^2]) -
 (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a
+ b*ArcCsc[c*x]))/(3*d^2*x) + (b*c^2*(2*c^2*d - 5*e)*x*Sqrt[1 - c^2*x^2]*Sq
rt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[c^2*x^2]*Sq
rt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d - 3*e)*(c^2*d + e)*x*Sq
rt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(
9*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
```

```
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{3d^2x^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{x^4\sqrt{-1+c^2x^2}}\,dx}{3d^2\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3} \\
&\quad + \frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x} - \frac{(bcx)\int \frac{d(2c^2d-5e)+(c^2d-6e)ex^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} \\
&\quad -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3} \\
&\quad +\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{(bcx)\int\frac{d(c^2d-6e)e-c^2d(2c^2d-5e)ex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^3\sqrt{c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} \\
&\quad -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3} \\
&\quad +\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}+\frac{(bc^3(2c^2d-5e)\,x)\int\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{9d^2\sqrt{c^2x^2}} \\
&\quad -\frac{(2bc(c^2d-3e)\,(c^2d+e)\,x)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{9d^2\sqrt{c^2x^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} \\
&\quad -\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x} \\
&\quad +\frac{\left(bc^3(2c^2d-5e)\,x\sqrt{1-c^2x^2}\right)\int\frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} \\
&\quad -\frac{\left(2bc(c^2d-3e)\,(c^2d+e)\,x\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{9d^2\sqrt{c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} \\
&\quad -\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x} \\
&\quad +\frac{\left(bc^3(2c^2d-5e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int\frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad -\frac{\left(2bc(c^2d-3e)\,(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

$$= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x} + \frac{bc^2(2c^2d-5e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,|-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{2b(c^2d-3e)\,(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.53 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.69

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2-5ex^2)+3a(d-2ex^2)+3b(d-2ex^2)\csc^{-1}(cx)\right)}{9d^2x^3} + \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(2c^2d-5e)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\,|-\frac{e}{c^2d}\right)+2(-c^4d^2+2c^2de+3e^2)\,\text{EllipticF}\right.}{9\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/9*(Sqrt[d + e*x^2]*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 - 5*e*x
^2) + 3*a*(d - 2*e*x^2) + 3*b*(d - 2*e*x^2)*ArcCsc[c*x]))/(d^2*x^3) + ((I/9
)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d - 5*e)*El
lipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(-(c^4*d^2) + 2*c^2*d*e
+ 3*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2
*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\text{arccsc}\,(cx)}{x^4\sqrt{e\,x^2+d}}dx$$

```
[In] int((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.55

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$= \frac{\left(6\,acdex^2 - 3\,acd^2 + 3\,(2\,bcdex^2 - bcd^2)\,\operatorname{arccsc}\left(cx\right) - (bcd^2 + (2\,bc^3d^2 - 5\,bcde)x^2)\sqrt{c^2x^2-1}\right)\sqrt{ex^2+d}\cdots}{9\cdots}$$

```
[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*((6*a*c*d*e*x^2 - 3*a*c*d^2 + 3*(2*b*c*d*e*x^2 - b*c*d^2)*arccsc(c*x) -
 (b*c*d^2 + (2*b*c^3*d^2 - 5*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 +
d) - ((2*b*c^6*d^2 - 5*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d)) -
 (2*b*c^6*d^2 - (5*b*c^4 - b*c^2)*d*e - 6*b*e^2)*x^3*elliptic_f(arcsin(c*x)
, -e/(c^2*d)))*sqrt(-d))/(c*d^3*x^3)
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{acsc}\left(cx\right)}{x^4\sqrt{d+ex^2}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex^2+dx^4}}\,dx$$

[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2+d}}\,dx$$

[In] int((a + b*asin(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)

[Out] int((a + b*asin(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)

3.147 $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 252

$$\begin{aligned}\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx &= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} \\ &\quad - \frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} \\ &\quad + \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{b(9c^2d-e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{c^2x^2}}\end{aligned}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3+8/3*b*c*d^(3/2)*x*arctan((e*x^2+d
)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)-1/6*b*(9*c^2*d-e)*x*
arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(5/2)/(c^2*x^2)^
(1/2)-d^2*(a+b*arccsc(c*x))/e^3/(e*x^2+d)^(1/2)-2*d*(a+b*arccsc(c*x))*(e*x^
2+d)^(1/2)/e^3+1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/e^2/(c^2*x^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules

used = {272, 45, 5347, 12, 1629, 163, 65, 223, 212, 95, 210}

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{bx(9c^2d-e)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{c^2x^2}} + \frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}}$$

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e^2*Sqrt[c^2*x^2]) - (d^2*(a
+ b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d + e*x^2]*(a + b*ArcCs
c[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + (8*b*c*d^(
3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*e^3*Sqrt[c^
2*x^2]) - (b*(9*c^2*d - e)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d
 + e*x^2])])/(6*c^2*e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}\\
&\quad+\frac{\left(d+ex^2\right)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}+\frac{(bcx)\int\frac{-8d^2-4dex^2+e^2x^4}{3e^3x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}}\\
&= -\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}\\
&\quad+\frac{\left(d+ex^2\right)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}+\frac{(bcx)\int\frac{-8d^2-4dex^2+e^2x^4}{x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3e^3\sqrt{c^2x^2}}\\
&= -\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}\\
&\quad+\frac{\left(d+ex^2\right)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}+\frac{(bcx)\text{Subst}\left(\int\frac{-8d^2-4dex+e^2x^2}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{6e^3\sqrt{c^2x^2}}\\
&= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}\\
&\quad+\frac{\left(d+ex^2\right)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}+\frac{(bx)\text{Subst}\left(\int\frac{-8c^2d^2e-\frac{1}{2}(9c^2d-e)e^2x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{6ce^4\sqrt{c^2x^2}}\\
&= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}\\
&\quad+\frac{\left(d+ex^2\right)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}-\frac{(4bcd^2x)\,\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3e^3\sqrt{c^2x^2}}\\
&\quad-\frac{(b(9c^2d-e)\,x)\,\text{Subst}\left(\int\frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{12ce^2\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} - \frac{(8bcd^2x)\,\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{(b(9c^2d-e)\,x)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{6c^3e^2\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{(b(9c^2d-e)\,x)\,\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{6c^3e^2\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} + \frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} - \frac{b(9c^2d-e)\,x\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.53 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.04

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{16bd^2\sqrt{1+\frac{d}{ex^2}}(-1+c^2x^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right) + be(-9c^2d+e)\sqrt{\ }}{}$$

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (16*b*d^2*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c^
2*x^2), -(d/(e*x^2))] + b*e*(-9*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1
 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)
/d)] + 2*x*(-1 + c^2*x^2)*(b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) - 2*a*c*
(8*d^2 + 4*d*e*x^2 - e^2*x^4) - 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*ArcCsc[
c*x]))/(12*c*e^3*x*(-1 + c^2*x^2)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^5(a + b \operatorname{arccsc}(cx))}{(e\,x^2 + d)^{\frac{3}{2}}} dx$$

[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)

[Out] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 1480, normalized size of antiderivative = 5.87

$$\int \frac{x^5(a + b\csc^{-1}(cx))}{(d + ex^2)^{3/2}} \, dx = \text{Too large to display}$$

[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")

[Out] [-1/24*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 16*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x) + (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/24*(32*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x) + (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 8*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 2*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x) + (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*(16*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (9*b

```
*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*
x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 -
c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16
*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x)
+ (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2
+ c^3*d*e^3)]
```

Sympy [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^5(a+b\operatorname{acsc}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**5*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^5}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.148 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} - \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}} + \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}$$

```
[Out] b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(3/2)/(c^2*x^2)^
(1/2)-2*b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e^2
/(c^2*x^2)^(1/2)+d*(a+b*arccsc(c*x))/e^2/(e*x^2+d)^(1/2)+(a+b*arccsc(c*x))*
(e*x^2+d)^(1/2)/e^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {272, 45, 5347, 12, 587, 163, 65, 223, 212, 95, 210}

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} - \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{e^2\sqrt{c^2x^2}} + \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}$$

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (d*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Arc
Csc[c*x]))/e^2 - (2*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 +
```

```
 c^2*x^2])])/(e^2*Sqrt[c^2*x^2]) + (b*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2]
)/(c*Sqrt[d + e*x^2])])/(e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}+\frac{(bcx)\int\frac{2d+ex^2}{e^2x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}+\frac{(bcx)\int\frac{2d+ex^2}{x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{e^2\sqrt{c^2x^2}}\\
&= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}+\frac{(bcx)\text{Subst}\left(\int\frac{2d+ex}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e^2\sqrt{c^2x^2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} \\
&\quad + \frac{(bcdx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{e^2\sqrt{c^2x^2}} \\
&\quad + \frac{(bcx)\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e\sqrt{c^2x^2}} \\
&= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} \\
&\quad + \frac{(2bcdx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}} \\
&\quad + \frac{(bx)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{ce\sqrt{c^2x^2}} \\
&= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} \\
&\quad - \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}} + \frac{(bx)\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{ce\sqrt{c^2x^2}} \\
&= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} \\
&\quad - \frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}} + \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04

$$
\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{-\frac{2bd\sqrt{1+\frac{d}{ex^2}}\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cx} - \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x^3\sqrt{1+\frac{ex^2}{d}}\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)}{\sqrt{1-c^2x^2}}}{2e^2\sqrt{d+ex^2}}
$$

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] ((-2*b*d*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x
^2))])/(c*x) - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1 + (e*x^2)/d]*AppellF
1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2] + 2*(2*d + e*x^
2)*(a + b*ArcCsc[c*x]))/(2*e^2*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^3(a+b\ \mathrm{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 1072, normalized size of antiderivative = 6.87

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(
c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sq
rt(e*x^2 + d)*sqrt(e) + e^2) + 2*(b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*d^2
 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2
*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(a*c*e*x^2 +
2*a*c*d + (b*c*e*x^2 + 2*b*c*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 +
c*d*e^2), -1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)
*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 -
d*e)*x^2 - d^2)) - (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*
c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^
2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(a*c*e*x^2 + 2*a*c*d + (b*c*e
*x^2 + 2*b*c*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), -1/2*(
(b*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2
- 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)
) - (b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(
c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
 + d)*sqrt(-d) + 8*d^2)/x^4) - 2*(a*c*e*x^2 + 2*a*c*d + (b*c*e*x^2 + 2*b*c*
d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), -1/2*(2*(b*c*e*x^2
+ b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt
(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (b*e*x^2 +
 b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(a*c
*e*x^2 + 2*a*c*d + (b*c*e*x^2 + 2*b*c*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c*e
^3*x^2 + c*d*e^2)]
```

Sympy [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3(a+b\,\mathrm{acsc}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**3*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.149 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}$$

```
[Out] b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/d^(1/2)/(c^2*x^2)
^(1/2)+(-a-b*arccsc(c*x))/e/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5345, 457, 95, 210}

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}$$

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcCsc[c*x])/(e*Sqrt[d + e*x^2])) + (b*c*x*ArcTan[Sqrt[d + e*x^2]/
(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[d]*e*Sqrt[c^2*x^2])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{1}{x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{e\sqrt{c^2x^2}} \\
&= -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{2e\sqrt{c^2x^2}} \\
&= -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{(bcx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} \\
&= -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}} + \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{b\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)-2cx(a+b\csc^{-1}(cx))}{2cex\sqrt{d+ex^2}}$$

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))]
- 2*c*x*(a + b*ArcCsc[c*x]))/(2*c*e*x*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x(a+b\,\text{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.58

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \left[-\frac{(bex^2+bd)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de+e^2)x^4-8\,(c^2d^2-de)x^2+4\,\sqrt{c^2x^2-1}((c^2d-e)x^2-2\,d)\sqrt{ex^2+}}{x^4}\right.}{4\,(de^2x^2+d^2e)}\right.$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*e*x^2 + b*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*(b*d*arccsc(c*x) + a*d))/(d
*e^2*x^2 + d^2*e), 1/2*((b*e*x^2 + b*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 -
1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2
- d*e)*x^2 - d^2)) - 2*sqrt(e*x^2 + d)*(b*d*arccsc(c*x) + a*d))/(d*e^2*x^2
+ d^2*e)]
```

Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x(a+b\operatorname{acsc}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] -(sqrt(e*x^2 + d)*c^2*e*integrate(x*e^(-1/2*log(e*x^2 + d) + 1/2*log(c*x +
1) + 1/2*log(c*x - 1))/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c
*x - 1)) - e), x) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(sqrt(e*x^2
+ d)*e) - a/(sqrt(e*x^2 + d)*e)
```

Giac [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x\left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.150 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\operatorname{arccsc}\left(cx\right)}{x\left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\operatorname{arccsc}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x),
 x)
```

Sympy [N/A]

Not integrable

Time = 75.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\operatorname{acsc}(cx)}{x\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(3/2),x)
[Out] Integral((a + b*acsc(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\operatorname{arccsc}(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

3.151 $\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 11.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 5.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\ \mathrm{arccsc}\left(cx\right)}{x^3\left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
[Out] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^3\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.152 $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 14.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\,\mathrm{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 +
 d^2), x)
```

Sympy [N/A]

Not integrable

Time = 122.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(a+b\,\mathrm{acsc}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a + b\csc^{-1}(cx))}{(d + ex^2)^{3/2}}\, dx = \int \frac{x^4\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^{3/2}}\, dx$$

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.153 $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{x^2(a + b\csc^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx$$

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a + b\,\mathrm{arccsc}\,(cx))}{(e\,x^2 + d)^{\frac{3}{2}}}dx$$

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
[Out] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^2}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 +
 d^2), x)
```

Sympy [N/A]

Not integrable

Time = 32.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^2(a+b\,\mathrm{acsc}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.154 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

```
[Out] x*(a+b*arccsc(c*x))/d/(e*x^2+d)^(1/2)+b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(
-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*
x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {197, 5337, 12, 432, 430}

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCsc[c*x]))/(d*Sqrt[d + e*x^2]) + (b*x*Sqrt[1 - c^2*x^2]*Sqrt[1
 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-
1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 5337

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{(bcx)\int \frac{1}{d\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{(bcx)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}}\\
&= \frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{\left(bcx\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{d+ex^2}}\\
&= \frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{\left(bcx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\\
&= \frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}} + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\left(-c+c^3x^2\right)\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCsc[c*x]))/(d*Sqrt[d + e*x^2]) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*
Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])
/(d*(-c + c^3*x^2)*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\mathrm{arccsc}\,(cx)}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.70

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = -\frac{(bex^2+bd)\sqrt{-d}F(\arcsin(cx)\mid -\frac{e}{c^2d}) - (bcdx\,\mathrm{arccsc}\,(cx)+acdx)\sqrt{ex^2+d}}{cd^2ex^2+cd^3}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -((b*e*x^2 + b*d)*sqrt(-d)*elliptic_f(arcsin(c*x), -e/(c^2*d)) - (b*c*d*x*a
rccsc(c*x) + a*c*d*x)*sqrt(e*x^2 + d))/(c*d^2*e*x^2 + c*d^3)
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\operatorname{acsc}(cx)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{b\operatorname{arccsc}(cx)+a}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

3.155 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 275

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx = &-\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}\\&-\frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\\&-\frac{b(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

```
[Out] (-a-b*arccsc(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arccsc(c*x))/d^2/(e*x^2+d
)^(1/2)-b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)+b*c^2*x*E
llipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*
x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-b*(c^2*d+2*e)*x*EllipticF(c*
x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2
)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules

used = {277, 197, 5347, 12, 597, 538, 438, 437, 435, 432, 430}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = -\frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{bx\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}} - \frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] -((b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[c^2*x^2])) - (a + b*Ar
cCsc[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcCsc[c*x]))/(d^2*Sqrt[d
+ e*x^2]) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x
], -(e/(c^2*d))])/(d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]
) - (b*(c^2*d + 2*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcS
in[c*x], -(e/(c^2*d))])/(d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
```

```
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-d-2ex^2}{d^2x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-d-2ex^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} \\
&\quad - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-2de+c^2dex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^3\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} \\
&\quad + \frac{(bc^3x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{d^2\sqrt{c^2x^2}} - \frac{(bc(c^2d+2e)\,x)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{c^2x^2}} \\
&= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} \\
&\quad + \frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} - \frac{\left(bc(c^2d+2e)\,x\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d^2\sqrt{c^2x^2}\sqrt{d+ex^2}} \\
&= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} \\
&\quad - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&\quad - \frac{\left(bc(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}}$$
$$+ \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$- \frac{b(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.96 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.77

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx = \frac{-bc\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)-a(d+2ex^2)-b(d+2ex^2)\csc^{-1}(cx)}{d^2x\sqrt{d+ex^2}}$$
$$+ \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\left|-\frac{e}{c^2d}\right.\right)-(c^2d+2e)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right),-\frac{e}{c^2d}\right)\right)}{\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (-(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)) - a*(d + 2*e*x^2) - b*(d + 2*e*
x^2)*ArcCsc[c*x])/(d^2*x*Sqrt[d + e*x^2]) + (I*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*
Sqrt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]
 - (c^2*d + 2*e)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-
c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\text{arccsc}\,(cx)}{x^2\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx =$$
$$-\frac{\left(2\,acdex^2+acd^2+\left(2\,bcdex^2+bcd^2\right)\operatorname{arccsc}\left(cx\right)+\left(bcdex^2+bcd^2\right)\sqrt{c^2x^2-1}\right)\sqrt{ex^2+d}+\left(\left(bc^4dex^3+bc^4c\right.\right.}{cd^3ex^3+cd^4x}$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -((2*a*c*d*e*x^2 + a*c*d^2 + (2*b*c*d*e*x^2 + b*c*d^2)*arccsc(c*x) + (b*c*d
*e*x^2 + b*c*d^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) + ((b*c^4*d*e*x^3 + b*
c^4*d^2*x)*elliptic_e(arcsin(c*x), -e/(c^2*d)) - ((b*c^4*d*e + 2*b*e^2)*x^3
 + (b*c^4*d^2 + 2*b*d*e)*x)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/
(c*d^3*e*x^3 + c*d^4*x)
```

Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\operatorname{acsc}\left(cx\right)}{x^2\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + b\csc^{-1}(cx)}{x^2\left(d + ex^2\right)^{3/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{(ex^2 + d)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{x^2\left(d + ex^2\right)^{3/2}}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2 + d\right)^{3/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.156 $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 243

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{c^2x^2}}$$

```
[Out] -1/3*d^2*(a+b*arccsc(c*x))/e^3/(e*x^2+d)^(3/2)+b*x*arctanh(e^(1/2)*(c^2*x^2
-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(5/2)/(c^2*x^2)^(1/2)-8/3*b*c*x*arctan((e*x^
2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e^3/(c^2*x^2)^(1/2)+2*d*(a+b*
arccsc(c*x))/e^3/(e*x^2+d)^(1/2)+1/3*b*c*d*x*(c^2*x^2-1)^(1/2)/e^2/(c^2*d+e
)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules

used = {272, 45, 5347, 12, 1628, 163, 65, 223, 212, 95, 210}

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{c^2x^2}} + \frac{bcdx\sqrt{c^2x^2-1}}{3e^2\sqrt{c^2x^2}\,(c^2d+e)\sqrt{d+ex^2}}$$

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*d*x*Sqrt[-1 + c^2*x^2])/(3*e^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^
2]) - (d^2*(a + b*ArcCsc[c*x]))/(3*e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b*Arc
Csc[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^
3 - (8*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/
(3*e^3*Sqrt[c^2*x^2]) + (b*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d
 + e*x^2])])/(e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
```

```
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d^2(a+b\csc^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} \\
&\quad+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}+\frac{(bcx)\int\frac{8d^2+12dex^2+3e^2x^4}{3e^3x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d^2(a+b\csc^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} \\
&\quad+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}+\frac{(bcx)\int\frac{8d^2+12dex^2+3e^2x^4}{x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3e^3\sqrt{c^2x^2}} \\
&= -\frac{d^2(a+b\csc^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} \\
&\quad+\frac{(bcx)\text{Subst}\left(\int\frac{8d^2+12dex+3e^2x^2}{x\sqrt{-1+c^2x}(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e^3\sqrt{c^2x^2}} \\
&= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} \\
&\quad+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}-\frac{(bcx)\text{Subst}\left(\int\frac{-4d^2(c^2d+e)-\frac{3}{2}de(c^2d+e)x}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3de^3\left(c^2d+e\right)\sqrt{c^2x^2}} \\
&= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} \\
&\quad+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}+\frac{(4bcdx)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3e^3\sqrt{c^2x^2}} \\
&\quad+\frac{(bcx)\text{Subst}\left(\int\frac{1}{\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e^2\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} + \frac{(8bcdx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{(bx)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{ce^2\sqrt{c^2x^2}}$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{(bx)\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx, x, \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{ce^2\sqrt{c^2x^2}}$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.47 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.99

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{\frac{2bcde\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)}{c^2d+e} + 2a(8d^2+12dex^2+3e^2x^4) + \frac{bc(d+ex^2)\left(-\frac{8d\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2}\cdots\right)}{c^2}\cdots\right.}{\cdots}}{6e\cdots}$$

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] ((2*b*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d + e) + 2*a*(8*d^2 +
 12*d*e*x^2 + 3*e^2*x^4) + (b*c*(d + e*x^2)*((-8*d*Sqrt[1 + d/(e*x^2)]*Appe
llF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/c^2 - (3*e*Sqrt[1 - 1/(c^2
*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/
d)])/Sqrt[1 - c^2*x^2]))/x + 2*b*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*ArcCsc[c*
x])/(6*e^3*(d + e*x^2)^(3/2))
```

Maple [F]

$$\int \frac{x^5(a+b\ \mathrm{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(205) = 410$.

Time = 0.50 (sec) , antiderivative size = 2119, normalized size of antiderivative = 8.72

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e
+ b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*
e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x
^2 + d)*sqrt(e) + e^2) + 8*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)
*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e
+ e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 -
 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(8*a*c^3*d^3 + 8*a*c*d^2*e
 + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 + (8*b*
c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b
*c*d*e^2)*x^2)*arccsc(c*x) + (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))
*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c
^3*d^2*e^4 + c*d*e^5)*x^2), -1/12*(16*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2
 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt
(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4
+ (c^2*d^2 - d*e)*x^2 - d^2)) - 3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 +
b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^
2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*s
qrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(8*a*c^3*d^3 + 8*a*c*d^
2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 + (8
*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e
+ b*c*d*e^2)*x^2)*arccsc(c*x) + (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 -
1))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2
*(c^3*d^2*e^4 + c*d*e^5)*x^2), -1/6*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x
^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*
x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 -
c*d*e + (c^3*d*e - c*e^2)*x^2)) - 4*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 +
```

```
 b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6
*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d -
 e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - 2*(8*a*c^3*d^3 + 8*
a*c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^
2 + (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*
d^2*e + b*c*d*e^2)*x^2)*arccsc(c*x) + (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*
x^2 - 1))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x
^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), -1/6*(8*(b*c^3*d^3 + b*c*d^2*e + (b*c^
3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*arctan(-1
/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d
*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)
*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*
e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4
- c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3
*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 + (8*b*c^3*d^3 + 8
*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x
^2)*arccsc(c*x) + (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4
+ c*d*e^5)*x^2)]
```

Sympy [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^5(a+b\operatorname{acsc}(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**5*(a + b*acsc(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.157 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcx\sqrt{-1+c^2x^2}}{3e(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\csc^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3\sqrt{d}e^2\sqrt{c^2x^2}}$$

```
[Out] 1/3*d*(a+b*arccsc(c*x))/e^2/(e*x^2+d)^(3/2)+2/3*b*c*x*arctan((e*x^2+d)^(1/2
)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/d^(1/2)/(c^2*x^2)^(1/2)+(-a-b*arccsc(c*x))
/e^2/(e*x^2+d)^(1/2)-1/3*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(c^2*x^2)^(1/2
)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {272, 45, 5347, 12, 587, 157, 95, 210}

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{d(a+b\csc^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} + \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3\sqrt{d}e^2\sqrt{c^2x^2}} - \frac{bcx\sqrt{c^2x^2-1}}{3e\sqrt{c^2x^2}(c^2d+e)\sqrt{d+ex^2}}$$

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*x*Sqrt[-1 + c^2*x^2])/(e*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2
]) + (d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcCsc[c*x]
```

```
)/(e^2*Sqrt[d + e*x^2]) + (2*b*c*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1
+ c^2*x^2])])/(3*Sqrt[d]*e^2*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{(bcx)\int\frac{-2d-3ex^2}{3e^2x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{(bcx)\int\frac{-2d-3ex^2}{x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3e^2\sqrt{c^2x^2}}\\
&= \frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{(bcx)\text{Subst}\left(\int\frac{-2d-3ex}{x\sqrt{-1+c^2x}(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e^2\sqrt{c^2x^2}}\\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{3e\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}}\\
&\quad-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}-\frac{(bcx)\text{Subst}\left(\int\frac{d(c^2d+e)}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3de^2\left(c^2d+e\right)\sqrt{c^2x^2}}\\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{3e\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}}\\
&\quad-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}-\frac{(bcx)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{3e^2\sqrt{c^2x^2}}\\
&= -\frac{bcx\sqrt{-1+c^2x^2}}{3e\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}}\\
&\quad-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}-\frac{(2bcx)\text{Subst}\left(\int\frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}}
\end{aligned}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3e\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3\sqrt{d}e^2\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{b(c^2d+e)\sqrt{1+\frac{d}{ex^2}}(d+ex^2)\,\mathrm{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right) - cx\left(bce\sqrt{1-\frac{1}{c^2x^2}}\right.}{3ce^2\left(c^2d+e\right)x\,(}$$

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*(c^2*d + e)*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(
c^2*x^2), -(d/(e*x^2))] - c*x*(b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) +
a*(c^2*d + e)*(2*d + 3*e*x^2) + b*(c^2*d + e)*(2*d + 3*e*x^2)*ArcCsc[c*x]))
/(3*c*e^2*(c^2*d + e)*x*(d + e*x^2)^(3/2))
```

Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arccsc}\,(cx))}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(137) = 274.

Time = 0.38 (sec) , antiderivative size = 663, normalized size of antiderivative = 4.07

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^{5/2}}\,dx = \left[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\left(bc^2d^2e+bde^2\right)x^2)\sqrt{-d}\log\left(\frac{(c^4d^2-6c^2de}{}\right.}{}\right.$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt
```

```
(-d) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)
*x^2 + (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arccsc(c*x
) + (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^4*e^
2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2), 1
/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d
*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sq
rt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (2*a*c^2
*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 + (2*b*c^2*d^3 + 2*b*d^2*e
 + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arccsc(c*x) + (b*d*e^2*x^2 + b*d^2*e)*sqr
t(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*
e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

Sympy [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^3(a+b\operatorname{acsc}(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx) + a)x^3}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x^3\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^{5/2}}\,dx$$

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.158 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{bcx\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{3e\,(d+ex^2)^{3/2}} + \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}$$

```
[Out] 1/3*(-a-b*arccsc(c*x))/e/(e*x^2+d)^(3/2)+1/3*b*c*x*arctan((e*x^2+d)^(1/2)/d
^(1/2)/(c^2*x^2-1)^(1/2))/d^(3/2)/e/(c^2*x^2)^(1/2)+1/3*b*c*x*(c^2*x^2-1)^(
1/2)/d/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5345, 457, 98, 95, 210}

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{a+b\csc^{-1}(cx)}{3e\,(d+ex^2)^{3/2}} + \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{c^2x^2}} + \frac{bcx\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}\,(c^2d+e)\sqrt{d+ex^2}}$$

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x*Sqrt[-1 + c^2*x^2])/(3*d*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2])
- (a + b*ArcCsc[c*x])/(3*e*(d + e*x^2)^(3/2)) + (b*c*x*ArcTan[Sqrt[d + e*x^
2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*d^(3/2)*e*Sqrt[c^2*x^2])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\csc^{-1}(cx)}{3e\,(d+ex^2)^{3/2}}-\frac{(bcx)\int\frac{1}{x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3e\sqrt{c^2x^2}}\\ &= -\frac{a+b\csc^{-1}(cx)}{3e\,(d+ex^2)^{3/2}}-\frac{(bcx)\mathrm{Subst}\left(\int\frac{1}{x\sqrt{-1+c^2x}(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e\sqrt{c^2x^2}}\end{aligned}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx, x, x^2\right)}{6de\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{3de\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} + \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{x(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{-\frac{2a}{e} + \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2)}{d(c^2d+e)} + \frac{b\sqrt{1+\frac{d}{ex^2}}\,(d+ex^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cdex} - \frac{2b\csc^{-1}(cx)}{e}}{6\left(d+ex^2\right)^{3/2}}$$

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] ((-2*a)/e + (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2))/(d*(c^2*d + e)) + (
b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d
/(e*x^2))])/(c*d*e*x) - (2*b*ArcCsc[c*x])/e)/(6*(d + e*x^2)^(3/2))
```

Maple [F]

$$\int \frac{x(a+b\,\mathrm{arccsc}\,(cx))}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(114) = 228.

Time = 0.37 (sec) , antiderivative size = 573, normalized size of antiderivative = 4.15

$$\int \frac{x(a+b\csc^{-1}(cx))}{\left(d+ex^2\right)^{5/2}}\,dx = \left[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de+e}{}\right.}{}\right.$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 -
 d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqr
t(-d) + 8*d^2)/x^4) + 4*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arccsc
(c*x) - (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^
5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2
), 1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d
)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 2*(a
*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arccsc(c*x) - (b*d*e^2*x^2 + b*d
^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e
^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x(a + b\operatorname{acsc}(cx))}{(d + ex^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arccsc}(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.159 $\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 16.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2
*e*x^3 + d^3*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

3.160 $\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\text{integral}=\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 18.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsc}\,(cx)}{x^3\,(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2
*e*x^5 + d^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^3\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.161 $\int \frac{x^6\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \operatorname{Int}\left(\frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

```
[In] Int[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 16.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

```
[In] Integrate[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^6(a+b\,\operatorname{arccsc}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\operatorname{arccsc}(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((b*x^6*arccsc(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4
 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**6*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^6\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

```
[In] int((x^6*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^6*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.162 $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Int}\left(\frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\,\mathrm{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4
 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^4\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.163 $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 276

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{3de(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

```
[Out] 1/3*x^3*(a+b*arccsc(c*x))/d/(e*x^2+d)^(3/2)+1/3*b*c*x^2*(c^2*x^2-1)^(1/2)/d
/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)-1/3*b*c^2*x*EllipticE(c*x,(-e/c^
2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/e/(c^2*d+e)/(c^2*x^2)^(1/2
)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/3*b*x*EllipticF(c*x,(-e/c^2/d)^(1/2
))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/e/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/
2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules

used = {270, 5347, 12, 482, 434, 438, 437, 435, 432, 430}

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{x^3(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\,(c^2d+e)\sqrt{\frac{ex^2}{d}+1}} + \frac{bcx^2\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}\,(c^2d+e)\sqrt{d+ex^2}}$$

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x^2*Sqrt[-1 + c^2*x^2])/(3*d*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2]
) + (x^3*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c^2*x*Sqrt[1 - c
^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d*e*(c^2*d
 + e)*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (b*x*Sqrt[1 -
 c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d*e*
Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
```

```
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{x^3(a + b\csc^{-1}(cx))}{3d\,(d + ex^2)^{3/2}} + \frac{(bcx)\int \frac{x^2}{3d\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{(bcx)\int \frac{x^2}{\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3d\sqrt{c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{(bcx)\int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\,dx}{3d(c^2d+e)\sqrt{c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{(bcx)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3de\sqrt{c^2x^2}} - \frac{(bc^3x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{3de(c^2d+e)\sqrt{c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{3de(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} + \frac{\left(bcx\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3de\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{3de(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(bcx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3de\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{3de(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.67

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{x^2\left(a(c^2d+e)\,x+bc\sqrt{1-\frac{1}{c^2x^2}}(d+ex^2)+b(c^2d+e)\,x\csc^{-1}(cx)\right)}{3d\,(c^2d+e)\,(d+ex^2)^{3/2}} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)|-\frac{c^2d}{e}\right)}{3d\sqrt{-\frac{e}{d}}\,(c^2d+e)\,\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (x^2*(a*(c^2*d + e)*x + b*c*Sqrt[1 - 1/(c^2*x^2)]*(d + e*x^2) + b*(c^2*d +
e)*x*ArcCsc[c*x]))/(3*d*(c^2*d + e)*(d + e*x^2)^(3/2)) - (b*c*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[-(e/d)]*x], -((c^2*d)/
e)])/(3*d*Sqrt[-(e/d)]*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{x^2(a+b\ \mathrm{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.03

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{((bc^3d^2e+bcde^2)x^3\ \mathrm{arccsc}\,(cx)+(ac^3d^2e+acde^2)x^3+(bcde^2x^3+bcd^2ex)\sqrt{c^2x^2-1}}{}$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((b*c^3*d^2*e + b*c*d*e^2)*x^3*arccsc(c*x) + (a*c^3*d^2*e + a*c*d*e^2)
*x^3 + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) + (
(b*c^4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x), -
e/(c^2*d)) - (b*c^4*d^3 + (b*c^4*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^4*d^
2*e + b*d*e^2)*x^2)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c^3*d^5
*e + c*d^4*e^2 + (c^3*d^3*e^3 + c*d^2*e^4)*x^4 + 2*(c^3*d^4*e^2 + c*d^3*e^3
)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x^
2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt
(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.164 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 296

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{2bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

```
[Out] 1/3*x*(a+b*arccsc(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arccsc(c*x))/d^2/(e*x^
2+d)^(1/2)-1/3*b*c*e*x^2*(c^2*x^2-1)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(e
*x^2+d)^(1/2)+1/3*b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2
)*(e*x^2+d)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/
d)^(1/2)+2/3*b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^
2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules

used = {198, 197, 5337, 12, 541, 538, 438, 437, 435, 432, 430}

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{x(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx), -\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\,(c^2d+e)\sqrt{\frac{ex^2}{d}+1}} - \frac{bcex^2\sqrt{c^2x^2-1}}{3d^2\sqrt{c^2x^2}\,(c^2d+e)\sqrt{d+ex^2}}$$

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*e*x^2*Sqrt[-1 + c^2*x^2])/(d^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d +
 e*x^2]) + (x*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*Ar
cCsc[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e
*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*(c^2*d + e)*Sqrt[c^2*x^2
]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (2*b*x*Sqrt[1 - c^2*x^2]*Sqrt[1
 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*Sqrt[c^2*x^2]*Sq
rt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 5337

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
```

```
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\text{integral} = \frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{3d+2ex^2}{3d^2\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{3d+2ex^2}{\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}\,dx}{3d^2\sqrt{c^2x^2}}$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{d(3c^2d+2e)+c^2dex^2}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^3(c^2d+e)\sqrt{c^2x^2}}$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{(2bcx)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{3d^2\sqrt{c^2x^2}} + \frac{(bc^3x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\,dx}{3d^2(c^2d+e)\sqrt{c^2x^2}}$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\,dx}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} + \frac{\left(2bcx\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d^2\sqrt{c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)\int \frac{\sqrt{1+\frac{ex^2}{d}}}{\sqrt{1-c^2x^2}}\,dx}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{\left(2bcx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\csc^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}}$$
$$+ \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,|-\frac{e}{c^2d}\right)}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}$$
$$+ \frac{2bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.84

$$\int \frac{a+b\csc^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(-bce\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)+a(c^2d+e)\left(3d+2ex^2\right)+b(c^2d+e)\left(3d+2ex^2\right)\csc^{-1}\right.}{3d^2\left(c^2d+e\right)\left(d+ex^2\right)^{3/2}}$$
$$+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)+2(c^2d+e)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right),-\frac{e}{c^2d}\right)\right)}{3\sqrt{-c^2}d^2\left(c^2d+e\right)\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (x*(-(b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)) + a*(c^2*d + e)*(3*d + 2*e
*x^2) + b*(c^2*d + e)*(3*d + 2*e*x^2)*ArcCsc[c*x]))/(3*d^2*(c^2*d + e)*(d +
 e*x^2)^(3/2)) + ((I/3)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^
2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(c^2*d + e)*Ellipt
icF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*(c^2*d + e)*Sq
rt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]

$$\int \frac{a+b\,\text{arccsc}\,(cx)}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.18

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex^2)^{5/2}}\,dx = \frac{(2\,(ac^3d^2e + acde^2)x^3 + 3\,(ac^3d^3 + acd^2e)x + (2\,(bc^3d^2e + bcde^2)x^3 + 3\,(bc^3d^3 + bcd^2$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*(a*c^3*d^2*e + a*c*d*e^2)*x^3 + 3*(a*c^3*d^3 + a*c*d^2*e)*x + (2*(b
*c^3*d^2*e + b*c*d*e^2)*x^3 + 3*(b*c^3*d^3 + b*c*d^2*e)*x)*arccsc(c*x) - (b
*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((b*c^4*d*
e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x), -e/(c^2*d)
) - (((b*c^4 - 3*b*c^2)*d*e^2 - 2*b*e^3)*x^4 + (b*c^4 - 3*b*c^2)*d^3 - 2*b*
d^2*e + 2*((b*c^4 - 3*b*c^2)*d^2*e - 2*b*d*e^2)*x^2)*elliptic_f(arcsin(c*x)
, -e/(c^2*d)))*sqrt(-d))/(c^3*d^6 + c*d^5*e + (c^3*d^4*e^2 + c*d^3*e^3)*x^4
 + 2*(c^3*d^5*e + c*d^4*e^2)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex^2)^{5/2}}\,dx = \int \frac{b\operatorname{arccsc}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*
x^2 + d)), x)
```

Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^{5/2}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(5/2),x)
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

3.165 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 585

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\,dx$$

$$= \frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right) x}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

$$+\frac{be^2\left(e(5+m)^2+3c^2d(42+13m+m^2)\right)x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

$$+\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}$$

$$+\frac{3d^2e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)}$$

$$+\frac{3de^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\csc^{-1}(cx)\right)}{f^7(7+m)}$$

$$+\frac{b\left(\frac{c^6d^3(2+m)(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{(3+m)(5+m)(7+m)}\right)x(f}{c^5f(1+m)(2+m)(4+m)(6+m)\sqrt{c^2x^2}\sqrt{-1+c}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arccsc(c
*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arccsc(c*x))/f^5/(5+m)+e^3*(f*x)^(7
+m)*(a+b*arccsc(c*x))/f^7/(7+m)+b*(c^6*d^3*(2+m)*(4+m)*(6+m)/(1+m)+e*(1+m)*
(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+1
79*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2
+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^5/f/(1+m)/(2+m)/(4+m)/(6+
m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m
)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*x*(f*x)^(1+m)*(
c^2*x^2-1)^(1/2)/c^5/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+71*m+105)/(c^2*x^2)^(1
```

```
/2)+b*e^2*(e*(5+m)^2+3*c^2*d*(m^2+13*m+42))*x*(f*x)^(3+m)*(c^2*x^2-1)^(1/2)
/c^3/f^3/(4+m)/(5+m)/(6+m)/(7+m)/(c^2*x^2)^(1/2)+b*e^3*x*(f*x)^(5+m)*(c^2*x
^2-1)^(1/2)/c/f^5/(6+m)/(7+m)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {276, 5347, 1823, 1281, 470, 372, 371}

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{d^3(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}$$

$$+\frac{3d^2e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}$$

$$+\frac{e^3(fx)^{m+7}\left(a+b\csc^{-1}(cx)\right)}{f^7(m+7)}+\frac{be^3x\sqrt{c^2x^2-1}(fx)^{m+5}}{cf^5(m+6)(m+7)\sqrt{c^2x^2}}$$

$$+\frac{be^2x\sqrt{c^2x^2-1}(fx)^{m+3}\left(3c^2d(m^2+13m+42)+e(m+5)^2\right)}{c^3f^3(m+4)(m+5)(m+6)(m+7)\sqrt{c^2x^2}}$$

$$+\frac{bcx\sqrt{1-c^2x^2}(fx)^{m+1}\left(\frac{e\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^6(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)}+\frac{d^3}{(m+1)^2}\right.}{f\sqrt{c^2x^2}\sqrt{c^2x^2-1}}$$

$$+\frac{bex\sqrt{c^2x^2-1}(fx)^{m+1}\left(3c^4d^2(m^4+22m^3+179m^2+638m+840)+3c^2de(m+3)^2\left(m^2+13m+42\right)\right.}{c^5f(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)\sqrt{c^2x^2}}$$

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^
4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*x*(f*x)^(1 + m)*Sqrt[-1 + c^2
*x^2])/(c^5*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^2]
) + (b*e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*x*(f*x)^(3 + m)*Sqrt[-
1 + c^2*x^2])/(c^3*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^2]) + (b*
e^3*x*(f*x)^(5 + m)*Sqrt[-1 + c^2*x^2])/(c*f^5*(6 + m)*(7 + m)*Sqrt[c^2*x^2
]) + (d^3*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(
3 + m)*(a + b*ArcCsc[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*A
rcCsc[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCsc[c*x]))/(f^7*(
7 + m)) + (b*c*(d^3/(1 + m)^2 + (e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 +
 m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))
)/(c^6*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)))*x*(f*x)^(1 + m)*Sq
rt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*S
qrt[c^2*x^2]*Sqrt[-1 + c^2*x^2])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d^3(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}+\frac{3d^2e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\csc^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad+\frac{(bcx)\int\frac{(fx)^m\left(\frac{d^3}{1+m}+\frac{3d^2ex^2}{3+m}+\frac{3de^2x^4}{5+m}+\frac{e^3x^6}{7+m}\right)}{\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= \frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} \\
&\quad+\frac{3d^2e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\csc^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad+\frac{(bx)\int\frac{(fx)^m\left(\frac{c^2d^3(6+m)}{1+m}+\frac{3c^2d^2e(6+m)x^2}{3+m}+\frac{e^2\left(e(5+m)^2+3c^2d\left(42+13m+m^2\right)\right)x^4}{(5+m)(7+m)}\right)}{\sqrt{-1+c^2x^2}}\,dx}{c(6+m)\sqrt{c^2x^2}} \\
&= \frac{be^2\left(e(5+m)^2+3c^2d\left(42+13m+m^2\right)\right)x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}} \\
&\quad+\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} \\
&\quad+\frac{3d^2e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{3de^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\csc^{-1}(cx)\right)}{f^7(7+m)} \\
&\quad+\frac{(bx)\int\frac{(fx)^m\left(\frac{c^4d^3(4+m)(6+m)}{1+m}+\frac{e\left(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2\left(840+638m+179m^2+22m^3+m^4\right)\right)x^2}{(3+m)(5+m)(7+m)}\right)}{\sqrt{-1+c^2x^2}}\,dx}{c^3(4+m)(6+m)\sqrt{c^2x^2}}
\end{aligned}$$

$$= \frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$
$$+\frac{be^2\left(e(5+m)^2+3c^2d(42+13m+m^2)\right)x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$
$$+\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}$$
$$+\frac{3d^2e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)}$$
$$+\frac{3de^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\csc^{-1}(cx)\right)}{f^7(7+m)}$$
$$+\frac{\left(b\left(\frac{c^4d^3(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{c^2(2+m)(3+m)(5+m)(7+m)}\right.\right.}{c^3(4+m)(6+m)\sqrt{c^2x^2}}$$

$$= \frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$
$$+\frac{be^2\left(e(5+m)^2+3c^2d(42+13m+m^2)\right)x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$
$$+\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}$$
$$+\frac{3d^2e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)}$$
$$+\frac{3de^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\csc^{-1}(cx)\right)}{f^7(7+m)}$$
$$+\frac{\left(b\left(\frac{c^4d^3(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{c^2(2+m)(3+m)(5+m)(7+m)}\right.\right.}{c^3(4+m)(6+m)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

$$= \frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2(840+638m+179m^2+22m^3\right.}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$
$$+\frac{be^2(e(5+m)^2+3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$
$$+\frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}}+\frac{d^3(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}$$
$$+\frac{3d^2e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)}$$
$$+\frac{3de^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\csc^{-1}(cx)\right)}{f^7(7+m)}$$
$$+\frac{b\left(\frac{c^6d^3}{(1+m)^2}+\frac{e\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)}\right)x(fx)^{1+m}\sqrt{}}{c^5f\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.69

$$\int (fx)^m\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= x(fx)^m\left(\frac{ad^3}{1+m}+\frac{3ad^2ex^2}{3+m}+\frac{3ade^2x^4}{5+m}+\frac{ae^3x^6}{7+m}+\frac{bd^3\csc^{-1}(cx)}{1+m}+\frac{3bd^2ex^2\csc^{-1}(cx)}{3+m}\right.$$
$$+\frac{3bde^2x^4\csc^{-1}(cx)}{5+m}+\frac{be^3x^6\csc^{-1}(cx)}{7+m}$$
$$-\frac{bcd^3\sqrt{1-\frac{1}{c^2x^2}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}}$$
$$-\frac{3bcd^2e\sqrt{1-\frac{1}{c^2x^2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2\sqrt{1-c^2x^2}}$$
$$-\frac{3bcde^2\sqrt{1-\frac{1}{c^2x^2}}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2\sqrt{1-c^2x^2}}$$
$$\left.-\frac{bce^3\sqrt{1-\frac{1}{c^2x^2}}x^7\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7+m}{2},\frac{9+m}{2},c^2x^2\right)}{(7+m)^2\sqrt{1-c^2x^2}}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] x*(f*x)^m*((a*d^3)/(1 + m) + (3*a*d^2*e*x^2)/(3 + m) + (3*a*d*e^2*x^4)/(5 +
 m) + (a*e^3*x^6)/(7 + m) + (b*d^3*ArcCsc[c*x])/(1 + m) + (3*b*d^2*e*x^2*Ar
```

```
cCsc[c*x])/(3 + m) + (3*b*d*e^2*x^4*ArcCsc[c*x])/(5 + m) + (b*e^3*x^6*ArcCs
c[c*x])/(7 + m) - (b*c*d^3*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, (
1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) - (3*b*c*d^2*e
*Sqrt[1 - 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2
*x^2])/((3 + m)^2*Sqrt[1 - c^2*x^2]) - (3*b*c*d*e^2*Sqrt[1 - 1/(c^2*x^2)]*x
^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^2*Sqrt[1
 - c^2*x^2]) - (b*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x^7*Hypergeometric2F1[1/2, (7
 + m)/2, (9 + m)/2, c^2*x^2])/((7 + m)^2*Sqrt[1 - c^2*x^2]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^3 \left(a+b\,\mathrm{arccsc}\left(cx\right)\right)dx$$

```
[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^3 (b\,\mathrm{arccsc}\left(cx\right)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccsc(c*x))*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\,dx=\text{Timed out}$$

```
[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*acsc(c*x)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (fx)^m (d+ex^2)^3 (a+b\csc^{-1}(cx)) dx = \int (ex^2+d)^3 (b\,\mathrm{arccsc}(cx)+a)(fx)^m dx$$

[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="maxima")

[Out] a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + (((b*e^3*f^m*m^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 9*b*e^3*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 23*b*e^3*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 15*b*e^3*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^7 + 3*(b*d*e^2*f^m*m^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 11*b*d*e^2*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 31*b*d*e^2*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 21*b*d*e^2*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^5 + 3*(b*d^2*e*f^m*m^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 13*b*d^2*e*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 47*b*d^2*e*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 35*b*d^2*e*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^3 + (b*d^3*f^m*m^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 15*b*d^3*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 71*b*d^3*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 105*b*d^3*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x)*x^m + (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*integrate(-(b*d^3*f^m*m^3 + 15*b*d^3*f^m*m^2 + (b*e^3*f^m*m^3 + 9*b*e^3*f^m*m^2 + 23*b*e^3*f^m*m + 15*b*e^3*f^m)*x^6 + 71*b*d^3*f^m*m + 105*b*d^3*f^m + 3*(b*d*e^2*f^m*m^3 + 11*b*d*e^2*f^m*m^2 + 31*b*d*e^2*f^m*m + 21*b*d*e^2*f^m)*x^4 + 3*(b*d^2*e*f^m*m^3 + 13*b*d^2*e*f^m*m^2 + 47*b*d^2*e*f^m*m + 35*b*d^2*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(m^4 + 16*m^3 - (c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^2 + 86*m^2 + 176*m + 105), x))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)

Giac [F]

$$\int (fx)^m (d+ex^2)^3 (a+b\csc^{-1}(cx)) dx = \int (ex^2+d)^3 (b\,\mathrm{arccsc}(cx)+a)(fx)^m dx$$

[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^3*(b*arccsc(c*x) + a)*(f*x)^m, x)

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\,dx = \int (f\,x)^m \left(e\,x^2+d\right)^3 \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((f*x)^m*(d + e*x^2)^3*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^3*(a + b*asin(1/(c*x))), x)
```

3.166 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 371

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right) \, dx$$
$$= \frac{be(e(3+m)^2+2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(3+m)(4+m)(5+m)\sqrt{c^2x^2}}$$
$$+ \frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}$$
$$+ \frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)}$$
$$+ \frac{b(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right))\,x(fx)^{1+m}\sqrt{1-}}{c^3f(1+m)^2(2+m)(3+m)(4+m)(5+m)\sqrt{c^2x^2}\sqrt{-1+c}}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arccsc(c*x
))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arccsc(c*x))/f^5/(5+m)+b*(c^4*d^2*(2+m)*(
3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20)))*x*(f*x)^(1+m)*
hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^3/f/(1
+m)^2/(2+m)/(3+m)/(4+m)/(5+m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*(e*(3+m
)^2+2*c^2*d*(m^2+9*m+20))*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c^3/f/(4+m)/(5+m)
/(m^2+5*m+6)/(c^2*x^2)^(1/2)+b*e^2*x*(f*x)^(3+m)*(c^2*x^2-1)^(1/2)/c/f^3/(4
+m)/(5+m)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {276, 5347, 12, 1281, 470, 372, 371}

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{d^2(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)} + \frac{2de(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)} + \frac{e^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)} + \frac{be^2x\sqrt{c^2x^2-1}(fx)^{m+3}}{cf^3(m+4)(m+5)\sqrt{c^2x^2}} + \frac{bcx\sqrt{1-c^2x^2}(fx)^{m+1}\left(\frac{e(2c^2d(m^2+9m+20)+e(m+3)^2)}{c^4(m+2)(m+3)(m+4)(m+5)}+\frac{d^2}{(m+1)^2}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{f\sqrt{c^2x^2}\sqrt{c^2x^2-1}} + \frac{bex\sqrt{c^2x^2-1}(fx)^{m+1}\left(2c^2d(m^2+9m+20)+e(m+3)^2\right)}{c^3f(m+2)(m+3)(m+4)(m+5)\sqrt{c^2x^2}}$$

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2))*x*(f*x)^(1 + m)*Sqrt[-1 + c^2
*x^2])/(c^3*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*Sqrt[c^2*x^2]) + (b*e^2*x*(f*
x)^(3 + m)*Sqrt[-1 + c^2*x^2])/(c*f^3*(4 + m)*(5 + m)*Sqrt[c^2*x^2]) + (d^2
*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a +
 b*ArcCsc[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCsc[c*x]))/(f
^5*(5 + m)) + (b*c*(d^2/(1 + m)^2 + (e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m
^2)))/(c^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)))*x*(f*x)^(1 + m)*Sqrt[1 - c^2*x
^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*Sqrt[c^2*x^2]
*Sqrt[-1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}+\frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)}\\&\quad+\frac{e^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)}\\&\quad+\frac{(bcx)\int\frac{(fx)^m\left(d^2(15+8m+m^2)+2de(5+6m+m^2)x^2+e^2(3+4m+m^2)x^4\right)}{(1+m)(3+m)(5+m)\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}\end{aligned}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)} + \frac{(bcx)\int \frac{(fx)^m\left(d^2(15+8m+m^2)+2de(5+6m+m^2)x^2+e^2(3+4m+m^2)x^4\right)}{\sqrt{-1+c^2x^2}}\,dx}{(15+23m+9m^2+m^3)\sqrt{c^2x^2}}$$

$$= \frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)} + \frac{(bx)\int \frac{(fx)^m\left(c^2d^2(3+m)(4+m)(5+m)+e(1+m)\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)x^2\right)}{\sqrt{-1+c^2x^2}}\,dx}{c(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{c^2x^2}}$$

$$= \frac{be\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} + \frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)} + -\frac{\left(b\left(-c^4d^2(2+m)(3+m)(4+m)(5+m)-e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)\right)x\right)\int \cdots}{c^3(2+m)(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{c^2x^2}}$$

$$= \frac{be\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} + \frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)} + -\frac{\left(b\left(-c^4d^2(2+m)(3+m)(4+m)(5+m)-e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)\right)x\sqrt{1 \cdots}\right.}{c^3(2+m)(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

$$= \frac{be\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} + \frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)} + \frac{b\left(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)\right)x(fx)^{1+\cdots}}{c^3f(1+m)(2+m)(4+m)\left(15+23m+9m^2+m^3\right)\sqrt{c^2x\cdots}}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.79

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right) \, dx$$
$$= x(fx)^m \left(\frac{ad^2}{1+m}+\frac{2adex^2}{3+m}+\frac{ae^2x^4}{5+m}+\frac{bd^2\csc^{-1}(cx)}{1+m}+\frac{2bdex^2\csc^{-1}(cx)}{3+m} + \frac{be^2x^4\csc^{-1}(cx)}{5+m}-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}x \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}} - \frac{2bcde\sqrt{1-\frac{1}{c^2x^2}}x^3 \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2\sqrt{1-c^2x^2}} - \frac{bce^2\sqrt{1-\frac{1}{c^2x^2}}x^5 \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2\sqrt{1-c^2x^2}}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] x*(f*x)^m*((a*d^2)/(1 + m) + (2*a*d*e*x^2)/(3 + m) + (a*e^2*x^4)/(5 + m) +
(b*d^2*ArcCsc[c*x])/(1 + m) + (2*b*d*e*x^2*ArcCsc[c*x])/(3 + m) + (b*e^2*x^
4*ArcCsc[c*x])/(5 + m) - (b*c*d^2*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1
[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) - (2*b*
c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2
, c^2*x^2])/((3 + m)^2*Sqrt[1 - c^2*x^2]) - (b*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*
x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^2*Sqrt[
1 - c^2*x^2]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^2 \left(a+b\,\mathrm{arccsc}\,(cx)\right) dx$$

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)
```

Fricas [F]

$$\int (fx)^m (d+ex^2)^2 (a+b\csc^{-1}(cx)) \, dx = \int (ex^2+d)^2 (b \operatorname{arccsc}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccsc(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m (d+ex^2)^2 (a+b\csc^{-1}(cx)) \, dx = \int (fx)^m (a+b \operatorname{acsc}(cx)) (d+ex^2)^2 \, dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsc(c*x))*(d + e*x**2)**2, x)
```

Maxima [F]

$$\int (fx)^m (d+ex^2)^2 (a+b\csc^{-1}(cx)) \, dx = \int (ex^2+d)^2 (b \operatorname{arccsc}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d
^2/(f*(m + 1)) + (((b*e^2*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
 4*b*e^2*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*e^2*f^m*arctan
2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^5 + 2*(b*d*e*f^m*m^2*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1)) + 6*b*d*e*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1)) + 5*b*d*e*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^3 + (b*d^2*f^m
*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*b*d^2*f^m*m*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1)) + 15*b*d^2*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1)))*x)*x^m + (m^3 + 9*m^2 + 23*m + 15)*integrate(-(b*d^2*f^m*m^2 + 8*b*d
^2*f^m*m + (b*e^2*f^m*m^2 + 4*b*e^2*f^m*m + 3*b*e^2*f^m)*x^4 + 15*b*d^2*f^m
 + 2*(b*d*e*f^m*m^2 + 6*b*d*e*f^m*m + 5*b*d*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(
c*x - 1)*x^m/(m^3 - (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c^2)*x^2 + 9*m^2 +
 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^2 (b\,\mathrm{arccsc}\,(cx)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \int (f\,x)^m \left(e\,x^2+d\right)^2 \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

```
[In] int((f*x)^m*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

3.167 $\int (fx)^m (d+ex^2) (a+b\csc^{-1}(cx)) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 215

$$\int (fx)^m (d+ex^2) (a+b\csc^{-1}(cx)) \, dx = \frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf(6+5m+m^2)\sqrt{c^2x^2}} + \frac{d(fx)^{1+m}(a+b\csc^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m}(a+b\csc^{-1}(cx))}{f^3(3+m)} + \frac{b(e(1+m)^2+c^2d(2+m)(3+m))\,x(fx)^{1+m}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{cf(1+m)^2(2+m)(3+m)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

```
[Out] d*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccsc(c*x))/f^3
/(3+m)+b*(e*(1+m)^2+c^2*d*(2+m)*(3+m))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/
2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c/f/(1+m)^2/(2+m)/(3+m)/(c^2*x
^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c/f/(m^2+5*
m+6)/(c^2*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {14, 5347, 12, 470, 372, 371}

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\csc^{-1}(cx)\right) \, dx = \frac{d(fx)^{m+1} \left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3} \left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)} +\frac{bcx\sqrt{1-c^2x^2}(fx)^{m+1} \left(\frac{e}{c^2(m+2)(m+3)}+\frac{d}{(m+1)^2}\right) \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{f\sqrt{c^2x^2}\sqrt{c^2x^2-1}} +\frac{bex\sqrt{c^2x^2-1}(fx)^{m+1}}{cf\left(m^2+5m+6\right)\sqrt{c^2x^2}}$$

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*x*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c*f*(6 + 5*m + m^2)*Sqrt[c^2*x^2]
) + (d*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a
 + b*ArcCsc[c*x]))/(f^3*(3 + m)) + (b*c*(d/(1 + m)^2 + e/(c^2*(2 + m)*(3 +
m)))*x*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3
 + m)/2, c^2*x^2])/(f*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad+\frac{(bcx)\int\frac{(fx)^m\left(d(3+m)+e(1+m)x^2\right)}{(1+m)(3+m)\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)}+\frac{(bcx)\int\frac{(fx)^m\left(d(3+m)+e(1+m)x^2\right)}{\sqrt{-1+c^2x^2}}\,dx}{(3+4m+m^2)\sqrt{c^2x^2}} \\
&= \frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{c^2x^2}}+\frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} \\
&\quad+\frac{e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)}-\frac{\left(bc\left(-\frac{e(1+m)^2}{c^2(2+m)}-d(3+m)\right)x\right)\int\frac{(fx)^m}{\sqrt{-1+c^2x^2}}\,dx}{(3+4m+m^2)\sqrt{c^2x^2}} \\
&= \frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{c^2x^2}}+\frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} \\
&\quad+\frac{e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} \\
&\quad-\frac{\left(bc\left(-\frac{e(1+m)^2}{c^2(2+m)}-d(3+m)\right)x\sqrt{1-c^2x^2}\right)\int\frac{(fx)^m}{\sqrt{1-c^2x^2}}\,dx}{(3+4m+m^2)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}
\end{aligned}$$

$$=\frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{c^2x^2}}+\frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)}$$
$$+\frac{bc\left(\frac{e(1+m)^2}{c^2(2+m)}+d(3+m)\right)x(fx)^{1+m}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{f(1+m)\left(3+4m+m^2\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.80

$$\int (fx)^m\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=x(fx)^m\left(-\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}}\right.$$
$$\left.+\frac{\frac{(3+m)\left(d(3+m)+e(1+m)x^2\right)\left(a+b\csc^{-1}(cx)\right)}{1+m}-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}\,x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{\sqrt{1-c^2x^2}}}{(3+m)^2}\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] x*(f*x)^m*(-((b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2])) + (((3 + m)*(d*(3 +
m) + e*(1 + m)*x^2)*(a + b*ArcCsc[c*x]))/(1 + m) - (b*c*e*Sqrt[1 - 1/(c^2*x
^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/Sqrt[1 - c^
2*x^2])/(3 + m)^2)
```

Maple [F]

$$\int (fx)^m\left(e\,x^2+d\right)\left(a+b\,\text{arccsc}\,(cx)\right)dx$$

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

Fricas [F]

$$\int (fx)^m (d+ex^2) (a+b\csc^{-1}(cx)) \, dx = \int (ex^2+d)(b \operatorname{arccsc}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m (d+ex^2) (a+b\csc^{-1}(cx)) \, dx = \int (fx)^m (a+b \operatorname{acsc}(cx)) (d+ex^2) \, dx$$

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*acsc(c*x)),x)
[Out] Integral((f*x)**m*(a + b*acsc(c*x))*(d + e*x**2), x)
```

Maxima [F]

$$\int (fx)^m (d+ex^2) (a+b\csc^{-1}(cx)) \, dx = \int (ex^2+d)(b \operatorname{arccsc}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) + (((b*e*f^m*m*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + b*e*f^m*arctan2(1, sqrt(c*x + 1)*sqrt
(c*x - 1)))*x^3 + (b*d*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*
d*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x)*x^m + (m^2 + 4*m + 3)*int
egrate((b*d*f^m*m + 3*b*d*f^m + (b*e*f^m*m + b*e*f^m)*x^2)*sqrt(c*x + 1)*sq
rt(c*x - 1)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2 - m^2 - 4*m - 3), x))/(m^2
 + 4*m + 3)
```

Giac [F]

$$\int (fx)^m (d+ex^2) (a+b\csc^{-1}(cx)) \, dx = \int (ex^2+d)(b \operatorname{arccsc}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d + ex^2) (a + b\csc^{-1}(cx)) \, dx = \int (f\,x)^m \left(e\,x^2 + d\right) \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.168 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \operatorname{Int}\left(\frac{(fx)^m\,(a+b\csc^{-1}(cx))}{d+ex^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{d + ex^2}\,dx = \int \frac{(fx)^m (a + b\csc^{-1}(cx))}{d + ex^2}\,dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 4.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\,\mathrm{arccsc}\,(cx))}{e\,x^2 + d}dx$$

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{d + ex^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)(fx)^m}{ex^2 + d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 35.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{d + ex^2}\,dx = \int \frac{(fx)^m (a + b\,\mathrm{acsc}\,(cx))}{d + ex^2}\,dx$$

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d),x)
[Out] Integral((f*x)**m*(a + b*acsc(c*x))/(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{d + ex^2} dx = \int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{ex^2 + d} dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{d + ex^2} dx = \int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{ex^2 + d} dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{d + ex^2} dx = \int \frac{(f\,x)^m \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2 + d} dx$$

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

3.169 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^2}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^2}\,dx=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2} dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b \operatorname{arccsc}(cx))}{(e\,x^2 + d)^2} dx$$

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2} dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)(fx)^m}{(ex^2 + d)^2} \, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)(fx)^m}{(ex^2 + d)^2} \, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{(f\,x)^m \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^2} \, dx$$

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.170 $\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \text{Int}\left((fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$$

```
[In] Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] Defer[Int][(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\text{integral} = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\, dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\, dx$$

```
[In] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.73 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}\left(cx\right)\right) dx$$

```
[In] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\,\mathrm{arccsc}\left(cx\right)+a)(fx)^m\, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)*(f*x
)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int (ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int (ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int (f\,x)^m \left(e\,x^2+d\right)^{3/2} \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \, dx$$

```
[In] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

3.171 $\int (fx)^m \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx = \mathrm{Int}\Big((fx)^m \sqrt{d+ex^2}(a+b\csc^{-1}(cx)),x\Big)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx = \int (fx)^m \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$$

```
[In] Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\text{integral} = \int (fx)^m \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

```
[In] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m\left(a+b\,\mathrm{arccsc}\left(cx\right)\right)\sqrt{e\,x^2+d}dx$$

```
[In] int((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
[Out] int((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arccsc}\left(cx\right)+a)(fx)^m\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 51.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int (fx)^m\left(a+b\,\mathrm{acsc}\left(cx\right)\right)\sqrt{d+ex^2}\,dx$$

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b \csc^{-1}(cx)\right) \, dx = \int \sqrt{ex^2+d}(b \operatorname{arccsc}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b \csc^{-1}(cx)\right) \, dx = \int \sqrt{ex^2+d}(b \operatorname{arccsc}(cx)+a)(fx)^m \, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b \csc^{-1}(cx)\right) \, dx = \int (f\,x)^m\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \,dx$$

```
[In] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.172 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\mathrm{Int}\left(\frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(fx)^m (a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}} \, dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m (a + b \operatorname{arccsc}(cx))}{\sqrt{e\,x^2 + d}} dx$$

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}} \, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 21.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(fx)^m (a + b \operatorname{acsc}(cx))}{\sqrt{d + ex^2}} \, dx$$

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}} \, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}} \, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(f\,x)^m \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}} \, dx$$

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.173 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\mathrm{Int}\left(\frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

```
[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m (a + b \csc^{-1}(cx))}{(d + ex^2)^{3/2}} \, dx = \int \frac{(fx)^m (a + b \csc^{-1}(cx))}{(d + ex^2)^{3/2}} \, dx$$

```
[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m (a + b \operatorname{arccsc}(cx))}{(e\,x^2 + d)^{\frac{3}{2}}} dx$$

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(fx)^m (a + b \csc^{-1}(cx))}{(d + ex^2)^{3/2}} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 +
 d^2), x)
```

Sympy [N/A]

Not integrable

Time = 134.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(fx)^m\left(a+b\operatorname{acsc}\left(cx\right)\right)}{\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
[Out] Integral((f*x)**m*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(b\operatorname{arccsc}\left(cx\right)+a)(fx)^m}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(b\operatorname{arccsc}\left(cx\right)+a)(fx)^m}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.174 $\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 401

$$\begin{aligned}\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = &-\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{7b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}\\&-\frac{13b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{3b\sqrt{1-c^2x^2}(1+c^2x^2)^{7/2}}{70c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}\\&-\frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{9/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}}\\&+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}}\\&-\frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}}\\&+\frac{4b\sqrt{1-c^2x^2}\,\mathrm{arctanh}\left(\sqrt{1+c^2x^2}\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}\end{aligned}$$

```
[Out] 1/3*(-c^4*x^4+1)^(3/2)*(a+b*arccsc(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+b*
arccsc(c*x))/c^12+7/90*b*(c^2*x^2+1)^(3/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c
^2/x^2)^(1/2)-13/150*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2
/x^2)^(1/2)+3/70*b*(c^2*x^2+1)^(7/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2
)^(1/2)-1/90*b*(c^2*x^2+1)^(9/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2)^(1
/2)+4/15*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^
2)^(1/2)-4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2)^(
1/2)-1/2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {272, 45, 5355, 12, 6853, 6874, 862, 52, 65, 214, 797}

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{(1-c^4x^4)^{5/2}\,(a+b\csc^{-1}(cx))}{10c^{12}} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{4b\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{c^2x^2+1}\right)}{15c^{13}x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{b\sqrt{1-c^2x^2}(c^2x^2+1)^{9/2}}{90c^{13}x\sqrt{1-\frac{1}{c^2x^2}}} + \frac{3b\sqrt{1-c^2x^2}(c^2x^2+1)^{7/2}}{70c^{13}x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{13b\sqrt{1-c^2x^2}(c^2x^2+1)^{5/2}}{150c^{13}x\sqrt{1-\frac{1}{c^2x^2}}} + \frac{7b\sqrt{1-c^2x^2}(c^2x^2+1)^{3/2}}{90c^{13}x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{4b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{15c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}$$

```
[In] Int[(x^11*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[1 - 1/(c^2*x^2)]*x
) + (7*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(90*c^13*Sqrt[1 - 1/(c^2*x^
2)]*x) - (13*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(150*c^13*Sqrt[1 - 1/
(c^2*x^2)]*x) + (3*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(7/2))/(70*c^13*Sqrt[1
 - 1/(c^2*x^2)]*x) - (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(9/2))/(90*c^13*Sqr
t[1 - 1/(c^2*x^2)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/(2*c^12) +
((1 - c^4*x^4)^(3/2)*(a + b*ArcCsc[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)*(
a + b*ArcCsc[c*x]))/(10*c^12) + (4*b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2
*x^2]])/(15*c^13*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}} \\
&\quad -\frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}}+\frac{b\int \frac{\sqrt{1-c^4x^4}(-8-4c^4x^4-3c^8x^8)}{30c^{12}\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{c} \\
&= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}} \\
&\quad -\frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}}+\frac{b\int \frac{\sqrt{1-c^4x^4}(-8-4c^4x^4-3c^8x^8)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{30c^{13}} \\
&= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}} \\
&\quad -\frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}}+\frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}(-8-4c^4x^4-3c^8x^8)}{x\sqrt{1-c^2x^2}}\,dx}{30c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}} \\
&\quad -\frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}} \\
&\quad -\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}(8+4c^4x^2+3c^8x^4)}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{60c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} \\
&= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}} \\
&\quad -\frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}} \\
&\quad -\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{8\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}+\frac{4c^4x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}+\frac{3c^8x^3\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\right)dx,x,x^2\right)}{60c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}} - \frac{\left(2b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\,dx,x,x^2\right)}{15c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{x^3\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\,dx,x,x^2\right)}{20c^5\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}} - \frac{\left(2b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x}\,dx,x,x^2\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int x\sqrt{1+c^2x}\,dx,x,x^2\right)}{15c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int x^3\sqrt{1+c^2x}\,dx,x,x^2\right)}{20c^5\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}} - \frac{\left(2b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{\sqrt{1+c^2x}}{c^2}+\frac{(1+c^2x)^{3/2}}{c^2}\right)dx,x,x^2\right)}{15c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{\sqrt{1+c^2x}}{c^6}+\frac{3(1+c^2x)^{3/2}}{c^6}-\frac{3(1+c^2x)^{5/2}}{c^6}+\frac{(1+c^2x)^{7/2}}{c^6}\right)dx,x,x^2\right)}{20c^5\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{7b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{13b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{3b\sqrt{1-c^2x^2}(1+c^2x^2)^{7/2}}{70c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{9/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}} - \frac{\left(4b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{15c^{15}\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{7b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{13b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{3b\sqrt{1-c^2x^2}(1+c^2x^2)^{7/2}}{70c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{9/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}} + \frac{4b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.48

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{105a\sqrt{1-c^4x^4}(8+4c^4x^4+3c^8x^8) + \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(768+36c^2x^2+78c^4x^4+5c^6x^6+35c^8x^8)}{-1+c^2x^2} + 105b\sqrt{1-c^4x^4}(8}{3150c^{12}}$$

```
[In] Integrate[(x^11*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/3150*(105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*Sqrt[1
- 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6*x
^6 + 35*c^8*x^8))/(-1 + c^2*x^2) + 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 +
 3*c^8*x^8)*ArcCsc[c*x] + 840*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]*x)/Sqrt[1 -
 c^4*x^4]])/c^12
```

Maple [F]

$$\int \frac{x^{11}(a+b\,\mathrm{arccsc}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

[In] int(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)

[Out] int(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.59

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx =$$

$$-\frac{(35\,bc^8x^8+5\,bc^6x^6+78\,bc^4x^4+36\,bc^2x^2+768\,b)\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}-840\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c}}{\sqrt{c^2}}\right.}{}$$

[In] integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")

[Out] -1/3150*((35*b*c^8*x^8 + 5*b*c^6*x^6 + 78*b*c^4*x^4 + 36*b*c^2*x^2 + 768*b)*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1) - 840*(b*c^2*x^2 - b)*arctan(sqrt(-c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) + 105*(3*a*c^10*x^10 - 3*a*c^8*x^8 + 4*a*c^6*x^6 - 4*a*c^4*x^4 + 8*a*c^2*x^2 + (3*b*c^10*x^10 - 3*b*c^8*x^8 + 4*b*c^6*x^6 - 4*b*c^4*x^4 + 8*b*c^2*x^2 - 8*b)*arccsc(c*x) - 8*a)*sqrt(-c^4*x^4 + 1))/(c^14*x^2 - c^12)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Timed out}$$

[In] integrate(x**11*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^{11}}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*sq
rt(-c^4*x^4 + 1)/c^12) + 1/30*(30*c^12*integrate(1/30*(3*c^10*x^11 + 3*c^8*
x^9 + 4*c^6*x^7 + 4*c^4*x^5 + 8*c^2*x^3 + 8*x)*e^(-1/2*log(c^2*x^2 + 1) + 1
/2*log(c*x - 1))/(c^10*e^(log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1))
+ sqrt(-c*x + 1)*c^10), x) - (3*c^8*x^8*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
 1)) + 4*c^4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1)))*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1))*
b/c^12
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^{11}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((x^11*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((x^11*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.175 $\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$\begin{aligned}\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = &-\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}x}\\ &-\frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8}\\ &+\frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{6c^8}\\ &+\frac{b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x}\end{aligned}$$

```
[Out] 1/6*(-c^4*x^4+1)^(3/2)*(a+b*arccsc(c*x))/c^8+1/18*b*(c^2*x^2+1)^(3/2)*(-c^2
*x^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/30*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+
1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)+1/3*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x
^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^
(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/2)/c^
8
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {272, 45, 5355, 12, 6853, 6874, 862, 52, 65, 214, 797}

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{6c^8} - \frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8} + \frac{b\sqrt{1-c^2x^2}\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{3c^9x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{b\sqrt{1-c^2x^2}(c^2x^2+1)^{5/2}}{30c^9x\sqrt{1-\frac{1}{c^2x^2}}} + \frac{b\sqrt{1-c^2x^2}(c^2x^2+1)^{3/2}}{18c^9x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{3c^9x\sqrt{1-\frac{1}{c^2x^2}}}$$

```
[In] Int[(x^7*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/3*(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
+ (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(18*c^9*Sqrt[1 - 1/(c^2*x^2)]*x
) - (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(30*c^9*Sqrt[1 - 1/(c^2*x^2)]
*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/2
)*(a + b*ArcCsc[c*x]))/(6*c^8) + (b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*
x^2]])/(3*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} + \frac{b\int \frac{\left(-2-c^4x^4\right)\sqrt{1-c^4x^4}}{6c^8\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{c}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} + \frac{b\int \frac{\left(-2-c^4x^4\right)\sqrt{1-c^4x^4}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{6c^9}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\left(-2-c^4x^4\right)\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}\,dx}{6c^9\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}\left(2+c^4x^2\right)}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{12c^9\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{2\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}} + \frac{c^4x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\right)dx,x,x^2\right)}{12c^9\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{6c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{x\sqrt{1-c^4x^2}}{\sqrt{1-c^2x}}\,dx,x,x^2\right)}{12c^5\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x}\,dx,x,x^2\right)}{6c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int x\sqrt{1+c^2x}\,dx,x,x^2\right)}{12c^5\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{6c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{\sqrt{1+c^2x}}{c^2}+\frac{(1+c^2x)^{3/2}}{c^2}\right)dx,x,x^2\right)}{12c^5\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx,x,\sqrt{1+c^2x^2}\right)}{3c^{11}\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x} + \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8} + \frac{b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.59

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{15a\sqrt{1-c^4x^4}(2+c^4x^4)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(28+c^2x^2+3c^4x^4)}{-1+c^2x^2}+15b\sqrt{1-c^4x^4}(2+c^4x^4)\csc^{-1}(cx)+30b\,\mathrm{arc}}{90c^8}$$

```
[In] Integrate[(x^7*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/90*(15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*
Sqrt[1 - c^4*x^4]*(28 + c^2*x^2 + 3*c^4*x^4))/(-1 + c^2*x^2) + 15*b*Sqrt[1
- c^4*x^4]*(2 + c^4*x^4)*ArcCsc[c*x] + 30*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]
*x)/Sqrt[1 - c^4*x^4]])/c^8
```

Maple [F]

$$\int \frac{x^7(a+b\,\mathrm{arccsc}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

```
[In] int(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.68

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{(3\,bc^4x^4+bc^2x^2+28\,b)\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}-30\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}}{\sqrt{c^2x^2-1}}\right)+15\,(ac^6x^6-ac^4x^4+}{90\,(c^{10}x^2-c^8)}$$

```
[In] integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/90*((3*b*c^4*x^4 + b*c^2*x^2 + 28*b)*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1
) - 30*(b*c^2*x^2 - b)*arctan(sqrt(-c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) + 15*(a
*c^6*x^6 - a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^6*x^6 - b*c^4*x^4 + 2*b*c^2*x^2 -
 2*b)*arccsc(c*x) - 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 - c^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x**7*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^7}{\sqrt{-c^4x^4+1}}\,dx$$

```
[In] integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*(c^8*x^8*
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 6*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)
*sqrt(-c*x + 1)*c^8*integrate(1/6*(c^6*x^7 + c^4*x^5 + 2*c^2*x^3 + 2*x)*e^(
-1/2*log(c^2*x^2 + 1) + 1/2*log(c*x - 1))/(c^6*e^(log(c*x + 1) + log(c*x -
1) + 1/2*log(-c*x + 1)) + sqrt(-c*x + 1)*c^6), x) + c^4*x^4*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1)) - 2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(sq
rt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=\int \frac{x^7\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((x^7*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^7*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.176 $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 126

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}} - \frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4} + \frac{bx\arctan\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{-1+c^2x^2}}\right)}{2c^3\sqrt{c^2x^2}}$$

```
[Out] 1/2*b*x*arctan((-c^4*x^4+1)^(1/2)/(c^2*x^2-1)^(1/2))/c^3/(c^2*x^2)^(1/2)-1/
2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/2)/c^4-1/2*b*x*(-c^4*x^4+1)^(1/2)/c^3/(
c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {267, 5355, 12, 1586, 1266, 862, 52, 65, 214}

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4} + \frac{b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{c^2x^2+1}\right)}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}$$

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/2*(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(c^5*Sqrt[1 - 1/(c^2*x^2)]*x)
- (Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/(2*c^4) + (b*Sqrt[1 - c^2*x^2]*Ar
cTanh[Sqrt[1 + c^2*x^2]])/(2*c^5*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1586

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Dist[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(
x^mn*e)))^FracPart[q]))/x^(mn*FracPart[q]), Int[x^(m + mn*q)*(1 + d*(1/(x^m
n*e)))^q*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] && E
qQ[n2, -2*mn] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n2]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4}+\frac{b\int -\frac{\sqrt{1-c^4x^4}}{2c^4\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{c}\\
&= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4}-\frac{b\int \frac{\sqrt{1-c^4x^4}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}\,dx}{2c^5}\\
&= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}\,dx}{2c^5\sqrt{1-\frac{1}{c^2x^2}}x}\\
&= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{4c^5\sqrt{1-\frac{1}{c^2x^2}}x}\\
&= -\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4}-\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x}\,dx,x,x^2\right)}{4c^5\sqrt{1-\frac{1}{c^2x^2}}x}\\
&= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4}\\
&\quad -\frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+c^2x}}\,dx,x,x^2\right)}{4c^5\sqrt{1-\frac{1}{c^2x^2}}x}
\end{aligned}
$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4} - \frac{\left(b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{1+c^2x^2}\right)}{2c^7\sqrt{1-\frac{1}{c^2x^2}}x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4} + \frac{b\sqrt{1-c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{2c^5\sqrt{1-\frac{1}{c^2x^2}}x}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$
$$= \frac{\left(a-bc\sqrt{1-\frac{1}{c^2x^2}}x-ac^2x^2\right)\sqrt{1-c^4x^4} - b(-1+c^2x^2)\sqrt{1-c^4x^4}\csc^{-1}(cx) + (b-bc^2x^2)\arctan\left(\frac{c\sqrt{1-\frac{1}{c^2}}}{\sqrt{1-c^4}}\right.}{2c^4(-1+c^2x^2)}$$

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
[Out] ((a - b*c*Sqrt[1 - 1/(c^2*x^2)]*x - a*c^2*x^2)*Sqrt[1 - c^4*x^4] - b*(-1 +
c^2*x^2)*Sqrt[1 - c^4*x^4]*ArcCsc[c*x] + (b - b*c^2*x^2)*ArcTan[(c*Sqrt[1 -
 1/(c^2*x^2)]*x)/Sqrt[1 - c^4*x^4]])/(2*c^4*(-1 + c^2*x^2))
```

Maple [F]

$$\int \frac{x^3(a+b\,\text{arccsc}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

```
[In] int(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
[Out] int(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx =$$
$$-\frac{\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}b - (bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}}{\sqrt{c^2x^2-1}}\right) + \sqrt{-c^4x^4+1}(ac^2x^2+(bc^2x^2-b)\,\text{arccsc}\,(cx)}{2(c^6x^2-c^4)}$$

[In] integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")

[Out] -1/2*(sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1)*b - (b*c^2*x^2 - b)*arctan(sqrt(-c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) + sqrt(-c^4*x^4 + 1)*(a*c^2*x^2 + (b*c^2*x^2 - b)*arccsc(c*x) - a))/(c^6*x^2 - c^4)

Sympy [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^3(a+b\,\mathrm{acsc}\,(cx))}{\sqrt{-(cx-1)\,(cx+1)\,(c^2x^2+1)}}\,dx$$

[In] integrate(x**3*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)

[Out] Integral(x**3*(a + b*acsc(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)), x)

Maxima [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

[In] integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")

[Out] 1/2*(2*c^4*integrate(1/2*(c^2*x^3 + x)*e^(-1/2*log(c^2*x^2 + 1) + 1/2*log(c*x - 1))/(c^2*e^(log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + sqrt(-c*x + 1)*c^2), x) - sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/c^4 - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4

Giac [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

[In] integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((b*arccsc(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.177 $\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\,\mathrm{arccsc}\,(cx)}{x\sqrt{-c^4x^4+1}}dx$$

```
[In] int((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
[Out] int((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsc(c*x) + a)/(c^4*x^5 - x), x)
```

Sympy [N/A]

Not integrable

Time = 11.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{x\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^2x^2+1\right)}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x/(-c**4*x**4+1)**(1/2),x)
[Out] Integral((a + b*acsc(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int \frac{a + b\csc^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*inte
grate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x +
 1)*sqrt(-c*x + 1)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a + b\csc^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a + b\csc^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{1 - c^4\,x^4}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

3.178 $\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

```
[In] Int[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\text{integral} = \int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\ \mathrm{arccsc}\left(cx\right)}{x^5\sqrt{-c^4x^4+1}}dx$$

```
[In] int((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
[Out] int((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsc(c*x) + a)/(c^4*x^9 - x^5), x)
```

Sympy [N/A]

Not integrable

Time = 83.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\mathrm{acsc}\left(cx\right)}{x^5\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^2x^2+1\right)}}\,dx$$

```
[In] integrate((a+b*acsc(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
[Out] Integral((a + b*acsc(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.31

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) + 2
*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^5), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

```
[In] integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

Mupad [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^5\,\sqrt{1-c^4\,x^4}}\,dx$$

```
[In] int((a + b*asin(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                   ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                  end if
              else #result contains complex but optimal is not
                  if debug then
                          print("result contains complex but optimal is not");
                  fi;
                  return "C","Result contains complex when optimal does not.";
              fi;
          else # result do not contain complex
               # this assumes optimal do not as well. No check is needed here.
              if debug then
                       print("result do not contain complex, this assumes optimal do not as well")
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  if debug then
                      print("leaf_count_result<=2*leaf_count_optimal");
                  fi;
                  return "A"," ";
              else
                  if debug then
                      print("leaf_count_result>2*leaf_count_optimal");
                  fi;
                  return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
      print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```